Non–Abelian discrete R symmetries

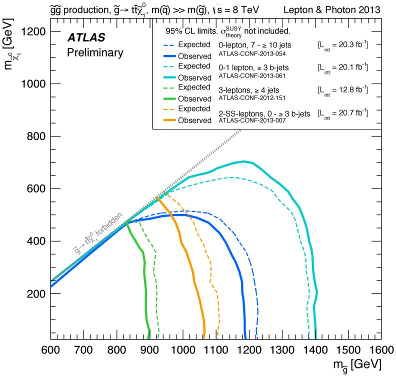
and the flavor puzzle

Particle Theory Seminar, LANL

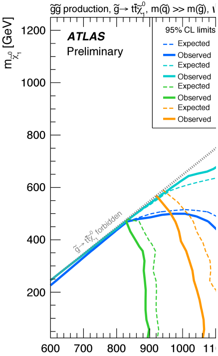
Andreas Trautner

based on arXiv:1306.5112 with: Prof. M.–C. Chen (UCI), Prof. M. Ratz (TUM)

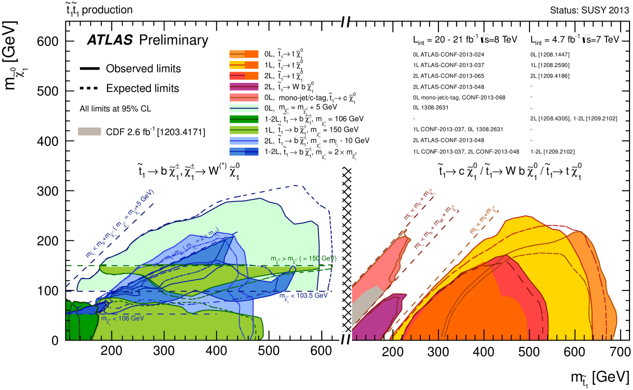
9/30/13



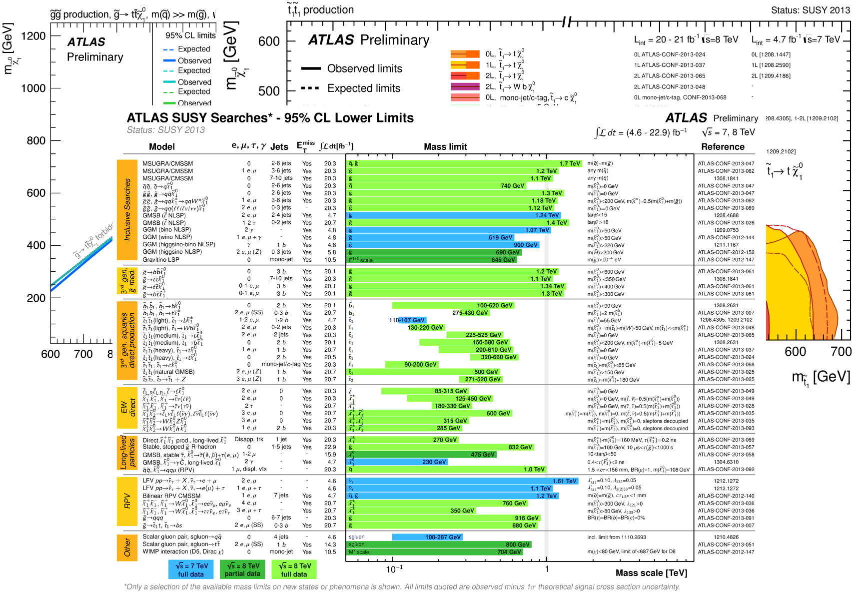
$\tilde{g}\tilde{g}$ production, $\tilde{g}\to t\bar{t}\tilde{\chi}^0_1$, $m(\tilde{q}) >> m(\tilde{g})$, $\sqrt{s}$ = 8 TeV
Lepton & Photon 2013
ATLAS
Preliminary
95% CL limits. σ^{SUSY}_{theory} not included.
Expected
Observed
0-lepton, 7 - ≥ 10 jets
ATLAS-CONF-2013-054
[L_{int} = 20.3 fb^{-1}]
Expected
Observed
0-1 lepton, ≥ 3 b-jets
ATLAS-CONF-2013-061
[L_{int} = 20.1 fb^{-1}]
Expected
Observed
3-leptons, ≥ 4 jets
ATLAS-CONF-2012-151
[L_{int} = 12.8 fb^{-1}]
Expected
Observed
2-SS-leptons, 0 - ≥ 3 b-jets
ATLAS-CONF-2013-007
[L_{int} = 20.7 fb^{-1}]
$\tilde{g}\to t\bar{t}\tilde{\chi}^0_1$ forbidden
$m_{\tilde{\chi}^0_1}$ [GeV]
$m_{\tilde{g}}$ [GeV]



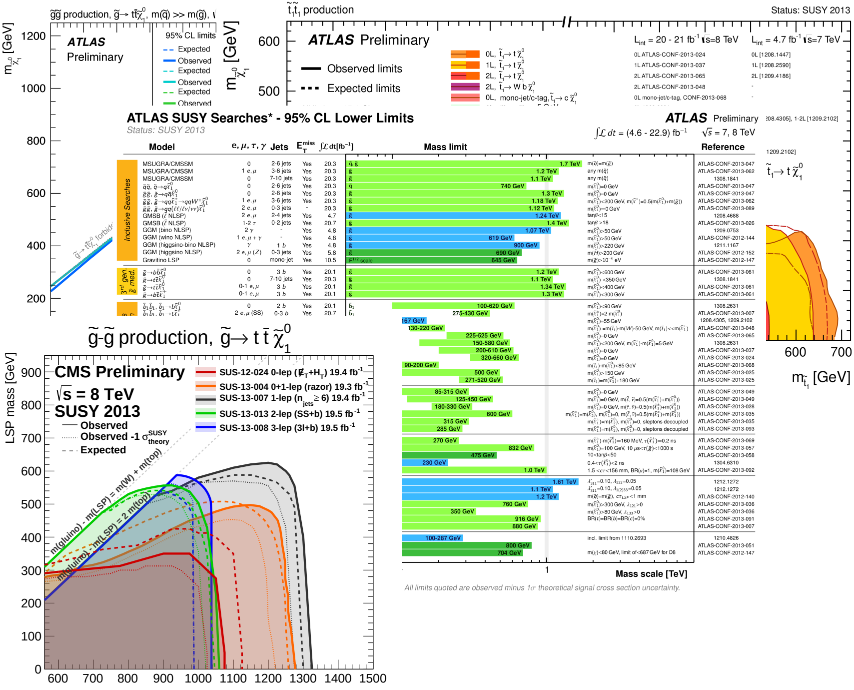
ATLAS
Preliminary
95% CL limits
Expected
Observed
Expected
Observed
Expected
Observed
Expected
Observed
forbidden
m [GeV]



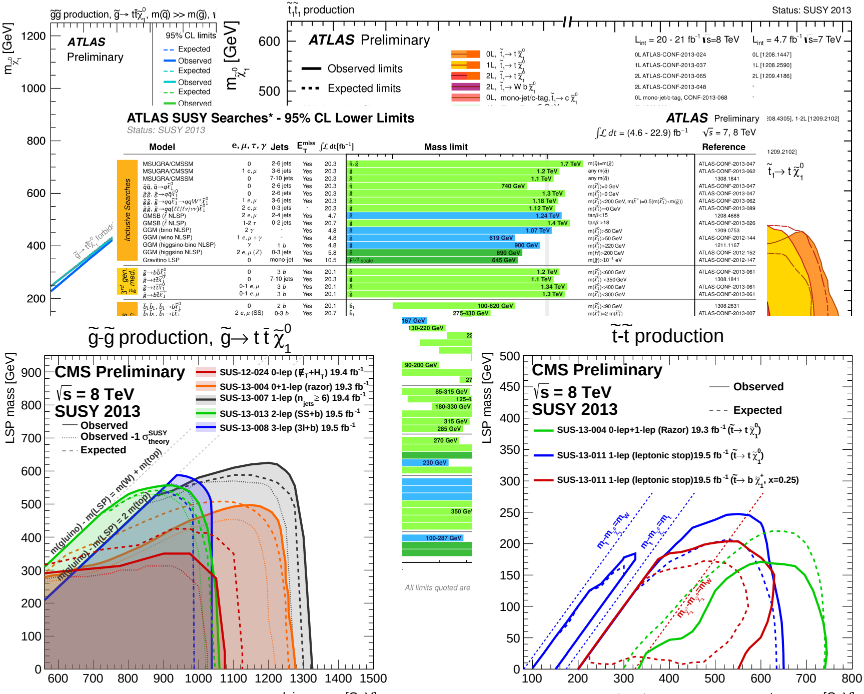
production
Status: SUSY 2013
ATLAS Preliminary
Observed limits
Expected limits
All limits at 95% CL
CDF 2.6 fb-1 [1203.4171]
0L ATLAS-CONF-2013-024
1L ATLAS-CONF-2013-037
2L ATLAS-CONF-2013-065
2L ATLAS-CONF-2013-048
0L mono-jet/c-tag, CONF-2013-068
0L 1308.2631
1L CONF-2013-037, 0L 1308.2631
2L ATLAS-CONF-2013-048
1L CONF-2013-037, 2L CONF-2013-048
0L [1208.1447]
1L [1208.2590]
2L [1209.4186]
2L [1208.4305], 1-2L [1209.2102]
1-2L [1209.2102]
m [GeV]
[GeV]



t̃1t̃1 production
Status: SUSY 2013
ATLAS Preliminary
95% CL limits
Observed limits
Expected limits
ATLAS SUSY Searches* - 95% CL Lower Limits
Status: SUSY 2013
ATLAS Preliminary
Model
Jets
Mass limit
Reference
Inclusive Searches
EW direct
Long-lived particles
RPV
Other
√s = 7 TeV full data
√s = 8 TeV partial data
√s = 8 TeV full data
Mass scale [TeV]
*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.



ATLAS SUSY Searches* - 95% CL Lower Limits
Status: SUSY 2013
ATLAS Preliminary
Model
Jets
Mass limit
Reference
Inclusive Searches
Mass scale [TeV]
All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.
t̃₁t̃₁ production
Status: SUSY 2013
ATLAS Preliminary
Observed limits
Expected limits
g̃g̃ production
95% CL limits
Expected
Observed
g̃-g̃ production, g̃→ t t̄ χ̃⁰₁
CMS Preliminary
√s = 8 TeV
SUSY 2013
Observed
Expected
SUS-12-024 0-lep (E̸T+HT) 19.4 fb⁻¹
SUS-13-004 0+1-lep (razor) 19.3 fb⁻¹
SUS-13-007 1-lep (njets ≥ 6) 19.4 fb⁻¹
SUS-13-013 2-lep (SS+b) 19.5 fb⁻¹
SUS-13-008 3-lep (3l+b) 19.5 fb⁻¹
m(gluino) - m(LSP) = m(W) + m(top)
m(gluino) - m(LSP) = 2 m(top)
LSP mass [GeV]
gluino mass [GeV]



g̃g̃ production, g̃→ tt̄χ̃⁰₁, m(q̃) >> m(g̃)
ATLAS Preliminary
95% CL limits
t̃₁t̃₁ production
Status: SUSY 2013
ATLAS Preliminary
Observed limits
Expected limits
ATLAS SUSY Searches* - 95% CL Lower Limits
Status: SUSY 2013
Model
Jets
Mass limit
Reference
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All limits quoted are
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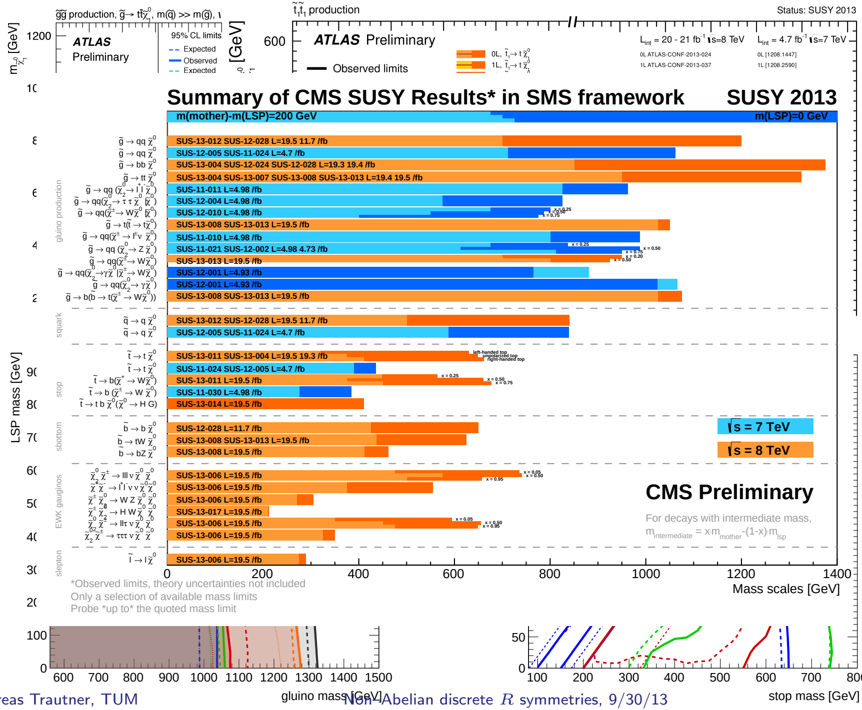
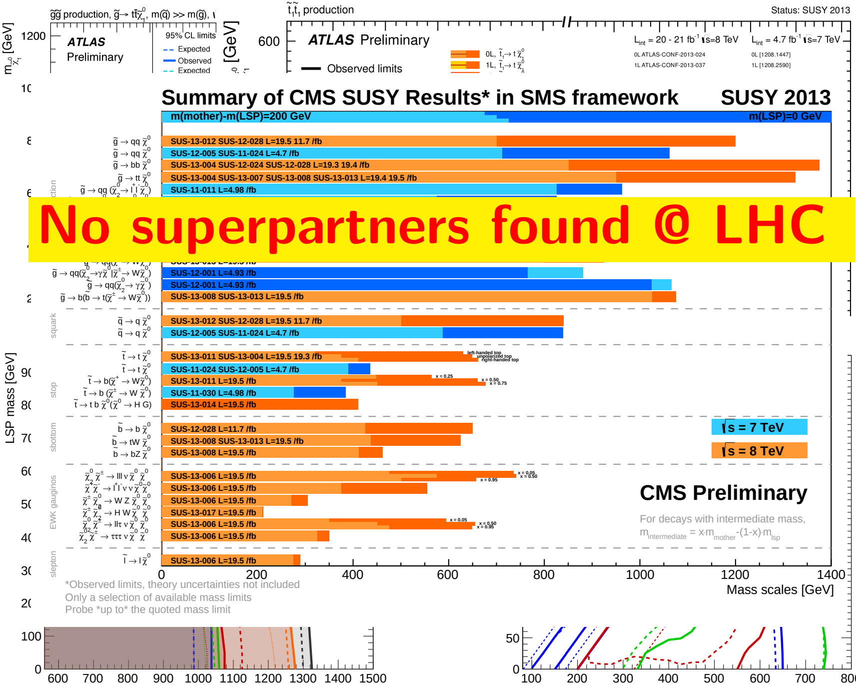
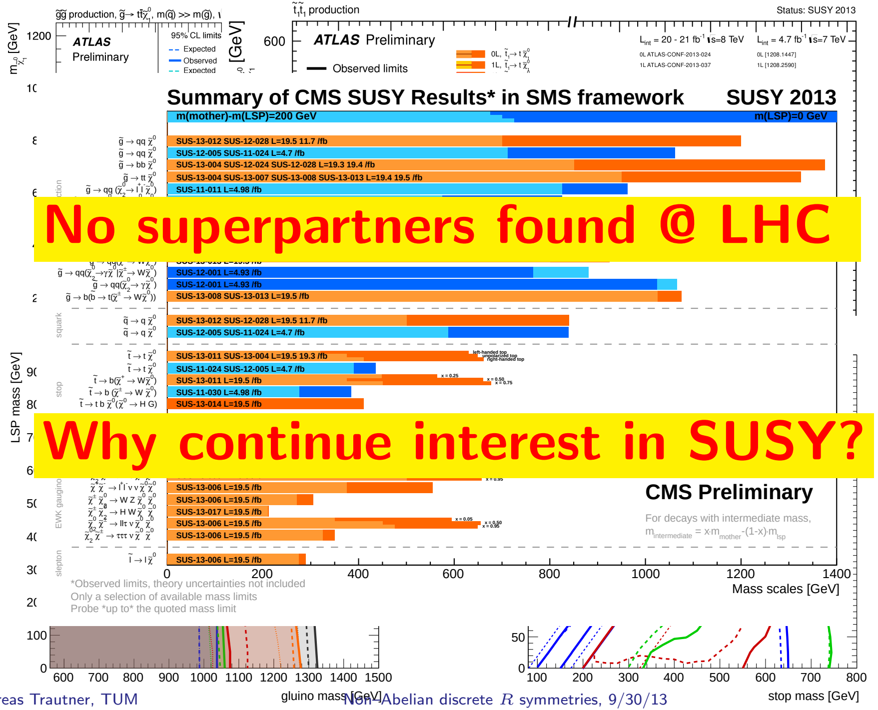
LSP mass [GeV]
gluino mass [GeV]
t̃-t̃ production
SUS-13-004 0-lep+1-lep (Razor) 19.3 fb⁻¹ (t̃→ t χ̃⁰₁)
SUS-13-011 1-lep (leptonic stop)19.5 fb⁻¹ (t̃→ t χ̃⁰₁)
SUS-13-011 1-lep (leptonic stop)19.5 fb⁻¹ (t̃→ b χ̃⁺₁, x=0.25)
stop mass [GeV]



Summary of CMS SUSY Results* in SMS framework
SUSY 2013
m(mother)-m(LSP)=200 GeV
m(LSP)=0 GeV
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
SUS-13-004 SUS-12-024 SUS-12-028 L=19.3 19.4 /fb
SUS-13-004 SUS-13-007 SUS-13-008 SUS-13-013 L=19.4 19.5 /fb
SUS-11-011 L=4.98 /fb
SUS-12-004 L=4.98 /fb
SUS-12-010 L=4.98 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-11-010 L=4.98 /fb
SUS-11-021 SUS-12-002 L=4.98 4.73 /fb
SUS-13-013 L=19.5 /fb
SUS-12-001 L=4.93 /fb
SUS-12-001 L=4.93 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-13-012 SUS-12-028 L=19.5 11.7 /fb
SUS-12-005 SUS-11-024 L=4.7 /fb
SUS-13-011 SUS-13-004 L=19.5 19.3 /fb
SUS-11-024 SUS-12-005 L=4.7 /fb
SUS-13-011 L=19.5 /fb
SUS-11-030 L=4.98 /fb
SUS-13-014 L=19.5 /fb
SUS-12-028 L=11.7 /fb
SUS-13-008 SUS-13-013 L=19.5 /fb
SUS-13-008 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-017 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
SUS-13-006 L=19.5 /fb
gluino production
squark
stop
sbottom
EWK gauginos
slepton
√s = 7 TeV
√s = 8 TeV
CMS Preliminary
For decays with intermediate mass, m_intermediate = x·m_mother+(1-x)·m_lsp
0
200
400
600
800
1000
1200
1400
Mass scales [GeV]
*Observed limits, theory uncertainties not included
Only a selection of available mass limits
Probe *up to* the quoted mass limit
ATLAS Preliminary
Observed limits
Status: SUSY 2013
LSP mass [GeV]
gluino mass [GeV]
stop mass [GeV]


No superpartners found @ LHC
Summary of CMS SUSY Results* in SMS framework
SUSY 2013
m(mother)-m(LSP)=200 GeV
m(LSP)=0 GeV
CMS Preliminary
For decays with intermediate mass,
*Observed limits, theory uncertainties not included
Only a selection of available mass limits
Probe *up to* the quoted mass limit
Mass scales [GeV]
ATLAS Preliminary
Status: SUSY 2013
Observed limits
LSP mass [GeV]
gluino mass [GeV]
stop mass [GeV]



No superpartners found @ LHC

Why continue interest in SUSY?

SUSY

- *only* non-trivial extension of Poincaré-Algebra: [Haag et al., 1975]

$$\left\{ Q_\alpha, Q^\dagger_{\dot\beta} \right\} \;=\; 2\,\sigma^\mu_{\alpha\dot\beta}\,P_\mu$$

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- gauged SUSY implies gravity (SUGRA) [Sohnius, 1985]

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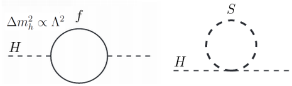
- non-negative spectrum of $\mathcal{H}$
- non-renormalization theorems
- most convincing solution to hierarchy problem



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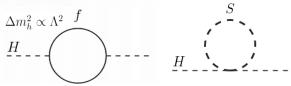
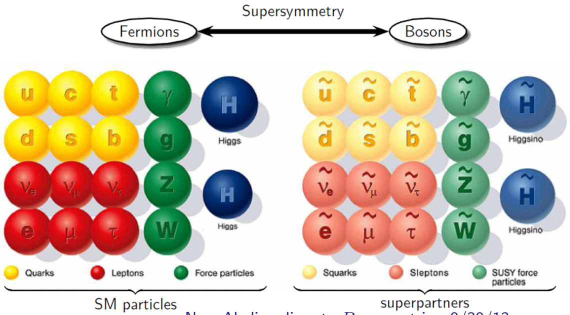
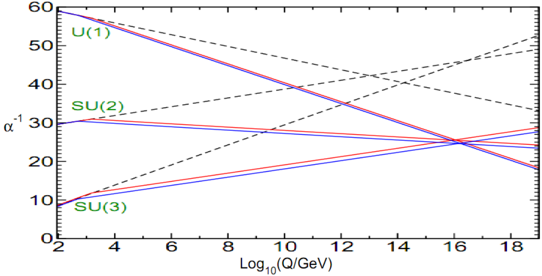


This talk: Minimal supersymmetric extension of the SM: MSSM



Running of the gauge couplings in the MSSM [Dimopoulos et al., 1981]



This suggests [Georgi and Glashow, 1974]

$$\begin{aligned}
\mathrm{SU}(5)/\mathrm{SO}(10) &\to G_{\mathrm{SM}} = \mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}} \\
\mathrm{SO}(10): \quad \mathbf{16} &\to (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\
&\oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{1})_{0}
\end{aligned}$$

at a scale $M_{\mathrm{GUT}} \sim 10^{16}$ GeV.

Assume: precision gauge coupling unification is *not* an accident.
$\curvearrowright$Throughout this talk:

MSSM + GUT.

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MSSM + GUT.

Several things have to be explained:

- 'Doublet-triplet splitting'
  Higgs: $\mathbf{10} \rightarrow (\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}$

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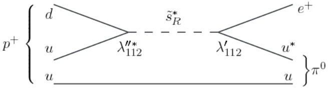
- Proton decay:
  - SUSY $\curvearrowright$ harmful tree level operators



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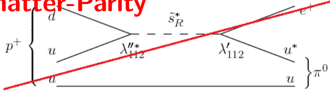
- Proton decay:
  - SUSY $\curvearrowright$ LSP Dark Matter $\curvearrowleft$ **R/matter-Parity**



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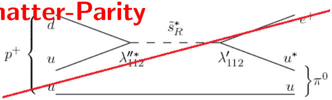
- Proton decay:
  - SUSY $\curvearrowright$ LSP Dark Matter $\curvearrowleft$ $\boldsymbol{R}$**/matter-Parity**
  - dim. 5 (Higgsino color triplets)



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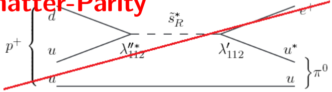
  - dim. 5 (Higgsino color triplets)
  - dim. 6 (New gauge bosons)



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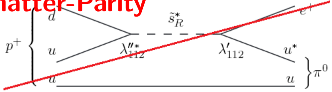
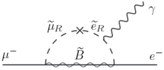
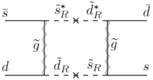


- No flavor and CP violation through sparticle interactions





Outline

Motivation and general framework ✓

Basics

- What is an R symmetry?
- Motivation for R symmetries
- Anomaly constraints for discrete R symmetries

Abelian discrete R symmetries

- MSSM μ problem and R symmetric solution

Interlude: Non–Abelian discrete flavor symmetries

Non–Abelian discrete R symmetries

- A minimal example model

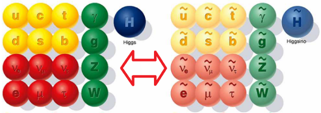
Conclusion

What is an R symmetry?

"Internal symmetry that does *not* commute with SUSY"

SUSY:

$$Q\,|\text{Boson}\rangle \quad = |\text{Fermion}\rangle$$
$$Q\,|\text{Fermion}\rangle \quad = |\text{Boson}\rangle$$



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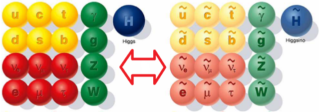

$Q\,|\text{Boson}\rangle \quad = |\text{Fermion}\rangle$

$Q\,|\text{Fermion}\rangle \quad = |\text{Boson}\rangle$

In general:

Q_i $_{(i\,=\,1\,\ldots\,\mathcal{N})}$ can be charged under any internal symmetry B, i.e.

$$[Q_i, B] = (b)_i^j Q_j .$$

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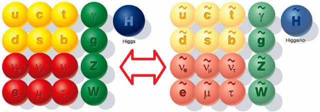

$$Q\,|\text{Boson}\rangle \quad = |\text{Fermion}\rangle$$
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If so, we call B an R symmetry.

What is an R symmetry?

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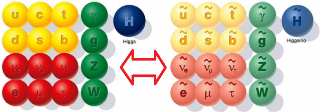

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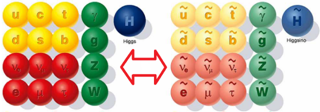
SUSY:



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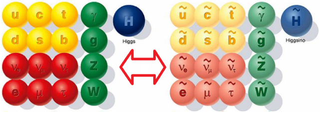
If so, we call B an R symmetry.

- Superpartners charged differently.
- Maximal possible R symmetry: $\mathrm{U}(\mathcal{N})$.

What is an R symmetry?

"Internal symmetry that does *not* commute with SUSY"

SUSY:



$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle$$
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In general:

Q_i $_{(i\,=\,1\,\ldots\,\mathcal{N})}$ can be charged under any internal symmetry B, i.e.

$$[Q_i, B] = (b)_i^j Q_j .$$

If so, we call B an R symmetry.

- Superpartners charged differently.
- Maximal possible R symmetry: $\mathrm{U}(\mathcal{N})$.

Note: by assumption $[Q, G_{SM}] = 0$ in MSSM.

What is an R symmetry?

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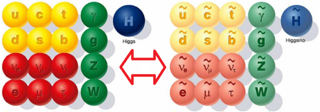

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throughout this talk: $\mathcal{N} = 1$ SUSY.

Superspace Formalism for $\mathcal{N} = 1$ theories

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'Chiral Superfields' $\Phi \supset (\phi, \psi, F)$ living on 'superspace' $(y^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha})$, where $\theta^\alpha, \theta^\dagger_{\dot\alpha}$ $(\alpha = 1, 2)$ are fermionic coordinates .

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$\mathcal{L}$ of a generic $\mathcal{N} = 1$ field theory defined by three functions of the superfields:

- $K\left(\Phi_i, \Phi^{*j}\right)$ Kähler potential $\in \mathbb{R}$
- $f_{ab}(\Phi_i)$ Gauge kinetic function (holomorphic)
- $\mathcal{W}(\Phi_i)$ Superpotential (holomorphic)

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In order to get back to $\mathcal{L}(x)$ in component fields, we have to integrate out the superspace, e.g.

$$\mathcal{L}(x) \supset \int d^2\theta\, \mathcal{W}(\Phi_i) + \text{c.c.}\ .$$

R symmetries in superspace

Example:
Consider a global $\mathrm{U}(1)_R$ with generator B_R, i.e. $[Q, B_R] = q_\theta\, Q$.
Then:

$$\Phi \to \mathrm{e}^{\mathrm{i}\, q_\Phi\, \alpha}\Phi, \quad \theta \to \mathrm{e}^{\mathrm{i}\, q_\theta\, \alpha}\theta\,,$$

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This is a major difference to non–R symmetries.
Completely analogue also for discrete R Symmetries.

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  - SUSY broken $\Rightarrow$ R symmetry exists.
  - Spontaneously broken R sym. $\Rightarrow$ SUSY broken.

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  But:
  - Very compelling soultion to doublet-triplet splitting from higher-dim. GUTs. [Kawamura, 2001]
  - Lorentz invariance of compact extra dimensions can lead to (anomaly free, discrete) R symmetries in 4D effective theory.

  [Kappl et al., 2011, Lee et al., 2011b, Cabo Bizet et al., 2013, Nilles et al., 2013]

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  [Freedman, 1977, Castaño et al., 1996]
  - Local $\mathrm{U}(1)_R$ anomalous with standard field content.
    [Chamseddine and Dreiner, 1996]
  - Local $\mathrm{U}(1)_R$ problematic, since the appearance of a Fayet-Illiopoulos term requires an exact global symmetry.
    [Fayet and Iliopoulos, 1974, Krauss and Wilczek, 1989, Komargodski and Seiberg, 2009, Banks and Seiberg, 2011]

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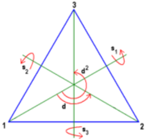
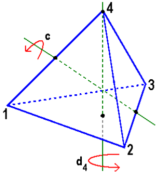
$\Rightarrow$ This talk: focus on anomaly free, discrete R symmetries.





Anomaly constraints for discrete R symmetries

- Anomaly calculation directly from the path integral measure:

[Fujikawa, 1980, Araki, 2007]

$$\psi \xrightarrow{R} \mathrm{e}^{2\pi\,\mathrm{i}\,q_\psi\,/\,M}\,\psi \qquad \curvearrowright \qquad \mathcal{D}\psi\,\mathcal{D}\overline{\psi} \xrightarrow{R} J^{-2}\,\mathcal{D}\psi\,\mathcal{D}\overline{\psi}\;,$$

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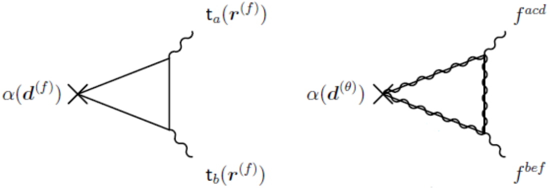
This corresponds to the vanishing of the typical triangle diagrams



Anomaly constraints for (non–)Abelian discrete (R) symmetries

Chiral superfield Φ transforming

- in representation $\boldsymbol{d}$ of a non–Abelian discrete R symmetry D,

$$\text{for } \Phi \;\to\; U(\boldsymbol{d})\,\Phi \;=\; \mathrm{e}^{2\pi\,\mathrm{i}\,\lambda(\boldsymbol{d})/M}\,\Phi\;,$$

$$\text{we define } \delta \;:=\; \mathrm{tr}[\lambda(\boldsymbol{d})] \;=\; \frac{M}{2\pi\,\mathrm{i}}\ln\det U(\boldsymbol{d})\;.$$

- with charge Q under Abelian factors of a gauge group G,
- in representation $\boldsymbol{r}$ under non–Abelian factors of G.

Anomaly coefficients of the a–generated subgroup $\mathbb{Z}_M$ of D are given as

[Chen, Ratz, AT]

$$\begin{aligned}
A_{G-G-\mathbb{Z}^R_{M\,(\mathsf{a})}} &= \sum_s \ell(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)}-\mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]+\ell(\mathrm{adj}\,G)\cdot\delta^{(\theta)}\;,\\
A_{\mathrm{U}(1)-\mathrm{U}(1)-\mathbb{Z}^R_{M\,(\mathsf{a})}} &= \sum_s (Q^{(s)})^2\,\dim(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)}-\mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\;,\\
A_{\mathrm{grav}-\mathrm{grav}-\mathbb{Z}^R_{M\,(\mathsf{a})}} &= -21\,\delta^{(\theta)}+\delta^{(\theta)}\sum_G \dim(\mathrm{adj}\,G)\\
&\quad+\sum_s \dim(\boldsymbol{r}^{(s)})\cdot\left[\delta^{(s)}-\mathsf{dim}(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\;.
\end{aligned}$$

Discrete Green-Schwarz anomaly cancellation

Even if the anomaly coefficients $A_{G_i - G_i - \mathbb{Z}_M^R} \neq 0 \mod \frac{M}{2}$, we can have an anomaly-free theory if there is an axion a coupling to

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For this to work with multiple gauge groups G_i (without spoiling gauge coupling unification) we have to impose 'anomaly universality': [Chen et al., 2012a]

$$A_{G^{(i)}-G^{(i)}-\mathbb{Z}_M^R} = \rho \mod \frac{M}{2} \quad \forall\, G^{(i)} ,$$

Abelian discrete R symmetries

- The MSSM μ Problem
- Only R symmetries can control the μ term
- $\mathbb{Z}_4^R$ symmetry solution to the μ Problem

The MSSM μ Problem

MSSM contains additional massive parameter μ

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Will show:

$\rightarrow$ *Only* R symmetries can control the μ term (assuming: GUT + anomaly universality)

$\rightarrow$ Discrete R symmetry allows for an explanation of $\mu \sim m_{\cancel{SUSY}}$.

Only R symmetries can control μ

Anomaly constraints for (discrete) non-R symmetries (+SU(5) relations):

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_M} = \frac{1}{2}\sum_{f=1}^{3}\left(3q_{\mathbf{10}}^{(f)} + q_{\overline{\mathbf{5}}}^{(f)}\right) ,$$

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$$

Non-R symmetry cannot forbid μ term!

[Babu et al., 2003, Lee et al., 2011a, Lee et al., 2011b]

R symmetry solution to the μ Problem

The MSSM superpotential, with effective B–L violating terms up to mass dimension 5:

$$
\begin{aligned}
\mathcal{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\
& + Y_e^{ij} H_d L_i E_j + Y_d^{ij} H_d Q_i \overline{D}_j + Y_u^{ij} H_u Q_i \overline{U}_j \\
& + \lambda^{(1)}_{ijk} L_i L_j \overline{E}_k + \lambda^{(2)}_{ijk} L_i Q_j \overline{D}_k + \lambda^{(3)}_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_l \\
& + \kappa^{(3)}_{ijk} Q_i Q_j Q_k H_d + \kappa^{(4)}_{ijk} Q_i \overline{U}_j \overline{E}_k H_d + \kappa^{(5)}_i L_i H_u H_u H_d \\
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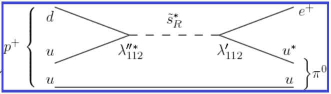

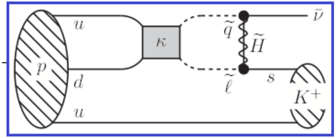

- Severely constrained by absence of proton decay.

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\mathcal{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\
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& + \lambda_{ijk}^{(1)} L_i L_j \overline{E}_k + \lambda_{ijk}^{(2)} L_i Q_j \overline{D}_k + \lambda_{ijk}^{(3)} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ijkl}^{(1)} Q_i Q_j Q_k L_l + \kappa_{ijkl}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_l \\
& + \kappa_{ijk}^{(3)} Q_i Q_j Q_k H_d + \kappa_{ijk}^{(4)} Q_i \overline{U}_j \overline{E}_k H_d + \kappa_i^{(5)} L_i H_u H_u H_d \\
& + \kappa_{ij}^{(6)} L_i H_u L_j H_u .
\end{aligned}
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- Severely constrained by absence of proton decay.
- Prohibited by MSSM R parity.

R symmetry solution to the μ Problem

The MSSM superpotential, with effective B–L violating terms up to mass dimension 5:

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$\curvearrowright$ Require an anomaly-free discrete R symmetry which:

- Prohibits μ,
- Allows for Yukawa couplings and an effective Neutrino mass,
- Contains R parity as a subgroup.

R symmetry solution to the μ Problem

Search for anomaly-free discrete R symmetry which:

- prohibits μ
- Yukawa couplings
  and Neutrino mass
- R parity subgroup

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$$\left.\begin{array}{l}\text{- prohibits } \mu \\ \text{- Yukawa couplings} \\ \quad\text{and Neutrino mass} \\ \text{- } R \text{ parity subgroup}\end{array}\right\} \Rightarrow \text{in } \mathrm{SU}(5) \text{ GUT : } \mathbb{Z}_{4\mathbb{N}}^{R} \text{ symmetries.}$$

[Babu et al., 2003, Lee et al., 2011a, Lee et al., 2011b, Chen et al., 2012b]

R symmetry solution to the μ Problem

Search for anomaly-free discrete R symmetry which:

- prohibits μ
- Yukawa couplings
  and Neutrino mass
- R parity subgroup

$\Rightarrow$ in $\mathrm{SO}(10)$ GUT : unique $\mathbb{Z}_4^R$.

[Babu et al., 2003, Lee et al., 2011a, Lee et al., 2011b, Chen et al., 2012b]

$\mathbb{Z}_4^R$ symmetry solution to the μ Problem

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$$\Rightarrow \mu \text{ problem solved}.$$

[Lee et al., 2011a, Lee et al., 2011b, Chen et al., 2012b, Chen et al., 2012a], cf. 'Retrofitting' in [Dine et al., 2006]

$\mathbb{Z}_4^R$ symmetry solution to the μ Problem

Further features:

- ☺ R parity contained as unbroken $\mathbb{Z}_2$ subgroup of the $\mathbb{Z}_4^R$,
- ☺ Automatic: Proton decay by dimension-5 operators under control:

$$\kappa^{(1)}, \kappa^{(2)} \sim \frac{m_{3/2}}{M_{\mathrm{P}}^2} \ .$$

(dominant p decay: dimension-6 operators).

- ☺ There are explicit models of orbifold compactifications of the heterotic string, featuring a MSSM spectrum + discrete R symmetry at low energy. [Kappl et al., 2011]

Connection of the R symmetry to other discrete Symmetries?

- Interlude: Non–Abelian discrete flavor symmetries

$\curvearrowright$ Non–Abelian discrete R symmetries

Interlude: Non-Abelian discrete flavor symmetries

- Popular: Non–Abelian discrete symmetries as flavor symmetries G_{F}, e.g. $\mathrm{S}_3, \mathrm{A}_4, \mathrm{T}', \Delta(27), \ldots$
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  [Kaplan 1994, Frampton 1995, Dermisek 1999, Aranda 2000, Carr 2000, Ma 2001, Ma 2002, Babu 2003, Kubo 2003, Kubo 2004, Kubo 2006, Chen 2004, Lavoura 2005, Dermisek 2005, Caravaglios 2005, Caravaglios 2005, Grimus 2006, Koide 2006, Teshima 2006, Haba 2006, Tanimoto 2006, Koide 2006, Morisi 2006, Picariello 2006, Mohapatra 2006, Kaneko 2007, Koide 2007, Chen 2008, Feruglio 2007, Grimus 2004, Adulpravitchai 2009, Blum 2004, Blum 2008, Everett 2009, Mohapatra 2004, Hagedorn 2006, Cai 2006, Zhang 2007, Ma 2007, Bazzocchi 2008, Ishimori 2009, Meloni 2009, Dutta 2009, Ding 2010, Morisi 2010, Hagedorn 2010, Ishimori 2010, Hirsch 2004, Ma 2004, Chen 2005, Altarelli 2005, Ma 2005, Hirsch 2005, Babu 2005, Zee 2005, Ma 2006, He 2006, Adhikary 2006, Altarelli 2006, Lavoura 2006, Ma 2007, Hirsch 2007, Luhn 2007, de Medeiros-Varzielas 2007, Altarelli 2007, Yin 2007, Bazzocchi 2008, Bazzocchi 2008, Honda 2008, Brahmachari 2008, Adhikary 2008, Hirsch 2008, Frampton 2008, Csaki 2008, Altarelli 2008, Morisi 2009, Lin 2009, Altarelli 2009, Morisi 2007, Grimus 2008, Ciafaloni 2009, Bazzocchi 2009, King 2009, Bazzocchi 2008, delAguila 2010, Kadosh 2010, Antusch 2010, ...]

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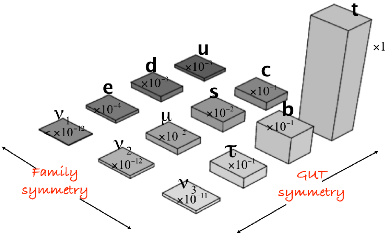
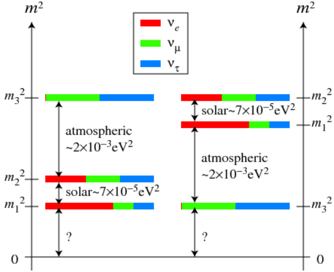
Quarks(CKM)	Leptons(PMNS)
$\theta_{12} = 13.04° \pm 0.05°$	$\theta_{12} \simeq 34° \pm 1°$
$\theta_{23} = 2.38° \pm 0.06°$	$\theta_{23} \simeq 40° \pm 3°$
$\theta_{13} = 0.201° \pm 0.011°$	$\theta_{13} \simeq 9° \pm 0.5°$
$\delta \quad = 1.20 \pm 0.08$ rad	$\delta \quad = 5.25^{+1.16}_{-2.42}$ rad

[Beringer et al., 2012, Gonzalez-Garcia et al., 2012, King and Luhn, 2013]

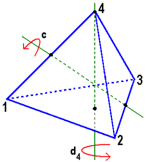




Interlude: Non-Abelian discrete flavor symmetries

Example: Lepton sector mixing from A_4 ($\equiv (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_3$).

[Altarelli and Feruglio, 2006]



A_4	e	T	T^2	S
$\mathbf{1}$	1	1	1	1
$\mathbf{1}'$	1	ω	ω^2	1
$\mathbf{1}''$	1	ω^2	ω	1
$\mathbf{3}$	3	0	0	-1

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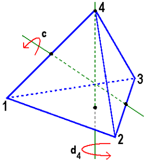

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charge assignment:

$$\left.\begin{array}{l}\left(\overline{D}_1, L_1\right)_{\overline{\mathbf{5}}}\\ \left(\overline{D}_2, L_2\right)_{\overline{\mathbf{5}}}\\ \left(\overline{D}_3, L_3\right)_{\overline{\mathbf{5}}}\end{array}\right\}\ \mathbf{3}. \qquad \begin{array}{l}\left(Q_1, \overline{U}_1, \overline{E}_1\right)_{\mathbf{10}}\ \}\ \mathbf{1}\\ \left(Q_2, \overline{U}_2, \overline{E}_2\right)_{\mathbf{10}}\ \}\ \mathbf{1'}\\ \left(Q_3, \overline{U}_3, \overline{E}_3\right)_{\mathbf{10}}\ \}\ \mathbf{1''}\end{array}$$

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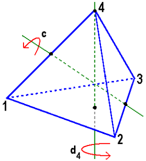

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Extra: 'Flavons' $\varphi_T : \mathbf{3}$, $\varphi_S : \mathbf{3}$, $\xi : \mathbf{1}$, and a $\mathbb{Z}_3$ symmetry.

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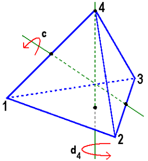

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$\mathbf{3}$	3	0	0	-1

charge assignment:

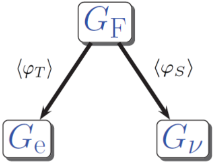
$$\left.\begin{array}{l} (\overline{D}_1, L_1)_{\overline{\mathbf{5}}} \\ (\overline{D}_2, L_2)_{\overline{\mathbf{5}}} \\ (\overline{D}_3, L_3)_{\overline{\mathbf{5}}} \end{array}\right\} \mathbf{3}. \qquad \begin{array}{l} (Q_1, \overline{U}_1, \overline{E}_1)_{\mathbf{10}} \,\}\, \mathbf{1} \\ (Q_2, \overline{U}_2, \overline{E}_2)_{\mathbf{10}} \,\}\, \mathbf{1'} \\ (Q_3, \overline{U}_3, \overline{E}_3)_{\mathbf{10}} \,\}\, \mathbf{1''} \end{array}$$

Extra: 'Flavons' $\varphi_T : \mathbf{3}$, $\varphi_S : \mathbf{3}$, $\xi : \mathbf{1}$, and a $\mathbb{Z}_3$ symmetry.

$$\mathcal{W}^{eff}_{lept} = (\varphi_T L)\,\overline{E}_1 + (\varphi_T L)'\,\overline{E}_2 + (\varphi_T L)''\,\overline{E}_3 + \xi\,(LL) + (\varphi_S LL) \ .$$

Interlude: Non-Abelian discrete flavor symmetries

Example: Lepton sector mixing from A_4. [Altarelli and Feruglio, 2006]



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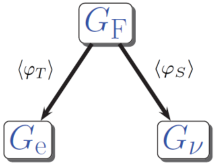

spontaneous breaking of G_F by

$\langle\varphi_T\rangle = (v_T, 0, 0)$

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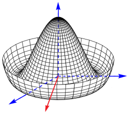
$\langle\xi\rangle = u$



crucial: 'vacuum alignment'!

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$$\begin{tikzcd} & \boxed{G_{\mathrm{F}}} \arrow[dl, "\langle\varphi_T\rangle"'] \arrow[dr, "\langle\varphi_S\rangle"] & \\ \boxed{G_{\mathrm{e}}} & & \boxed{G_{\nu}} \end{tikzcd}$$

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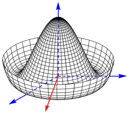

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$\curvearrowright$

$$M_l = v_d \frac{v_S}{\Lambda} \begin{pmatrix} y_e & & \\ & y_\mu & \\ & & y_\tau \end{pmatrix}$$

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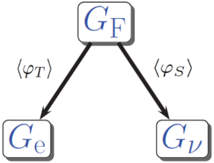

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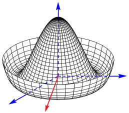

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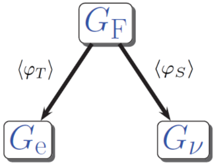

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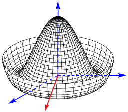

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$\curvearrowright$ Tri-Bi-Maximal mixing (TBM) [Harrison et al., 2002]

Leptons(*exp.*)	TBM
$\theta_{12} \simeq 34° \pm 1°$	$\theta_{12} \simeq 35.3°$
$\theta_{23} \simeq 40° \pm 3°$	$\theta_{23} = 45°$
$\theta_{13} \simeq 9° \pm 0.5°$	$\theta_{13} = 0°$

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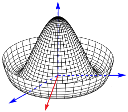

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However: Additional symmetries and fields needed to explain

- Vacuum alignment: $\mathrm{U}(1)_R$,
- Large mass hierarchies: $\mathrm{U}(1)_\mathrm{FN}$. [Froggatt and Nielsen, 1979]

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Dynamical explanation of vacuum alignment. [Altarelli and Feruglio, 2006]
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Requiring SUSY to be unbroken at the flavor scale we have:

$$\frac{\partial\,\mathcal{W}}{\partial\varphi_0^T} = \frac{\partial\,\mathcal{W}}{\partial\varphi_0^S} = \frac{\partial\,\mathcal{W}}{\partial\xi_0} \stackrel{!}{=} 0$$

Solving those 'drives' $\langle\varphi_T\rangle$, $\langle\varphi_S\rangle$, $\langle\xi\rangle$ to aligned directions.

Discrete symmetries of the superpotential

$$
\begin{aligned}
\mathcal{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\
& + Y_e^{ij} H_d L_i E_j + Y_d^{ij} H_d Q_i \overline{D}_j + Y_u^{ij} H_u Q_i \overline{U}_j \\
& + \lambda_{ijk}^{(1)} L_i L_j \overline{E}_k + \lambda_{ijk}^{(2)} L_i Q_j \overline{D}_k + \lambda_{ijk}^{(3)} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ijkl}^{(1)} Q_i Q_j Q_k L_l + \kappa_{ijkl}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_l \\
& + \kappa_{ijk}^{(3)} Q_i Q_j Q_k H_d + \kappa_{ijk}^{(4)} Q_i \overline{U}_j \overline{E}_k H_d + \kappa_i^{(5)} L_i H_u H_u H_d \\
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\left.\begin{array}{l}\text{Non–Abelian discrete} \\ + \;\text{discrete } R\end{array}\right\} \Rightarrow \boxed{\text{Non–Abelian discrete R symmetry.}}
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[Chen,Ratz,AT]

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[Chen,Ratz,AT]

Intriguing fact: Discrete R, non-Abelian discrete, as well as non-Abelian discrete R symmetries all can originate from extra compact dimensions.

[Kobayashi et al., 2007, Araki et al., 2008, Nilles et al., 2012], Ratz and Vaudrevange 2013 (PC)

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`GAP` scan:

$\mathcal{O}(G)$	G_F
24	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$
24	$\mathrm{S}_3 \times \mathbb{Z}_4$
32	$(\mathbb{Z}_8 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$
32	$(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2$
32	$\mathbb{Z}_8 \rtimes \mathbb{Z}_4$
	$\ldots$

[Chen, Ratz, AT]

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

The group $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$: [Chen, Ratz, AT]

	$\mathbb{1}$	v	v^2	v^4	u	v^3	v^5	v^6	uv^2	uv^4	v^7	uv^6
	1	3	1	1	2	3	3	1	2	2	3	2
$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	1a	8a	4a	2a	3a	8b	8c	4b	12a	6a	8d	12b
$\mathbf{1}_1$	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}_2$	1	-1	1	1	1	-1	-1	1	1	1	-1	1
$\mathbf{1}_3$	1	$-\mathrm{i}$	-1	1	1	i	$-\mathrm{i}$	-1	-1	1	i	-1
$\mathbf{1}_4$	1	i	-1	1	1	$-\mathrm{i}$	i	-1	-1	1	$-\mathrm{i}$	-1
$\mathbf{1}_5$	1	$-\tau$	i	-1	1	τ^*	τ	$-\mathrm{i}$	i	-1	$-\tau^*$	$-\mathrm{i}$
$\mathbf{1}_6$	1	τ^*	$-\mathrm{i}$	-1	1	$-\tau$	$-\tau^*$	i	$-\mathrm{i}$	-1	τ	i
$\mathbf{1}_7$	1	$-\tau^*$	$-\mathrm{i}$	-1	1	τ	τ^*	i	$-\mathrm{i}$	-1	$-\tau$	i
$\mathbf{1}_8$	1	τ	i	-1	1	$-\tau^*$	$-\tau$	$-\mathrm{i}$	i	-1	τ^*	$-\mathrm{i}$
$\mathbf{2}_1$	2	0	-2	2	-1	0	0	-2	1	-1	0	1
$\mathbf{2}_2$	2	0	2	2	-1	0	0	2	-1	-1	0	-1
$\mathbf{2}_3$	2	0	$-2\mathrm{i}$	-2	-1	0	0	$2\mathrm{i}$	i	1	0	$-\mathrm{i}$
$\mathbf{2}_4$	2	0	$2\mathrm{i}$	-2	-1	0	0	$-2\mathrm{i}$	$-\mathrm{i}$	1	0	i

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- 8 singlet and 4 doublet irreps $\mathbf{1}_{1-8}$, $\mathbf{2}_{1-4}$.

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$\Rightarrow$

θ	Matter	Higgses
$\mathbf{1}_5/\mathbf{1}_8$	$\mathbf{1}_5/\mathbf{1}_8/\mathbf{2}_4$	$\mathbf{1}_1/\mathbf{1}_2$

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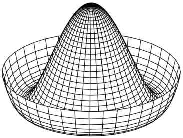
$\Rightarrow$

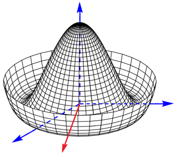
θ	Matter	Higgses
$\mathbf{1}_5/\mathbf{1}_8$	$\mathbf{1}_5/\mathbf{1}_8/\mathbf{2}_4$	$\mathbf{1}_1/\mathbf{1}_2$

- Spontaneous breaking:

$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$

$\langle \mathbf{1}_2 \rangle, \langle \mathbf{2}_2 \rangle \;\to\; \mathbb{Z}_4$ $\qquad$ $\langle \mathbf{2}_2 \rangle \propto (1, 0) \;\to\; \mathbb{Z}_8$





No alignment necessary

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

Minimal example model: [Chen, Ratz, AT]

$$\theta:\ \mathbf{1}_5\ \left(\Rightarrow\ \mathcal{W}:\ \mathbf{1}_4 = \mathbf{1}_5 \otimes \mathbf{1}_5\right)$$

$$\left.\begin{array}{l}\left(\overline{D}_1, L_1\right)_{\overline{\mathbf{5}}} \\ \left(\overline{D}_2, L_2\right)_{\overline{\mathbf{5}}}\end{array}\right\} \mathbf{2}_4 \qquad \left(Q_1, \overline{U}_1, \overline{E}_1\right)_{\mathbf{10}} \Big\}\ \mathbf{1}_5$$

$$\left(Q_2, \overline{U}_2, \overline{E}_2\right)_{\mathbf{10}} \Big\}\ \mathbf{1}_5$$

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$$H_u:\ \mathbf{1}_1 \qquad H_d:\ \mathbf{1}_2$$

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$$H_u: \; \mathbf{1}_1 \qquad H_d: \; \mathbf{1}_2$$

Spontaneous breaking sector: $\phi: \; \mathbf{1}_2$ $\qquad \chi: \; \mathbf{2}_2$.

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$$\left(\overline{D}_3, L_3\right)_{\overline{\mathbf{5}}} \ \} \ \mathbf{1}_5 \qquad \left(Q_3, \overline{U}_3, \overline{E}_3\right)_{\mathbf{10}} \ \} \ \mathbf{1}_8$$

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Spontaneous breaking sector: $\phi: \ \mathbf{1}_2$ $\qquad \chi: \ \mathbf{2}_2$.

- Neutrino masses from (effective) Weinberg operator.
- Spare 'shaping' symmetries (commonly used to generate hierarchies and split quarks & leptons).
- Focus discussion on general issues:
  - Consistent assignment of fields to representations,
  - Symmetry breaking,
  - Possible mass matrix structures,
  - Anomaly cancellation.

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

Spontaneous symmetry breaking:

$$\langle\chi\rangle \;=\; v\begin{pmatrix}\cos\theta_\chi\\ \sin\theta_\chi\end{pmatrix} \quad \text{and} \quad \langle\phi\rangle \;=\; v\,r_\phi\;,$$

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- 'Generate' VEVs $\curvearrowright$ include field ξ : $\mathbf{1}_4$ (recall $\mathcal{W}$: $\mathbf{1}_4$)

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  - No coupling to matter sector
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$\left.\right\}$ ξ =Driving field!

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$$\left.\begin{array}{l} \text{– No coupling to matter sector} \\ \text{– Only linear coupling to flavon fields} \end{array}\right\} \xi = \text{Driving field!}$$

- Aligned VEV $\Rightarrow G \to \mathbb{Z}_8^R$ with family dependent $\mathbb{Z}_8^R$.

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F-Term condition for ξ: $\quad 0 \stackrel{!}{=} \frac{\partial\,\mathcal{W}}{\partial\xi}$

$$\curvearrowright \text{LO} : v^2(\theta_\chi) \;=\; \frac{M^2}{g_1\,r_\phi^2 + g_2\,(2\cos^2(\theta_\chi)-1)}$$

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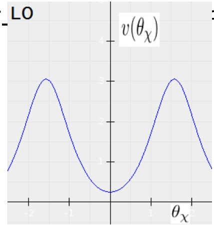
F- 

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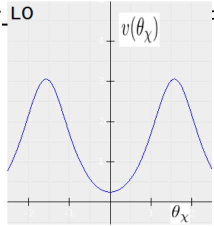
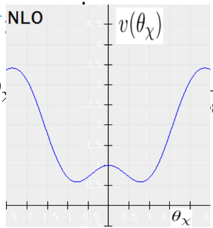
F- … for ξ:

$v^2(\theta_\chi$ … $\overline{\ldots s^2(\theta_\chi) - 1)}$





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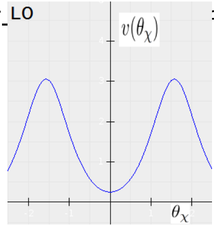

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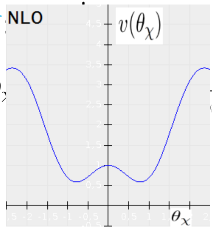
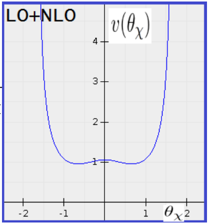


…$s^2(\theta_\chi$



A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

- Identify terms consistent with all symmetries ($\mathcal{W}$ charged!)

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

- Identify terms consistent with all symmetries ($\mathcal{W}$ charged!)
- SSB with small mis–alignment $\langle \chi \rangle \propto (1, \delta)^T$

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↷ Yukawa coupling (mass) matrix structures $\varepsilon := v/\Lambda$

$$Y_e \sim Y_d^T \sim \begin{pmatrix} \varepsilon & -\varepsilon\,\delta & \varepsilon\,r_\phi \\ \varepsilon & -\varepsilon\,\delta & \varepsilon\,r_\phi \\ -\varepsilon\,\delta & \varepsilon & 1 \end{pmatrix}, Y_u \sim \begin{pmatrix} 1 & 1 & \varepsilon\,r_\phi \\ 1 & 1 & \varepsilon\,r_\phi \\ \varepsilon\,r_\phi & \varepsilon\,r_\phi & 1 \end{pmatrix},$$

$$M_\nu \sim \begin{pmatrix} 1+\varepsilon & \varepsilon\,\delta & -\varepsilon\,\delta \\ \varepsilon\,\delta & -1+\varepsilon & \varepsilon \\ -\varepsilon\,\delta & \varepsilon & 1 \end{pmatrix}.$$

A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ minimal example model

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$$M_\nu \sim \begin{pmatrix} 1+\varepsilon & \varepsilon\,\delta & -\varepsilon\,\delta \\ \varepsilon\,\delta & -1+\varepsilon & \varepsilon \\ -\varepsilon\,\delta & \varepsilon & 1 \end{pmatrix}.$$

Model features:

- Can accomodate reality (with 28 parameters),
- Anomaly free by Green–Schwarz mechanism,
- Contains successfull $\mathbb{Z}_4^R$ solution to μ and proton decay problem,
- Vacuum alignment without enlarging the symmetry,
- Natural mass hierarchy requires additional symmetry e.g. $\mathrm{U}(1)_{\mathrm{FN}}$.

Structure of the soft SUSY breaking terms

$$\int \mathrm{d}^4\theta \, \frac{X^\dagger X}{\Lambda^2} \, Q^\dagger Q \xrightarrow{X \to F_X \, \theta^2} \widetilde{m}^2 \, |q|^2 \; , \tag{1}$$

$$\int \mathrm{d}^2\theta \, \frac{X}{\Lambda} \, y \, Q^3 \xrightarrow{X \to F_X \, \theta^2} A \, y \, q^3 \; , \tag{2}$$

$$\int \mathrm{d}^2\theta \, \frac{X}{\Lambda} \, W_\alpha W^\alpha \xrightarrow{X \to F_X \, \theta^2} M_\lambda \, \lambda\lambda \; . \tag{3}$$

- If matter fields Q are irrep of non-Abelian discrete symmetry $\curvearrowright$ LO breaking (1) is flavor diagonal ($\leftrightarrow$ MFV).
- If ~~SUSY~~ spurion X is R-charged: (2) and (3) not allowed (but pheno: $M_\lambda \neq 0$).
- If X is higher-dim. irrep of a NA discrete flavor symmetry: if F_X and $\langle \mathrm{Flavons} \rangle$ are not aligned $\curvearrowright$ dangerous flavor violation via (2)
- However, if NA flavor symmetry is also R, we can prohibit (2) and have the possibility to break flavor and SUSY with one single hidden sector.

Conclusion

- Have discussed MSSM GUTs with R symmetries to solve μ and proton decay problem,
- Assuming GUT relations and anomaly freedom we have seen that *only* R symmetries can solve the μ problem.
- Requiring
  (i) anomaly freedom (allowing for GS anomaly cancellation),
  (ii) μ term prohibited perturbatively,
  (iii) Yukawa coupling and Weinberg effective neutrino mass allowed,

  $\curvearrowright$ discrete $\mathbb{Z}_4^R$ can do the job (unique in $\mathrm{SO}(10)$).
  - unbroken R parity subgroup (dim. 4 proton decay absent),
  - dim. 5 proton decay operators highly suppressed,
  - non-perturbative reintroduction of $\mu \sim m_{3/2}$.
- In UV complete (heterotic orbifold) models, such symmetries have a geometrical origin,
- This also applies to non-Abelian discrete flavor symmetries, which we have shown to be useful in engineering specific flavor mixing patterns (i.e. reducing the number of free parameters of the flavor puzzle).

Conclusion

- Combining discrete R with non-Abelian discrete flavor symmetries, we have introduced non–Abelian discrete R symmetries in the framework of $\mathcal{N} = 1$ SUSY GUTs.
- A search for viable symmetries that include
  - solution to the μ and proton decay problem of the MSSM,
  - possible explanation of the flavor structure,

  reveals $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ as the smallest possible group with non–trivial embedding of the R symmetry.
- A $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ toy model shows generic features of such models:
  - the Abelian R symmetry lies in the center of the group,
  - VEV (mis-)alignment, which is neccessary to explain family mixing and $\cancel{CP}$, can be obtained w/o enlarging the symmetry group.

Outlook:

- There is no obstacle for the construction of possibly realistic models with non–Abelian discrete R symmetries. In such models, it will be interesting to investigate:
  - Implications for ~~SUSY~~, i.e. scalar sector soft masses,
  - Possible UV origin of the symmetry.

Thank You!

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