Dynamics of the neutron superfluid

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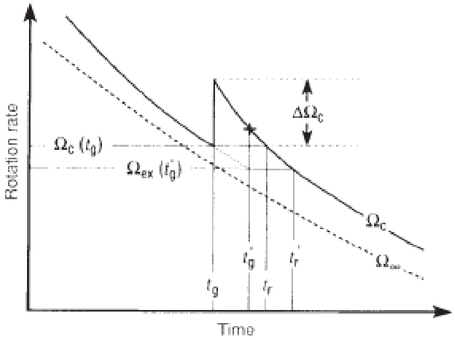
Neutron star glitches



Glitches

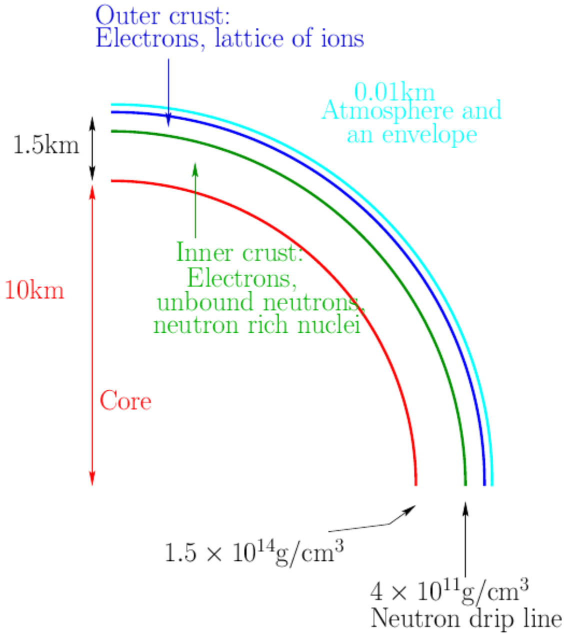
- $\frac{\Delta\Omega}{\Omega} \sim 10^{-11} - 10^{-5}$
- 315 glitches in 105 pulsars *Espinoza et. al. (2011)*
- A wide diversity in glitch behavior
- The bigger glitches are more regular and one hopes that there is a simple dynamical explanation for a class of glitches

Glitches from pinned vortices

- Neutron star glitches can result from transfer of angular momentum from the neutron superfluid to the rigid crust *Anderson and Itoh (1975)*
- Superfluid angular frequency proportional to the vortex density
- For the superfluid to slow down, the vortices need to dilute. Pinning sites may prevent free motion of vortices
- Stress develops till the angular velocity difference reaches a critical point, when a macroscopic number of vortices unpin and dilute, and transfer angular momentum to the crust

Unbound neutrons in the inner crust

Unbound neutrons in the inner crust are superfluid. The nuclei form a lattice.



Neutron Superfluidity

- Neutrons form Cooper pairs and condense, $\Delta \propto \langle \psi\psi \rangle$
- A BCS (Bardeen-Cooper-Schrieffer) or fermionic superfluid
- Interactions between neutrons specified by a scattering length $a \sim -18.6\text{fm}$ and effective range $r_e \sim 2.2\text{fm}$
- To be compared to inter-particle separation $(n)^{-1/3} = (3\pi^2)^{1/3}/k_F$
- At low densities ($k_F \sim 10^{-1}\text{fm}^{-1}$), $|k_F a| \gtrsim 1$, $k_F r_e \lesssim 1$
- Ground state properties calculated using *ab-initio* techniques *Gezerlis, Carlson (2007); Gandolfi, Carlson, Reddy (2011)*

Relation with the unitary Fermi gas

- Unitary Fermi gas, $k_F a \to \infty$, $k_F r_e \to 0$, has a scale symmetry
- $P = c_0 m^{3/2} \mu^{5/2}$ where c_0 is related to the Bertsch parameter ξ, $c_0 = \frac{2^{5/2}}{15\pi^2 \xi^{3/2}}$. Equivalently $\mathcal{E} = \xi \mathcal{E}_{FG}$ where

$$\mathcal{E}_{FG} = \frac{3}{5} n \frac{k_F^2}{2m} = \frac{3}{5} \frac{\hbar^2}{2m} (3\pi^2)^{2/3} (n)^{5/3}$$

- Monte-Carlo simulations used to calculate ξ reliably

Vortex configurations

- $\Delta = |\Delta| e^{i\phi}$
- Classic Onsager vortices in superfluids. The phase ϕ of the condensate winds around by $\kappa = 2\pi$.
- $|\Delta| = 0$ at the core
- Density of the fermions depleted near the core
- The gradient of the phase gives the superfluid velocity, and the velocity field curls around the vortex

The goal

- Solving space-time varying configurations using *ab-initio* techniques is difficult
- Therefore use a simpler model that captures the physics
- I will describe a bosonic, extended Thomas Fermi (ETF) model
- See its applications and limitations

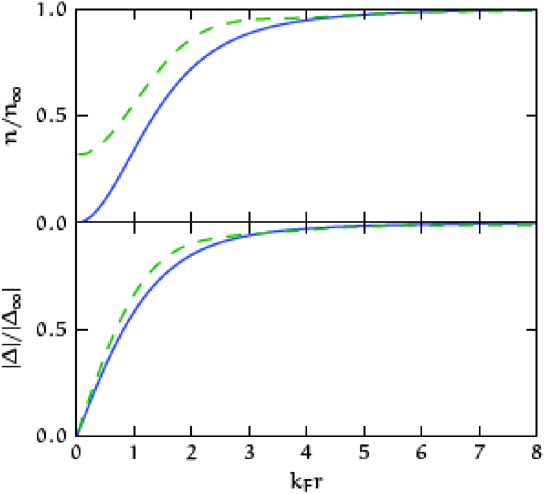
A simpler unitary bosonic theory

- We try a unitary bosonic model (Gross-Pitaevskii like equations) of time evolution
- $\mathcal{L} = \Psi^* i\partial_t \Psi - \left[\Psi^*(-\frac{\hbar^2\nabla^2}{4m_F} + 2(V(r) - \mu))\Psi + \xi\frac{3}{5}\frac{\hbar^2}{2m_F}(3\pi^2)^{2/3}(2\Psi^*\Psi)^{5/3}\right]$
- A Thomas-Fermi model "extended" by kinetic terms
- One can think of Ψ as a field describing a boson composed of Cooper pairs. $\rho_F = 2\Psi^*\Psi$
- Correct $c_s = \sqrt{\xi}3\frac{k_F}{m} = \sqrt{\xi}3v_F$

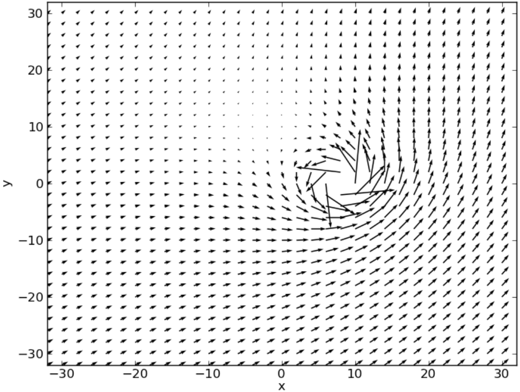
A simpler unitary bosonic theory

- Gives a good description of the collision of two fermionic clouds *(Salasnich, Toigo PRA (2008))*
- Gives a resonable agreement for the energy dependence on particle number with the fermionic theory for large particle numbers in a harmonic trap *(Forbes (2012))*
- The interaction term is fixed by scale invariance: the same as the expression for the fermionic system
- $V(r)$ can be seen as the effective potential for the pairing field *(Broglia et. al, Pizzocherro et. al.)*

Structure of a vortex



Vortex motion



- The well known Magnus force governs the dynamics

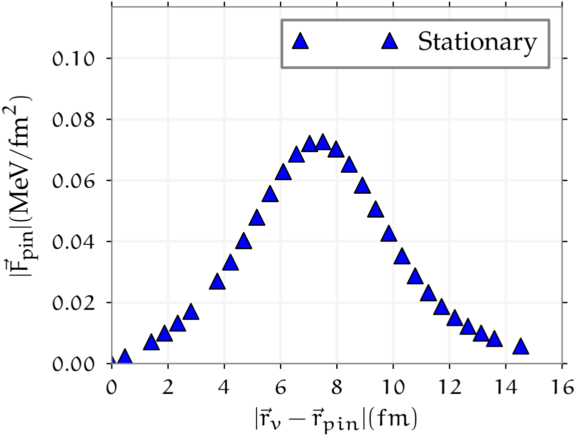
Vortex Motion

- Vortex motion can be described by the equation

$$M\ddot{\vec{r}} - \vec{f}qp = \rho_s \vec{\kappa} \times (\dot{\vec{r}}_v - \dot{\vec{v}}_s) + \vec{F}_v \tag{1}$$

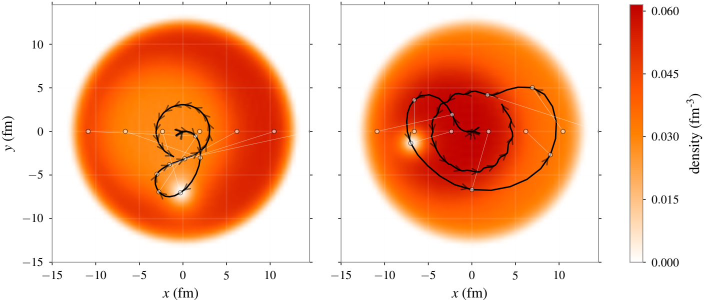
- $\vec{r}_v$ is the position on the vortex, $\vec{F}_v$ that is the key ingredient in the dynamics
- In our case time derivatives are small and the left hand side can be neglected
- Our main result is an efficient method to calculate $\vec{F}_v$
- The usual method is to calculate $F_v = (E_v - E_{\mathrm{nov}})/r_v$. Questions: Should the number of neutrons be kept fixed? Furthermore, it is not clear whether the "force" calculated this way describes vortex dynamics
- If the nucleus-vortex interaction is approximated by a potential, one can do a very well defined calculation by using $\vec{F} = -\langle \nabla V \rangle$

Force

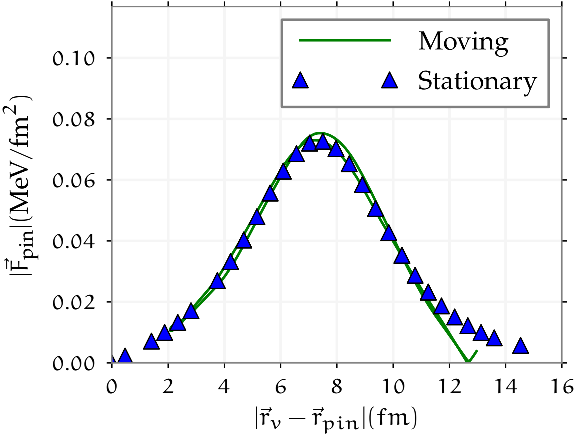


Stationary
0.10
0.08
0.06
0.04
0.02
0.00
$|\vec{F}_{pin}|(\mathrm{MeV/fm^2})$
0
2
4
6
8
10
12
14
16
$|\vec{r}_v - \vec{r}_{pin}|(\mathrm{fm})$

Vortex motion

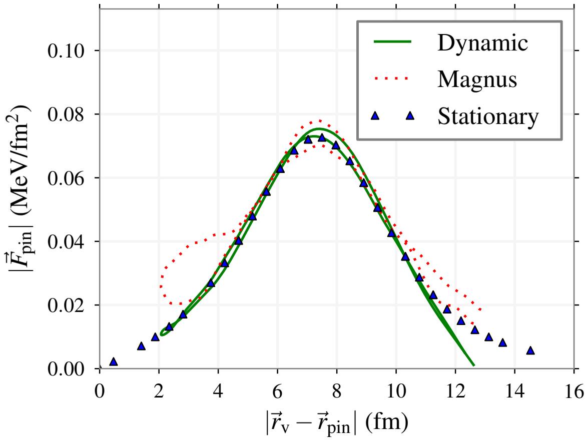


Force



Moving
Stationary
0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
10
12
14
16
$|\vec{F}_{pin}|(MeV/fm^2)$
$|\vec{r}_v - \vec{r}_{pin}|(fm)$

Force



- *Bulgac, Forbes, RS, PRL (2013)*

Comments on previous results

- There are two current sets of results:
- *(Broglia et. al.)* find that there is no pinning — the vortex is repelled from the nucleus. This is surprising because pairing inside the nucleus is weaker than in the bulk superfluid. Perhaps because they compare the energies of system with and without a vortex with the same total particle number
- Compared to *(Pizzocherro et. al.)* our maximum force is a factor of 50 larger at the same density. Partially because of a stronger potential V, partially because we have not yet done the $3-$d simulation, but (I think) mainly because we are not just looking at the end point configuration
- Systematics associated with using V from different models, and behaviour as a function of density not yet analyzed

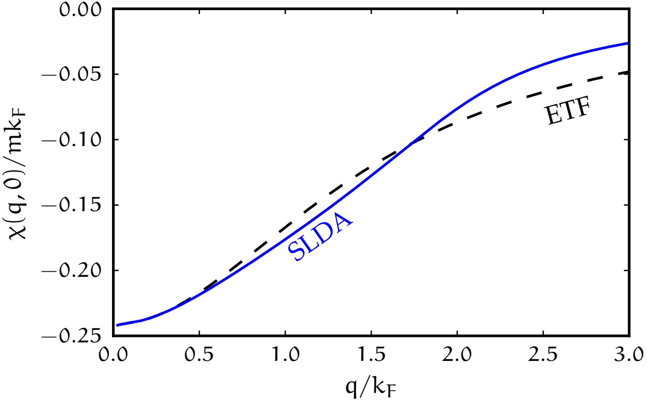
Validating the model

Superfluid Local Density Approximation (SLDA)

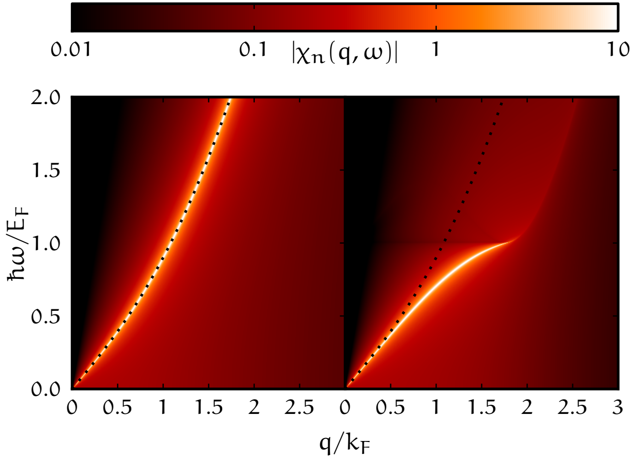
- Kohn-Sham theorem assures that there exists a functional of the density satisfying $d\mathcal{E}/dn = -\mu$
- The form of the functional highly constrained due to conformal symmetry
- $\mathcal{E}[n, \nu] = \alpha \frac{\hbar^2 \tau_r^2}{2m} - \frac{\hbar^2 \gamma}{mn^{1/3}} \nu_r^\dagger \nu_r + \beta \mathcal{E}_{FG}$
- $n = \langle \psi^\dagger \psi \rangle$ and $\nu = \langle \psi \psi \rangle$
- The values of $\alpha = 1$, $\beta = -0.3942$, $\gamma = -13.196$ set to reproduce results of *ab-initio* Monte-Carlo simulations [$\Delta/e_F = 0.502$, $\xi = 0.41$] *Forbes, Bulgac PRL (2008)*
- Larger ξ than the current best calculated and measured values *Forbes, Gandolfi, Gezerlis PRL (2011)*
- The SLDA has been successfully used to study the creation of vortices in unitary Fermi gases *Bulgac et. al. Science (2011)*

Linear response: small fluctuations

- For time independent fluctuations of the potential $V(r) = \delta \cos(qx)$, $\chi(q,0) = \delta n(q)/\delta$



Response of the unitary gas



Low energy constants

- These results can be used to extract the coefficients of the low energy theory. *(Forbes, RS) in preparation*
- *Son, Wingate Ann. Phys. (2002)*
- $\omega(q) = c_s q[1 + \frac{c_\omega}{24\xi}(\frac{q}{k_F})^2]$
- $\chi(q,0) = -\frac{mk_F}{\hbar^2\pi^2\xi}[1 - \frac{c_\chi}{12\xi}(\frac{q}{k_F})^2]$
- For the ETF $c_\omega = c_\chi = 9/4$
- For the SLDA $c_\chi = 7/3$, and is independent of β and γ. $c_\omega \simeq (-0.255, 0.055)$ as we vary ξ from the 0.41 to 0.37

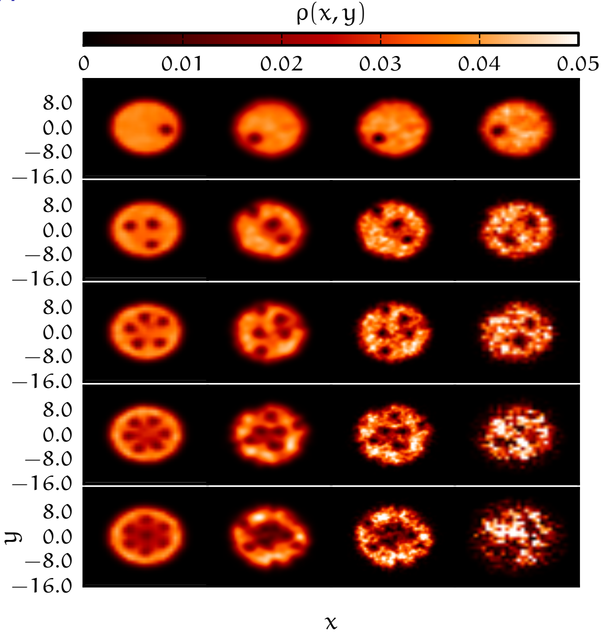
Comparing SLDA evolution with GPE evolution

- $v_{\mathrm{stir}} = 0.1 v_F$
- SLDA:Movie
- *(Bulgac et. al. Science (2011))* and *(http://www.phys.washington.edu/groups/qmbnt/UFG/)*
- GPE:Movie
- *(Forbes, RS)*

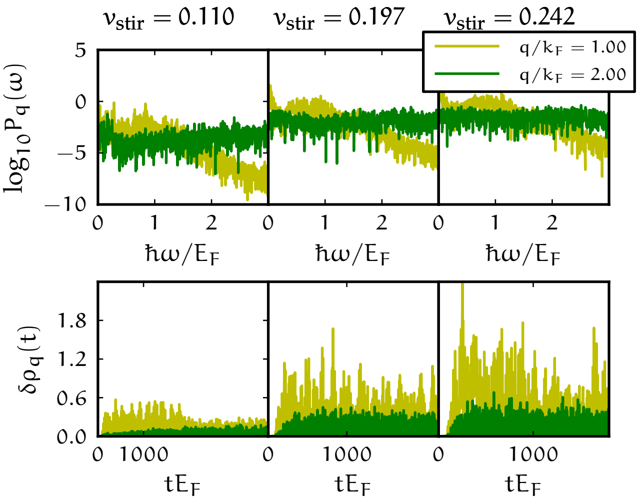
Summary of the comparison

- No fitting parameter. The ξ, the trapping and stirring potentials are precisely the same
- ETF gives roughly the correct of the number of vortices with the stirring velocity
- This is because this is governed essentially by hydrodynamics and long distance physics
- The details of the dynamics are different. The vortex is created later in the ETF
- But the most striking difference is that the ETF is more noisy

Comparing resolutions

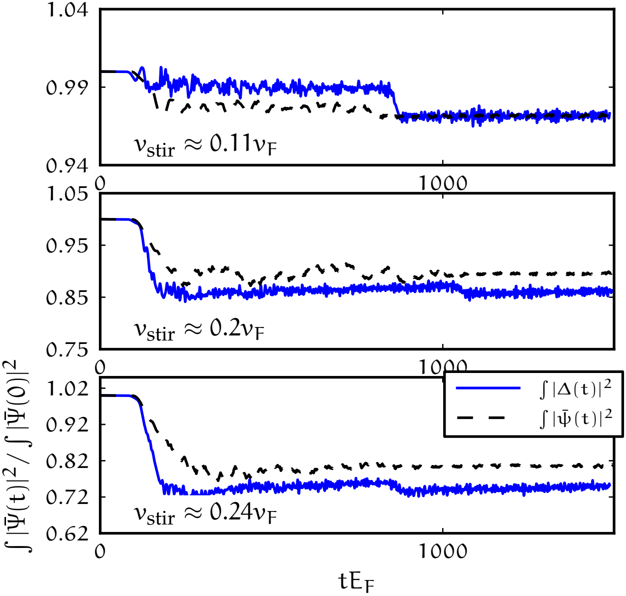


Spectra



$v_{stir} = 0.110$
$v_{stir} = 0.197$
$v_{stir} = 0.242$
$q/k_F = 1.00$
$q/k_F = 2.00$
$\log_{10} P_q(\omega)$
5
0
−5
−10
0
1
2
$\hbar\omega/E_F$
$\delta\rho_q(t)$
1.8
1.2
0.6
0.0
1000
tE_F

Spectra



1.04
0.99
0.94
$v_{stir} \approx 0.11 v_F$
0
1000
1.05
0.95
0.85
0.75
$v_{stir} \approx 0.2 v_F$
0
1000
1.02
0.92
0.82
0.72
0.62
$v_{stir} \approx 0.24 v_F$
0
1000
tE_F
$\int |\bar{\Psi}(t)|^2 / \int |\bar{\Psi}(0)|^2$
$\int |\Delta(t)|^2$
$\int |\bar{\psi}(t)|^2$

More realistic modelling of dynamics?

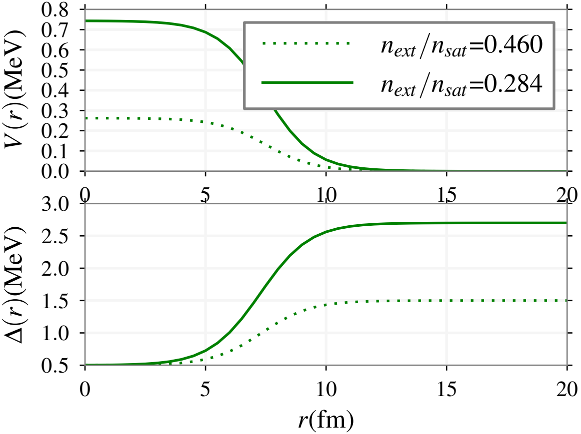
- Perhaps a model with decay or smoothing
- $\frac{1}{k_F a}$ and $k_F r_e$ corrections – easy to include
- The fermionic problem using SLDA – a step up, but has been done for the unitary gas. A nice feature is that V is self-consistently determined
- Realistic nuclear SLDA functionals – easy to include
- Three dimensional simulation
- The combined lattice and superfluid problem – hard even at the density functional level, let alone at the *ab-initio* level. May affect the number of unbound neutrons in the inner crust *(Chamel)*

Summary

- The existence of neutron superfluidity affects dynamical properties of neutron stars and can affect observables
- The unitary Fermi gas is a useful model system of superfluid neutrons in the inner crust
- It would be “cool” if we can definitively show from observations that the inner crusts of neutron stars contain superfluid neutrons, and having a realistic model for the dyanamics will be useful for this purpose

Backup Slides

Potential



$V(r)$(MeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
n_{ext}/n_{sat}=0.460
n_{ext}/n_{sat}=0.284
0
5
10
15
20
$\Delta(r)$(MeV)
3.0
2.5
2.0
1.5
1.0
0.5
0
5
10
15
20
r(fm)

Number of vortices versus v_{stir}

v_{stir}/v_F	SLDA (32^2)	ETF (64^2)	ETF (32^2)
0.1	1	0	0
0.11	-	1	1
0.197	-	3	2
0.2	3	4	3
0.242	-	5	2
0.25	5	6	2
0.3	6	5	(noise)
0.312	-	6	(noise)
0.35	7	7	(noise)
0.40	9	9	(noise)