arXiv:1309.6922,
PRL 2011, PRL 2010, PRA 2008

Cold Atoms from Few-body Physics: Application of Pionless EFT

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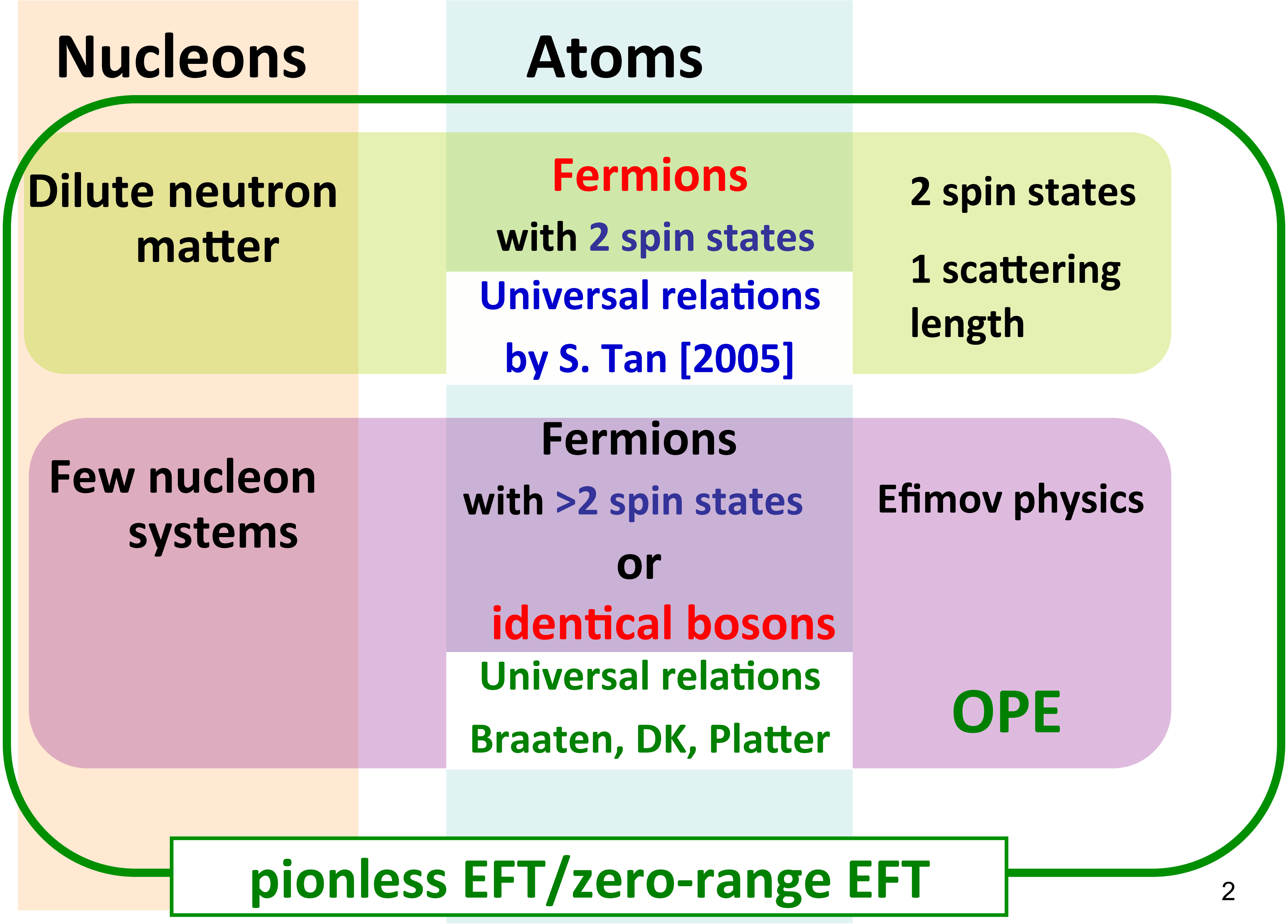


Nucleons

Atoms

Dilute neutron matter

Fermions
with 2 spin states

Universal relations
by S. Tan [2005]

2 spin states

1 scattering length

Few nucleon systems

Fermions
with >2 spin states
or
identical bosons

Universal relations
Braaten, DK, Platter

Efimov physics

OPE

pionless EFT/zero-range EFT

Outline

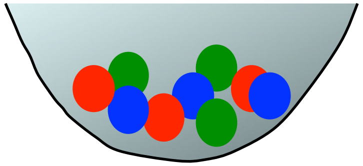
- Strongly interacting ultracold atoms
- Fermions with 2 spin states (2-body physics)
  - Universal relations and Contact
  - Operator Product Expansion (OPE)
- Identical bosons (3-body physics)
  - Efimov physics and Universal relations
  - Recent result on unitary Bose gas

Strongly interacting atoms

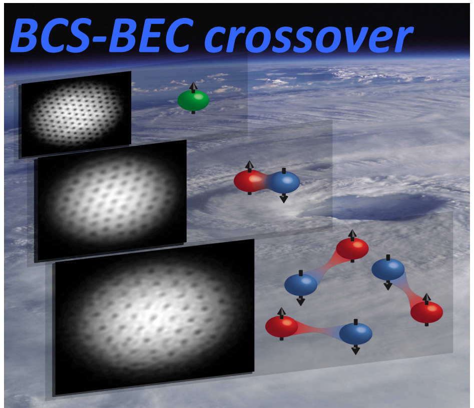
- What are they?

  Ultracold atoms with large scattering length **(*a*)**

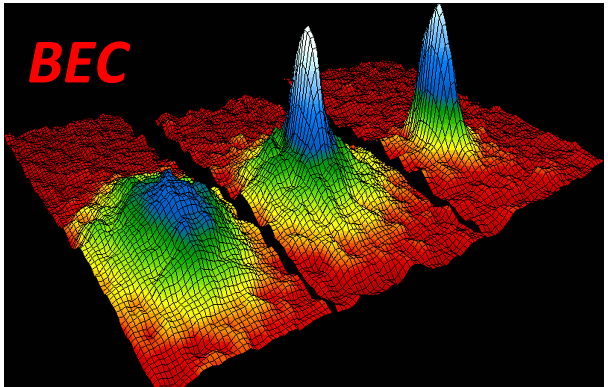
- Ultracold atoms?
  - Alkali atoms: **^{6}Li, ^{40}K**, **^{7}Li, ^{23}Na, ^{39}K, ^{41}K, ^{85}Rb, ^{87}Rb, ^{133}Cs**
  - Trapped in harmonic potential
  - Cooled to $T< 10^{-6}$ K while $T_{QGP}>10^{12}$ K
  - ***a*** controlled by B field



Fermi gas with 2 spin states



Bose gas



Strongly interacting atoms

- Quantum Mechanics at low energy

$$f(k) = \frac{1}{-\frac{1}{a} - ik + \xcancel{\frac{r_s}{2}k^2 + \cdots}}$$

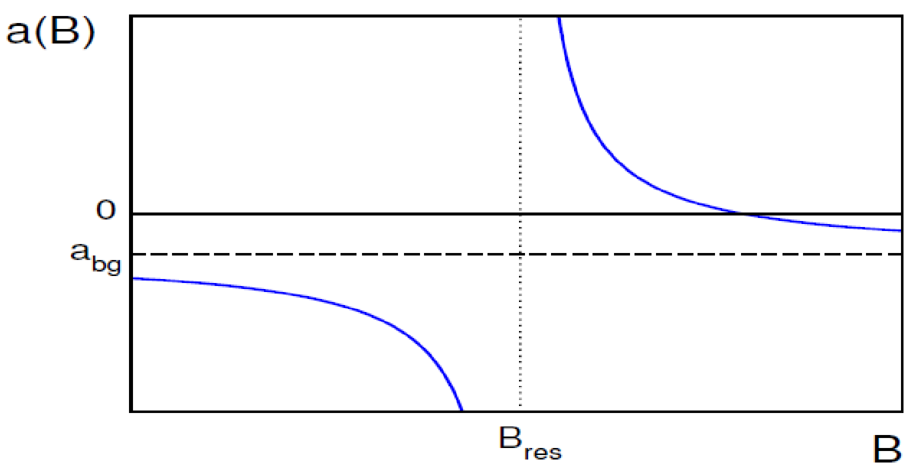
- At very low energy (k << 1/range),

  f(k) depends only on scattering length *a*

- For large |*a*|>>range

  f(k) is nonperturbative for |*a*|*k>1* !

Strongly interacting particles



- **For atoms,**
  - Near Feshbach resonance,
    a varies with the B field !

- **For nucleons,**
  - *a* = -19 fm (n-n) and *a* = +5.3 fm (n-p spin-triplet)
  - *a* varies with quark masses
    - *Tuning u* and *d* masses → *a* = $\pm\infty$ *for the both channels*

      Braaten, Hammer [PRL 2003]
    - Constraint on quark mass variation from BBN
      quark mass → a → binding energies → BBN

      Bedaque, Luu, Platter [PRC 2011]

$$1/m_\pi \approx 1.4 \text{ fm}$$

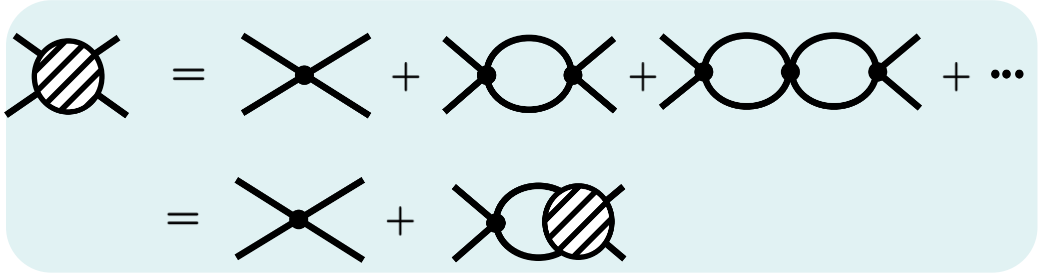
Effective Field Theory

$$\mathcal{L} = \psi_\sigma^\dagger i \frac{\partial}{\partial t} \psi_\sigma - \mathcal{H}$$

$$\mathcal{H} = \tfrac{1}{2} \nabla \psi_\sigma^\dagger \cdot \nabla \psi_\sigma + g\, \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$$

$\sigma = 1, 2$

2-body diagrams
(Lippmann-Schwinger eq.)



Renormalization
with hard cutoff Λ:

$$f(k) = -\frac{1}{1/a + ik}$$

$$\frac{1}{a} = \frac{4\pi}{g} + \frac{2}{\pi}\Lambda$$

Nonperturbative problem!!

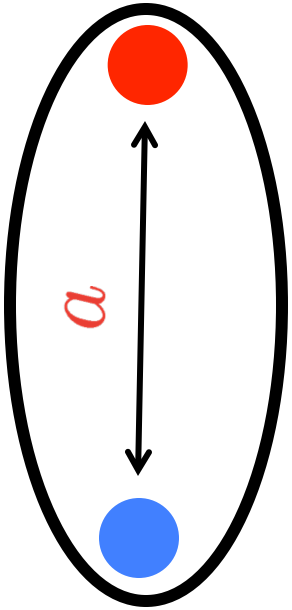
2-body: analytic solution

3- and 4-body: precise numerical solution

Many-body is challenging : Quantum Monte Carlo, Lattice, ...

2-body state

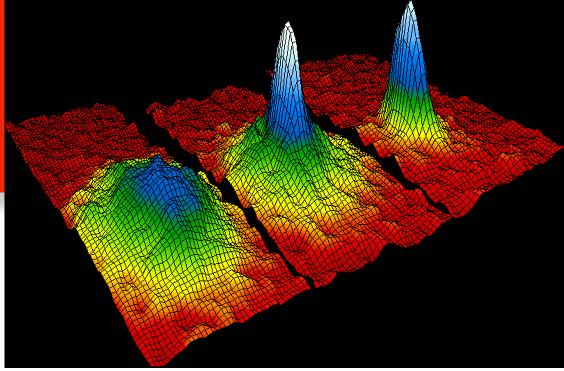
- Low energy amplitude $$f(k) = \frac{1}{-1/a - ik}$$
  - Cross section $$\sigma(k) = \frac{4\pi}{1/a^2 + k^2}$$
  - Molecule (when *a*>0)
    - Binding energy $$E = -\frac{1}{a^2}$$
    - Size $$\sqrt{\langle r^2 \rangle} = a/\sqrt{2}$$



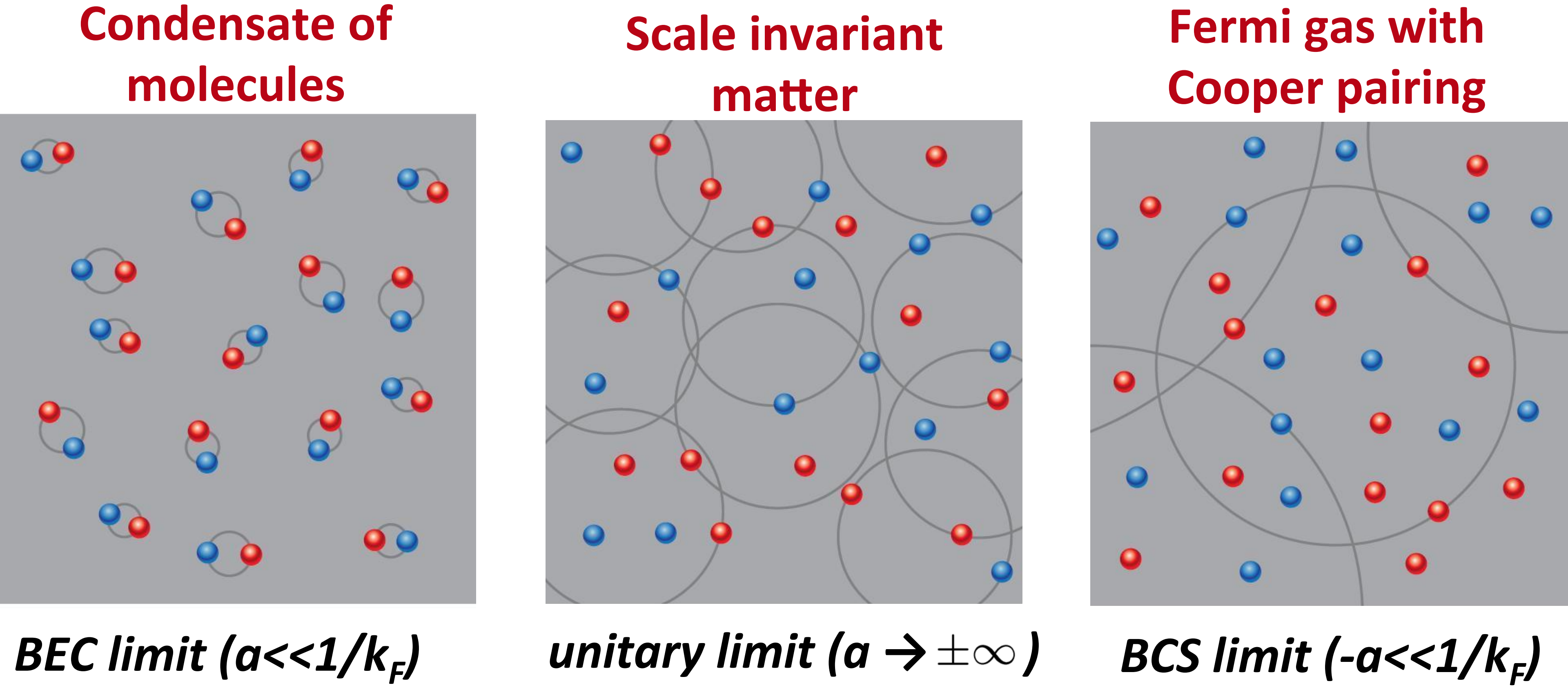
Scale invariance for *a* $\rightarrow \pm\infty$

Of course, free theory (a->0) is scale invariant!

Many-body states



- Identical Bose gas :
  Bose-Einstein Condensate (a>0)
- Fermi gas with 2 spin states



Condensate of molecules

Scale invariant matter

Fermi gas with Cooper pairing

BEC limit ($a<<1/k_F$)

unitary limit ($a \rightarrow \pm\infty$)

BCS limit ($-a<<1/k_F$)

Fermi momentum: $k_F=(3\pi^2 <n>)^{1/3}$

Tan [Annals of Physics 2008]

Universal Relations

for fermions with 2 spin states

- Hold for any state of the system

  e.g. few-body/ many-body, homogeneous/trapped,

  normal gas/superfluid, ground state/nonzero temperature, etc.

- Involve an extensive property of the system called a *contact (C)*
- Are determined by 2-body physics

Universal relations

- Adiabatic relation: variation of energy with scattering length

$$\frac{dE}{da} = \frac{C}{4\pi a^2}$$

Tan 2005

- Tail of the momentum distribution for large $k \gg k_F$

$$n(k) \to C/k^4$$

Tan 2005

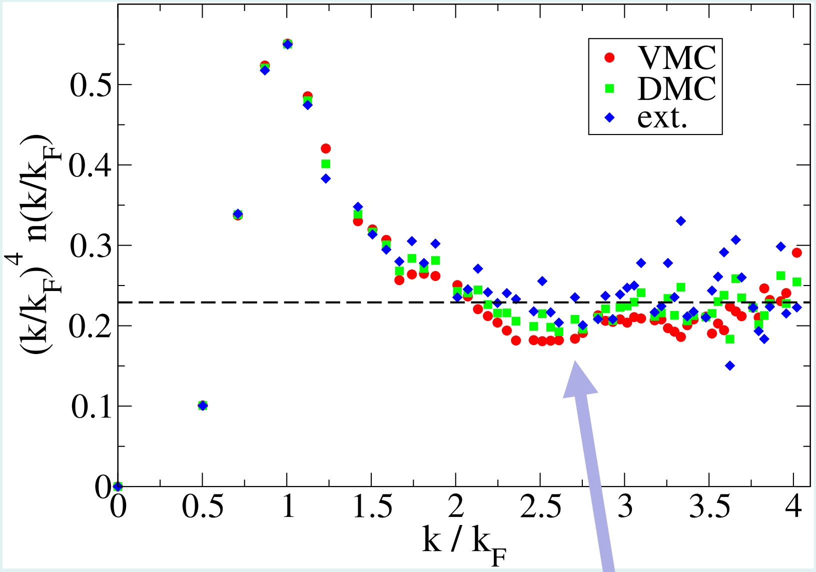
- Many more relations involving C

Virial theorem, Pressure relation, Energy relation by Tan [2005], Structure factors by Son + Thompson [PRA 2010], Hu, Liu + Drummond [EPL 2010], Goldberger + Rothstein[arXiv:1012], Correlation for viscosity by Taylor + Randeria [PRA2010], Enss, Haussmann + Zwerger [Annals Phys. 2011], Hard probe by Nishida [arXiv:1110], and more

C is a central quantity *relating various observables!*

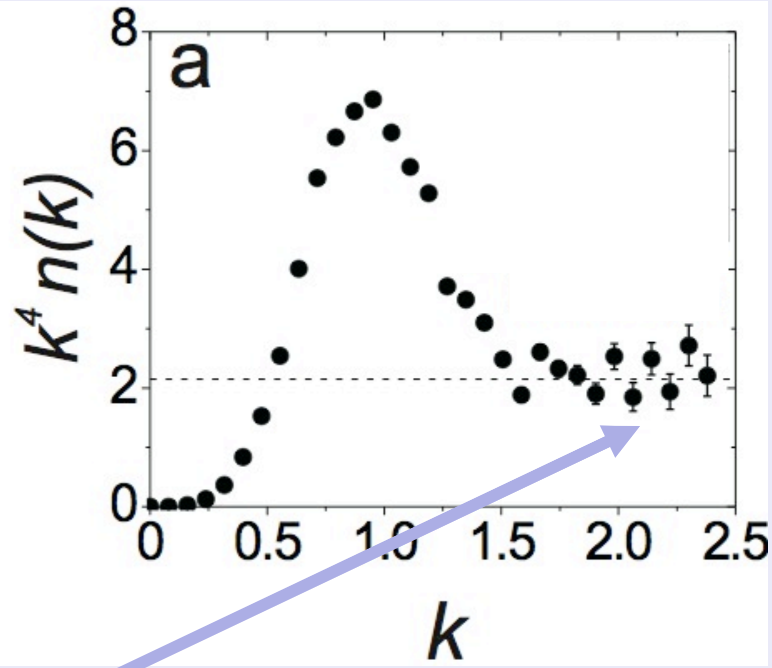
Verifying universal relation

Quantum Monte Carlo



Gandolfi, Schmidt, Carlson [PRA 2011]

Experiment



JILA group [PRL 2010]

scaled by Fermi momentum

$k_F=(3\pi^2 <n>)^{1/3}$

Plateau (1/k^4 tail) above 2 k_F!

Universal relations

- **Adiabatic relation** $C = 4\pi a^2 \frac{dE}{da}$
  - operational definition
  - contact density for given $\mathcal{H}_{\rm int} = g\, \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1$

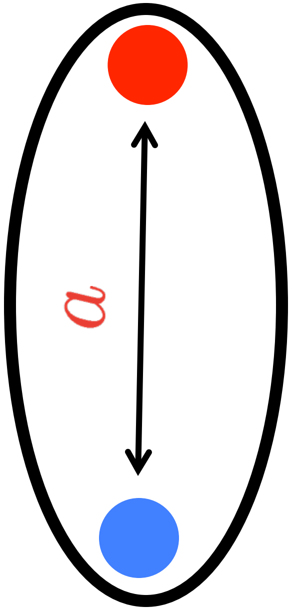
$$\frac{d\mathcal{E}}{da} = \langle \frac{d}{da} \mathcal{H}_{\rm int} \rangle = \frac{1}{4\pi a^2} \langle g^2\, \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1 \rangle$$

- **The contact *C***
  - is an extensive thermodynamic quantity conjugate to *1/a*
  - measures a probability for 2 atoms being close together
  - depends on the state
  - depends on scattering length (*a*), density (n), temperature (T), …

Relations for dimer

- Dimer contact : $C = 4\pi a^2 \frac{dE}{da} = \frac{8\pi}{a}$ $E = -\frac{1}{a^2}$

- Dimer wavefunction: $\tilde{\psi}(k) = \frac{\sqrt{8\pi/a}}{k^2+1/a^2}$



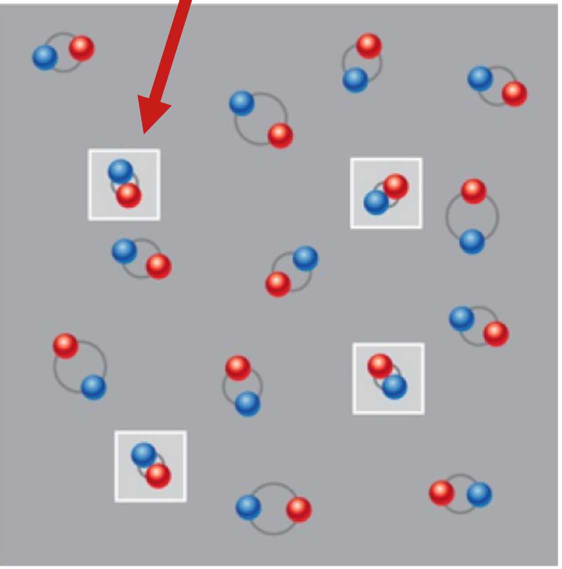
- Tail of momentum distribution:

$$n(k) = \tilde{\psi}^\dagger \tilde{\psi}(k) \rightarrow \frac{8\pi/a}{k^4}$$

Many-body states

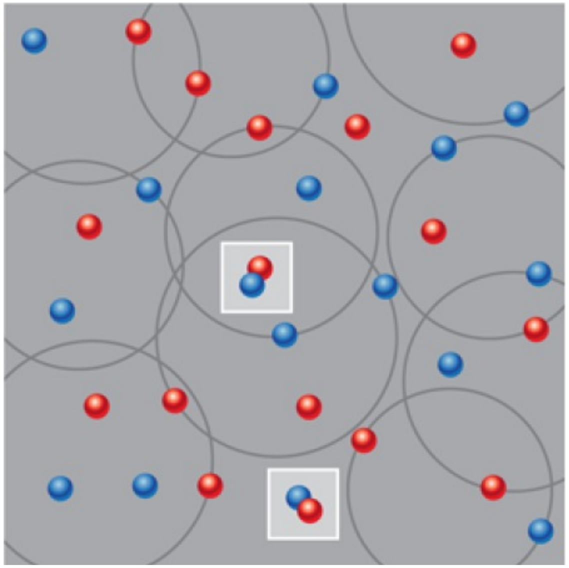
- **Contact density (C/V)**

for homogeneous gas at T=0



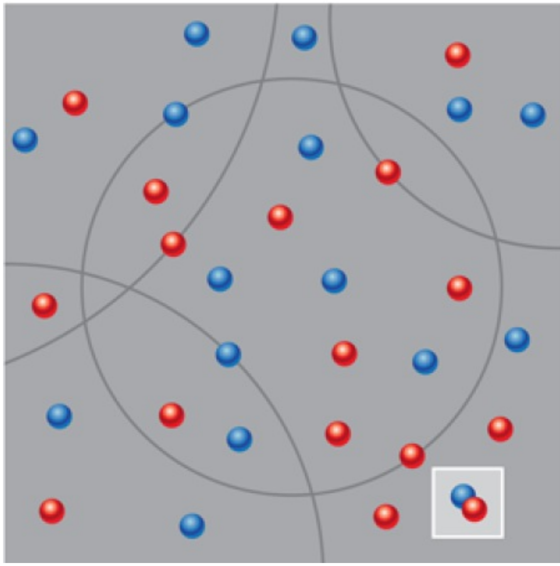
BEC limit ($a<<1/k_F$)

$$8\pi/a \times n/2$$



unitary limit ($a\to \pm\infty$)

$$10.51(3)\, n^{4/3}$$



BCS limit ($-a<<1/k_F$)

$$4\pi^2 a^2 n^2$$

Gandolfi, Schmidt, Carlson
[PRA 2011]

Proof of universal relation

- Operator Product Expansion

$$\hat{O}_A(\boldsymbol{r})\hat{O}_B(\mathbf{0}) = \sum_i c_i(\boldsymbol{r})\,\hat{O}_i(\mathbf{0})$$

  - lowest scaling dimension operators

$\psi_1^\dagger \psi_1,$	$\psi_1^\dagger \vec{\nabla} \psi_1,$	$\psi_1^\dagger i \frac{\partial}{\partial t} \psi_1,$	$\psi_1^\dagger \nabla^2 \psi_1,$	$g^2 \psi_1^\dagger \psi_2^\dagger \psi_1 \psi_2,$	$\cdots$
3	**4**	**5**	**5**	**6-2=4**	

  - Determine Wilson coeff. by matching few-body matrix elements
    Few-body problem can be solved exactly!
  - Operator identity is valid for any states → Universal relation

OPE reveals aspects of **many-body physics**
controlled by few-body physics!!

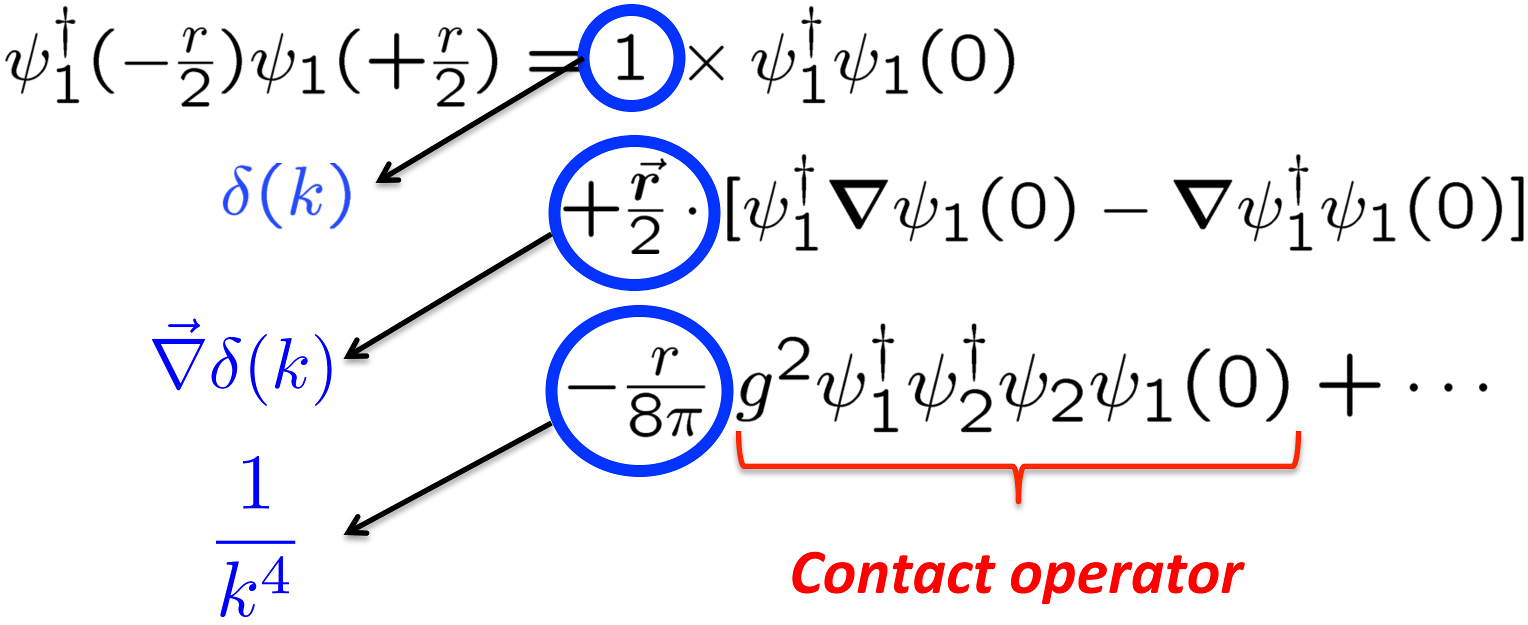
Operator product expansion

Braaten and Platter [PRL 2008]

$$n(k) = \langle \tilde{\psi}_1^\dagger \tilde{\psi}_1(k) \rangle$$

$$= \int_R \int_r e^{-ik\cdot r} \langle \psi_1^\dagger(R - \tfrac{1}{2}r)\psi_1(R + \tfrac{1}{2}r) \rangle$$

After matching for 1- and 2-atom states ...



$$\psi_1^\dagger(-\tfrac{r}{2})\psi_1(+\tfrac{r}{2}) = \boxed{1} \times \psi_1^\dagger\psi_1(0)$$

$$\boxed{+\frac{\vec{r}}{2}\cdot}[\psi_1^\dagger \nabla\psi_1(0) - \nabla\psi_1^\dagger\psi_1(0)]$$

$$\boxed{-\frac{r}{8\pi}} \underbrace{g^2\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1(0)}_{\text{Contact operator}} + \cdots$$

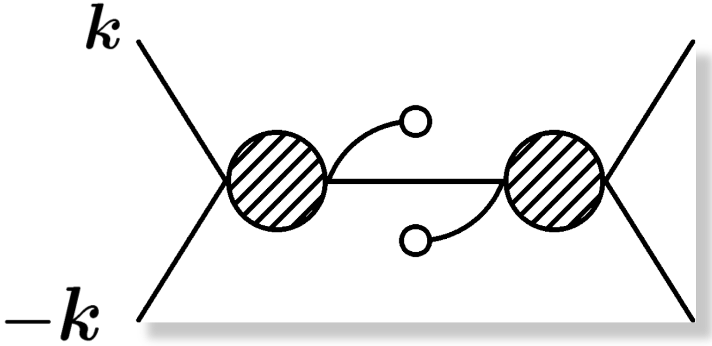
$$\delta(k)$$

$$\vec{\nabla}\delta(k)$$

$$\frac{1}{k^4}$$

Matching for 2-atom State

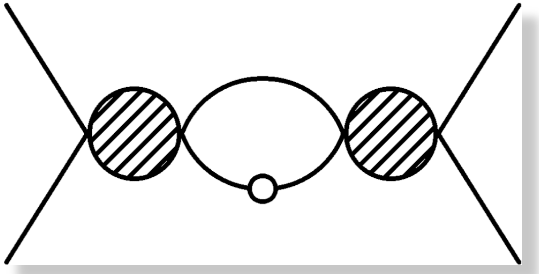
$\psi_1^\dagger(-\frac{r}{2})\psi_1(+\frac{r}{2})$



$i2\pi f(k)^2\,\frac{e^{ikr}}{k}$

$\psi_1^\dagger\psi_1(0)$

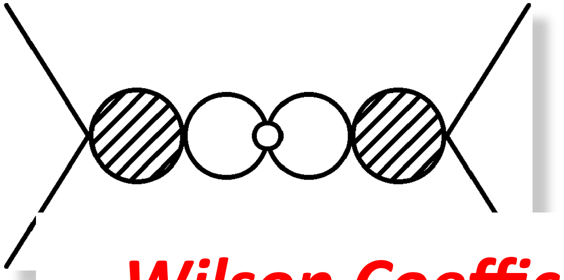
$\psi_1^\dagger\nabla^j\psi_1(0) - \nabla^j\psi_1^\dagger\psi_1(0)$



$i2\pi f(k)^2\,\frac{1}{k}$

0

$g^2\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1(0)$



$16\pi^2 f(k)^2$

Wilson Coefficient -> -r /(8π)

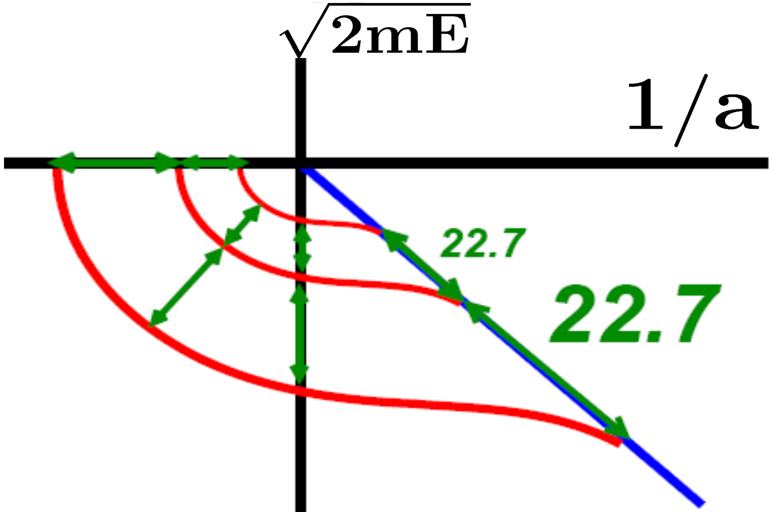
Identical Bosons

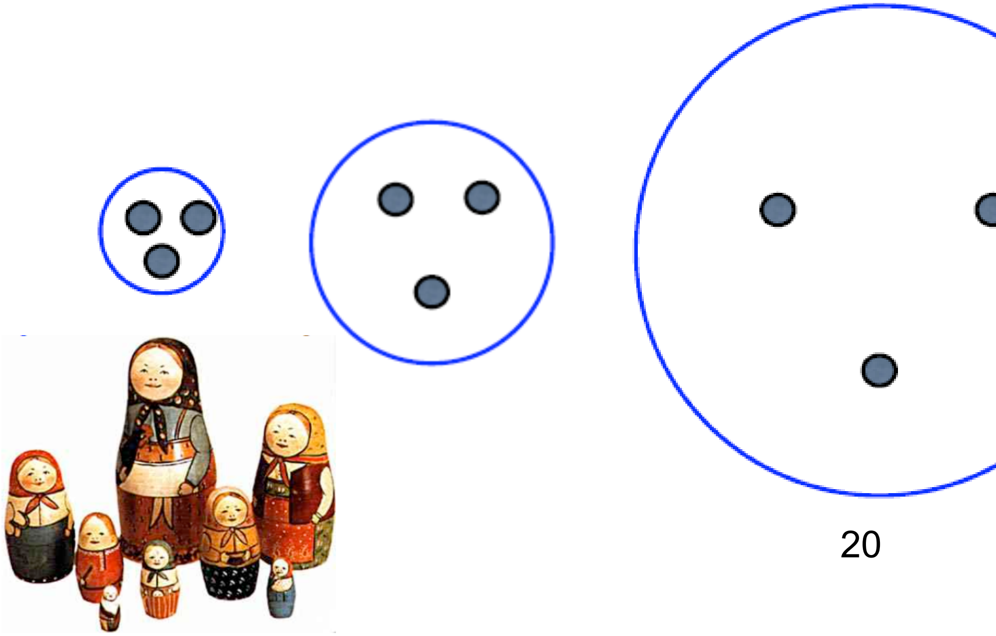
2- and 3-body physics

- **2-body :** Similar to fermions except for statistics
  Scale invariance when $a \to \pm\infty$
- **3-body :** Broken to discrete scale invariance !!!
  Log-periodic behavior !!!

} **Efimov physics**

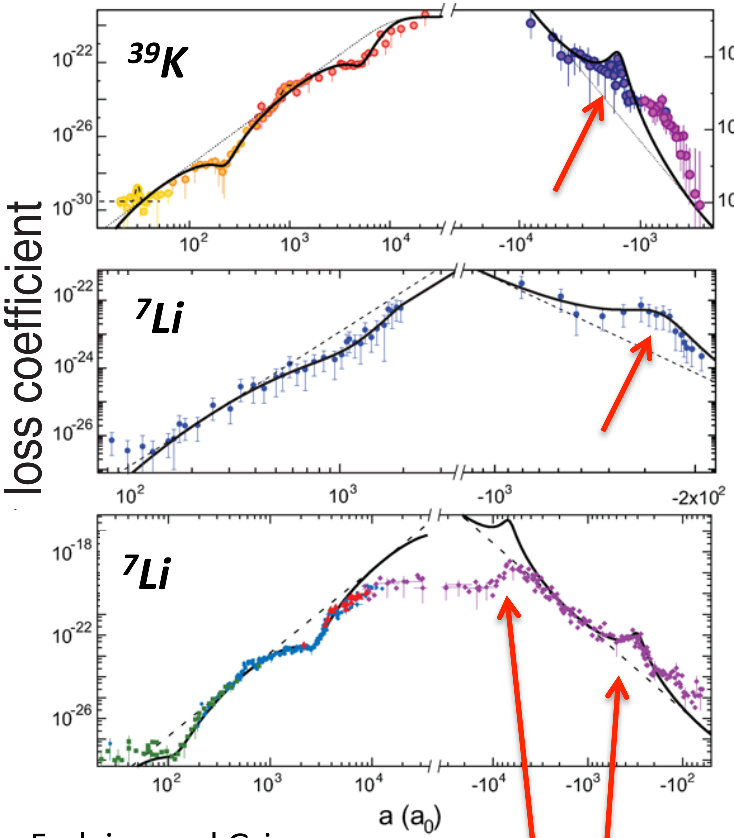
- **Efimov trimers:** $E_{n+1}/E_n = 22.7^2$



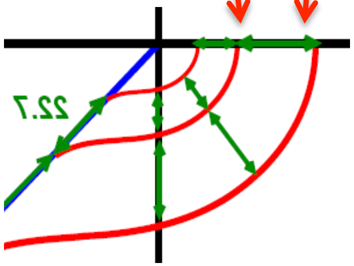


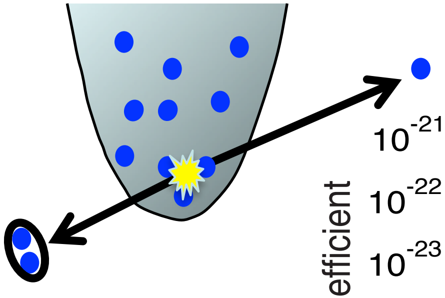
Efimov physics in atom loss

identical bosons

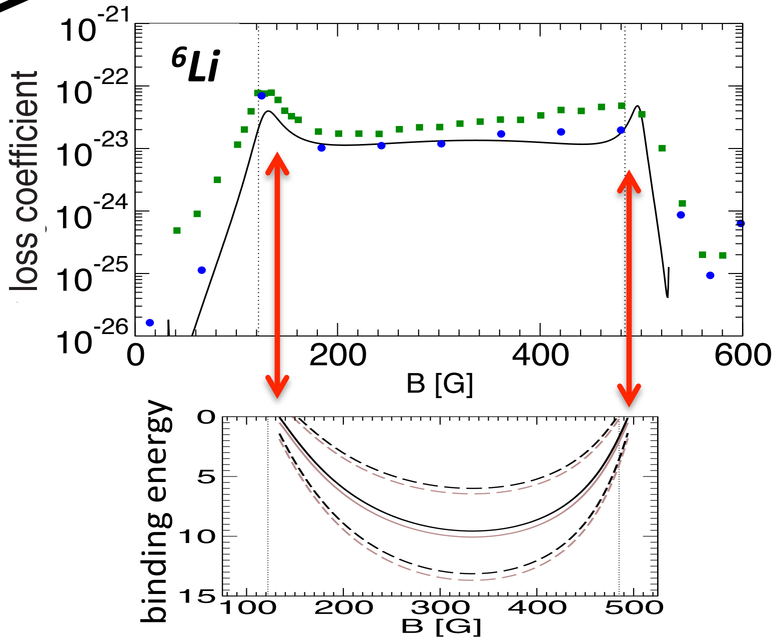


Ferlaino and Grimm
[Physics 2010]





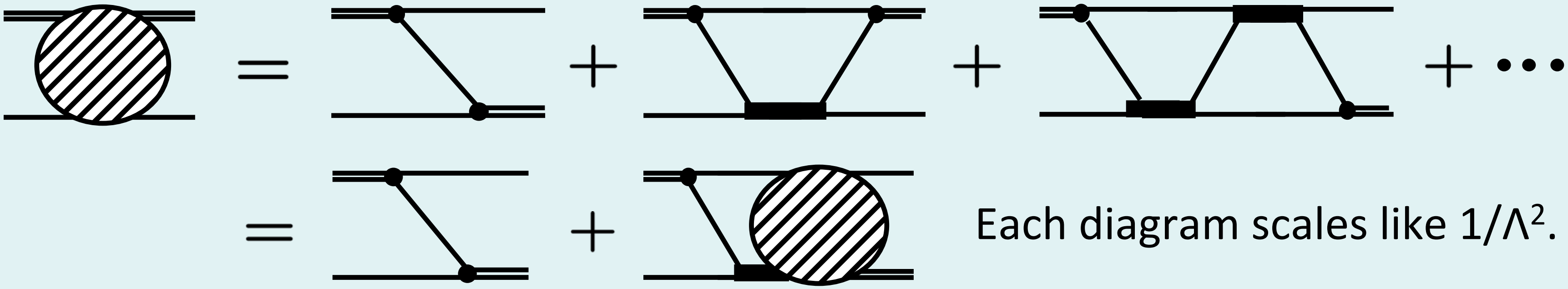
Fermions
with 3 spin states



Huckans et al, Ottenstein et al,
Braaten, Hammer, DK, Platter
[PRL 2008,2009]

EFT for bosons

- Interaction $\frac{g_2}{4}\left(\psi^\dagger\psi\right)^2$
- Integral equation for atom-diatom amplitude



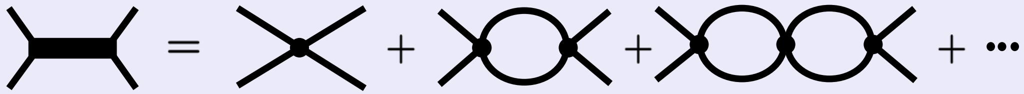
Each diagram scales like 1/Λ^2.

Their sum is finite
but has a nontrivial Λ dependence.

$$\propto \frac{\cos[s_0 \ln(k/\Lambda)]}{k}$$

Log-periodic behavior !!!

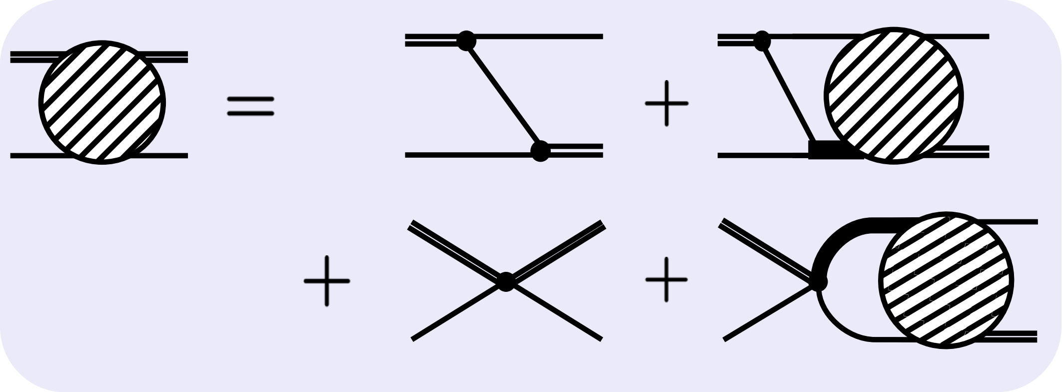
$s_0 \approx 1.006$



EFT for bosons

Bedaque, Hammer, and van Kolck [PRL 1999]

- interactions $\frac{g_2}{4}\left(\psi^\dagger\psi\right)^2 + \frac{g_3}{36}\left(\psi^\dagger\psi\right)^3$
- integral equation for atom-diatom amplitude



- renormalization $g_3 = -9\frac{g_2^2}{\Lambda^2}H_{BHvK}$

$$H_{BHvK} = -h_0\frac{\sin[s_0\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$

$h_0 \approx 0.879$

Braaten, DK , Platter [PRL 2011]

$s_0 \approx 1.006$

renormalized 3-body parameter

Braaten, DK , Platter
[PRL 2011]

Universal Relations

from 3-body physics

- Hold for any state of the system
- Involve 2- and *3-body contacts*

$$C_2 = \int_R \frac{g_2^2}{4} \langle \psi^\dagger \psi^\dagger \psi \psi(R) \rangle$$

$$C_3 = - \int_R \frac{g_2^2 \, H'}{8\Lambda^2} \langle \psi^\dagger \psi^\dagger \psi^\dagger \psi \psi \psi(R) \rangle$$

- Are characterized by log-periodic behavior
  (Efimov physics)

Relations for bosons

- Adiabatic relations: 2-body and 3-body contacts

$$a\frac{dE}{da} = \frac{C_2}{8\pi a} \qquad \kappa_*\frac{dE}{d\kappa_*} = -2C_3$$

κ_* is a binding mom. of trimer at unitarity and is chosen as a 3-body parameter.

- Tail of n*(k)* from OPE by

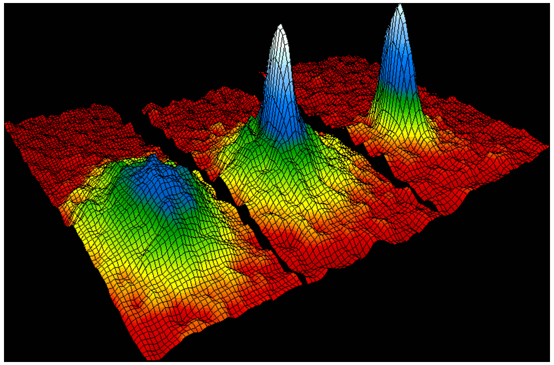
matching for 1-,2-, and 3- body states

$$\boldsymbol{n(k) \to \frac{C_2}{k^4} + F(k)\,\frac{C_3}{k^5}}$$

$$F(k) = 89.3\sin[2s_0\log(k/\kappa_*) - 1.34]$$

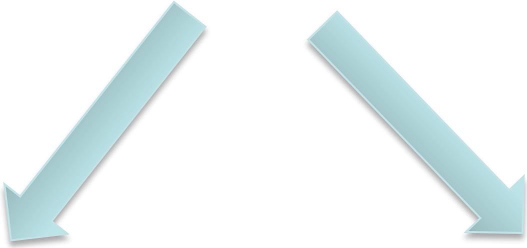
Log-periodic !!!

Many-body states



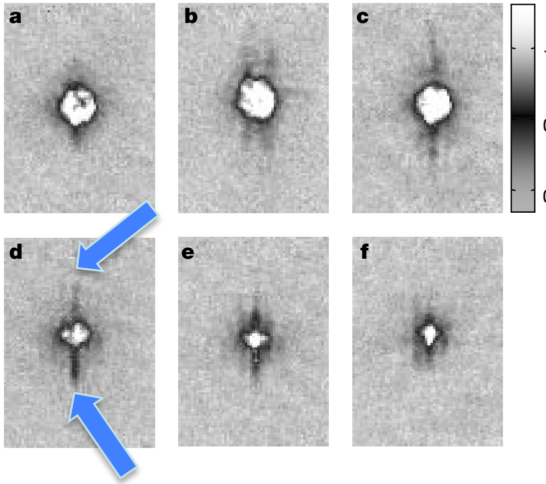
BEC ($a<<1/k_F$)

?? ($a \rightarrow \pm\infty$)



Thermal gas at unitarity

Degenerate unitary Bose gas



($-a<<1/k_F$)
Collapse by producing "dijets"

Atom loss rates

- dilute BEC (a>0)
  - 3-body loss rate: $dn/dt \propto a^4 n^3$

    Catastrophic loss rate as *a*→ infinity!
- thermal gas at unitarity

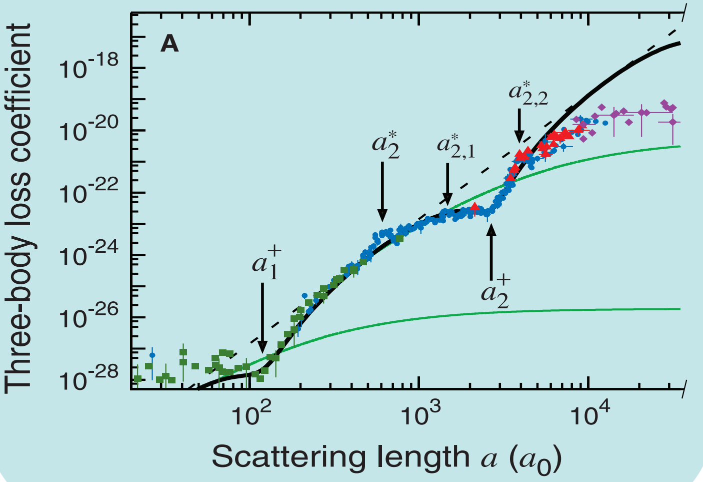
  Salomon group and Hadzibabic group [PRL 2013]
  - 3-body loss rate: $dn/dt \propto \lambda_T^4 n^3$

    $$\lambda_T = \sqrt{2\pi\hbar^2/mk_BT}$$
- **degenerate unitary Bose gas**

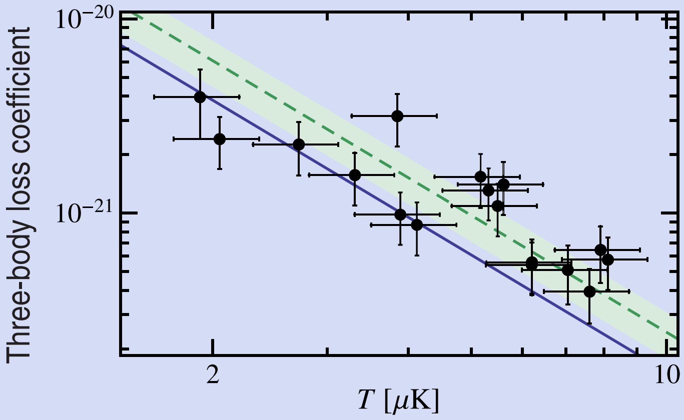
  very recently *by JILA group [arXiv:1308.3696]*
  - n(k) evolves and saturates before significant loss.

    t_{loss}= 0.63 ms >> $t_{\text{saturation}}$=0.1 ms
  - universal scaling in n(k) and $t_{\text{saturation}}$
  - JILA's conclusion: equilibrated metastable state!

Rice Group [Science 2009]



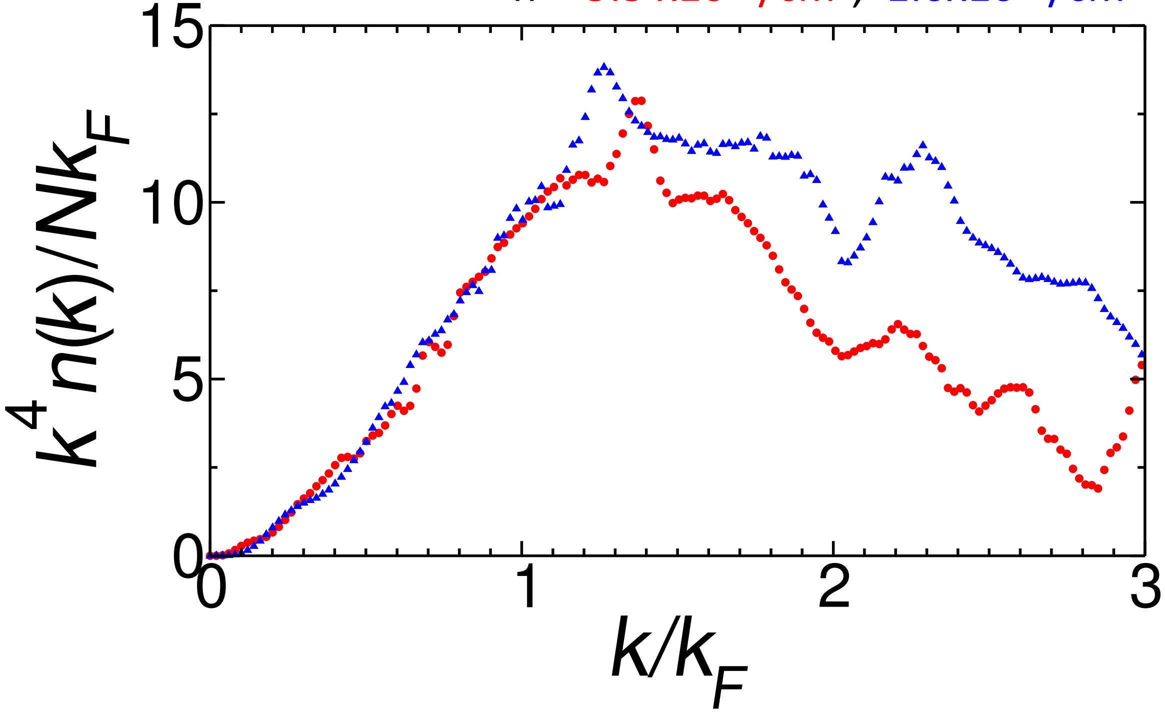
Salomon group[PRL 2013]



Unitary Bose gas

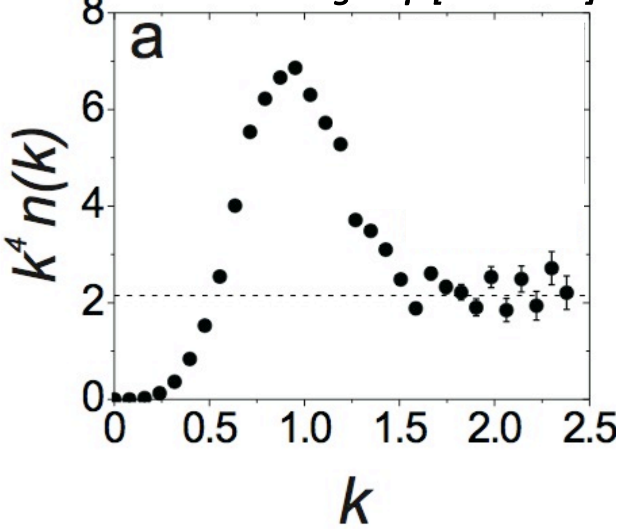
JILA group [arXiv:1308.3696]

<n>=5.5 x10^{12}/cm^3, 1.6x10^{12}/cm^3



Fermi gas

JILA group [PRL 2010]



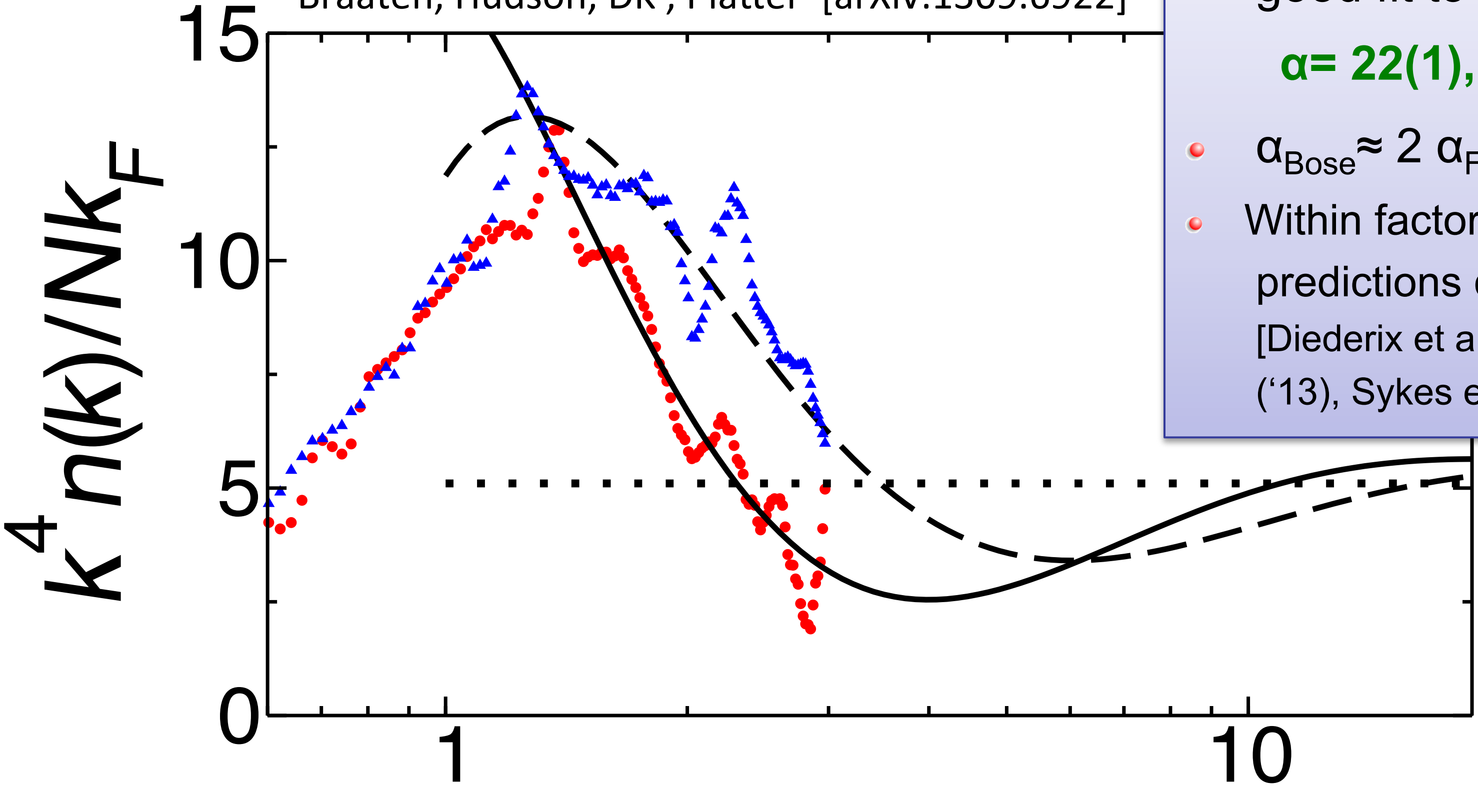
- Universal scaling for $k < k_F$
- Scaling violation for $k > k_F$!
- Why no plateau for $k > 2\ k_F$?

$k_F=(6\pi^2 <n>)^{1/3}$

- Efimov effect gives log-periodic scaling violations! $\sin[2s_0\log(k/\kappa_*)-1.34]$
    - C_3 term with $1/k^5$ tail. Absent in Fermi gas.
- Contact density at unitarity: $C_2 = \alpha\, n^{4/3}$ $C_3 = \beta\, n^{5/3}$
- 2 parameter fit to data in $1.5<k/k_F<3$.

$$n(k) \to \frac{C_2}{k^4} + F(k)\frac{C_3}{k^5}$$

Braaten, Hudson, DK , Platter [arXiv:1309.6922]



- good fit to data (solid line)

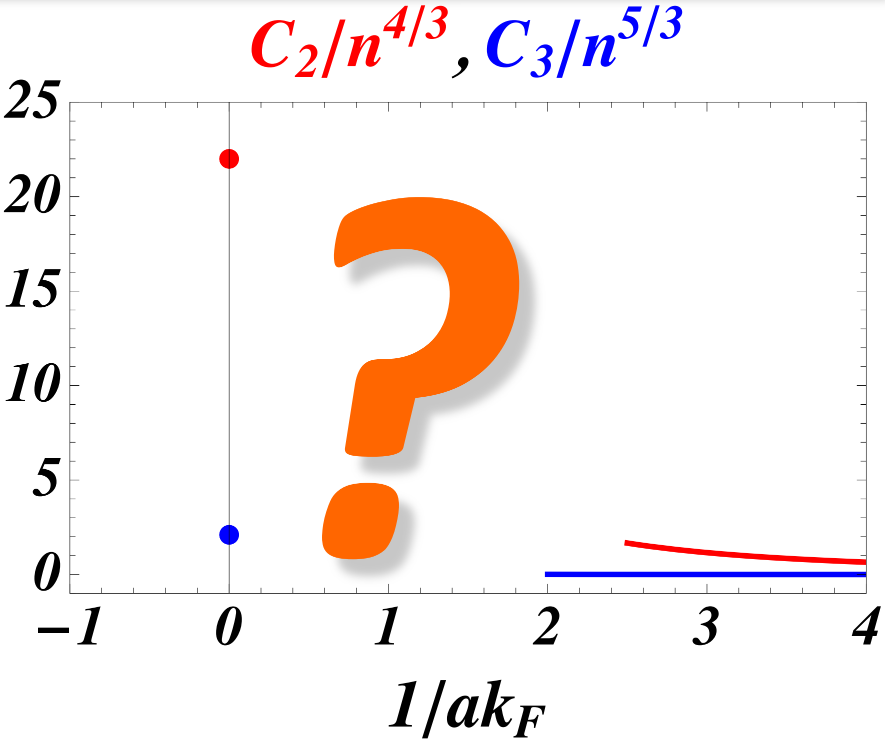
  α= 22(1), β=2.1(1)
- $\alpha_{Bose} \approx 2\, \alpha_{Fermi}$
- Within factors 2 of theoretical predictions α =10, 32, 12 [Diederix et al ('11), Heugten and Stoof ('13), Sykes et al ('13)]

Efimov effect plays an important role in understanding unitary Bose gas!

Contact densities

Braaten, Hudson, DK , Platter [arXiv:1309.6922]

	$\mathcal{C}_2/n^{4/3}$	$\mathcal{C}_3/n^{5/3}$
dilute BEC ($na^3<<1$)	$16\pi^2(na^3)^{2/3}$	$2.8\,(na^3)^{4/3}$
thermal gas at unitarity ($n\lambda_T^3<<1$)	$32\pi(n\lambda_T^3)^{2/3}$	$3\sqrt{3}s_0\,(n\lambda_T^3)^{4/3}$
Unitary gas ($T<T_C$)	$22(1)$	$2.1(1)$



- C_3 is parametrically suppressed for dilute BEC and for unitary thermal gas. Not for unitary gas below T_C!
- The contacts are unknown for $ak_F > \mathcal{O}(1)$. Well defined? Continuous or not?
  - Accessible by JILA group in experiment !
  - No available many-body simulations.

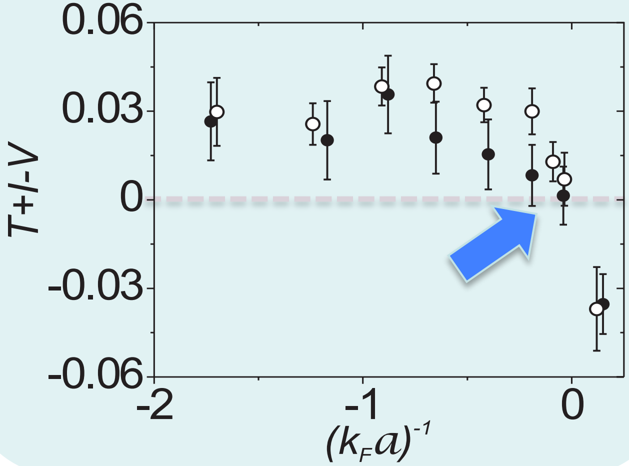
Virial theorem

$$(T+U)-V=-\frac{\hbar^2}{16\pi m a}C_2-\frac{\hbar^2}{m}C_3$$

Werner
[PRA 2008]

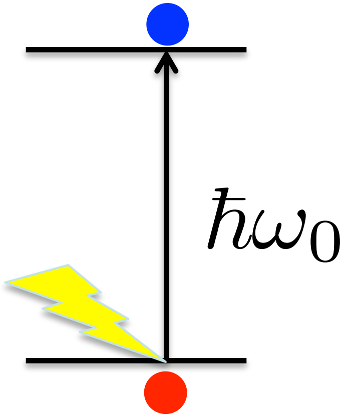
- Energies: T= kinetic, U= interaction, and V= potential.
- **T+U–V=0** for unitary Fermi gas
  - No C_3 term in Fermi gas
  - C_2 term vanishes
  - Verified by JILA group. [PRL 2010]

Fermi gas



- **T+U–V≠0** for unitary Bose gas
  - C_3 can be determined by measuring T+U and V!

Radio frequency spectroscopy



- Transition between hyperfine states by a rf photon
  - Probe on time scale much shorter than that for atom loss
- Rate without final- state interactions

Braaten, DK and Platter 2010, 2011

$$\Gamma(\omega) \to \Omega^2 \left[\frac{1}{4\pi\omega^{3/2}} C_2 + \frac{G(\omega)}{2\omega^2} C_3 \right]$$

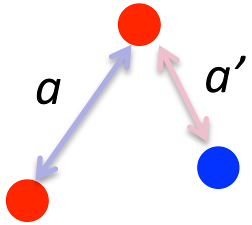
ω = frequency shift respect to the resonance ω_0

$$G(\omega) = 9.23 - 13.6 \sin[s_0 \ln(\omega/\kappa_*^2) + 2.66]$$

Log-periodic!

- With final-state interactions

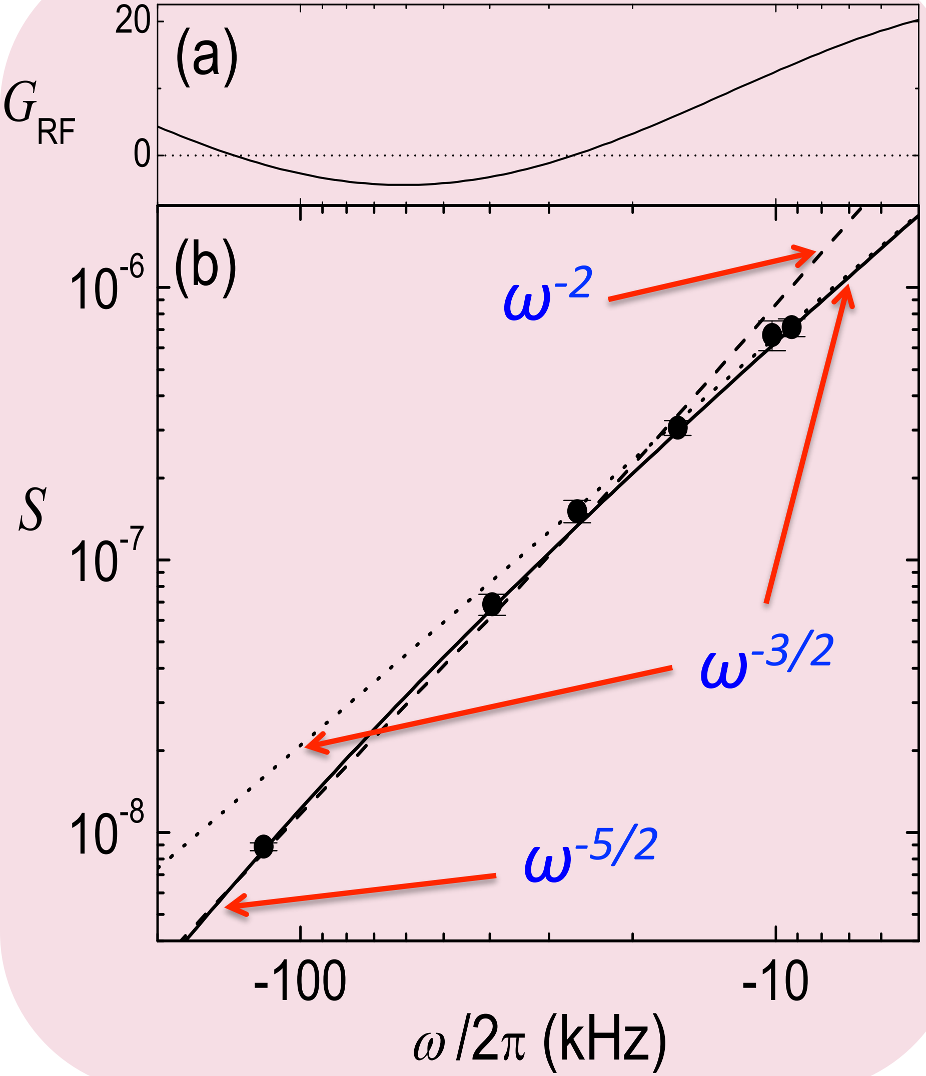
$$\Gamma(\omega) \to \Omega^2 \left[\frac{(1/a - 1/a')^2}{4\pi\omega^{3/2}\,(1/a'^2 + \omega)} C_2 + ?\, C_3 \right]$$



- Wilson coeff. of C_3 needs to be calculated!!

rf spectroscopy for ^{85}Rb BEC

Wild et al, PRL 108 2012



- **Only with C_2 term**
  Solid/Dotted line: rate with/ without final-state interaction
- **With C_2 and C_3 terms**
  Dashed line:
  rate for $C_3/N_0 = 0.1\ \mu m^{-2}$
- *C_3 effect is not identified!*
  upper limit: $C_3/N_0 < 0.07\ \mu m^{-2}$
- Consistent with our estimate
  $C_3/N_0 = 2.8\ a^4 \langle n^2 \rangle$
  $\approx$ (upper limit)/30
- C_3 contribution should be visible for larger *a*!

a=982 a0, <n>=10^{13} cm^{-3}

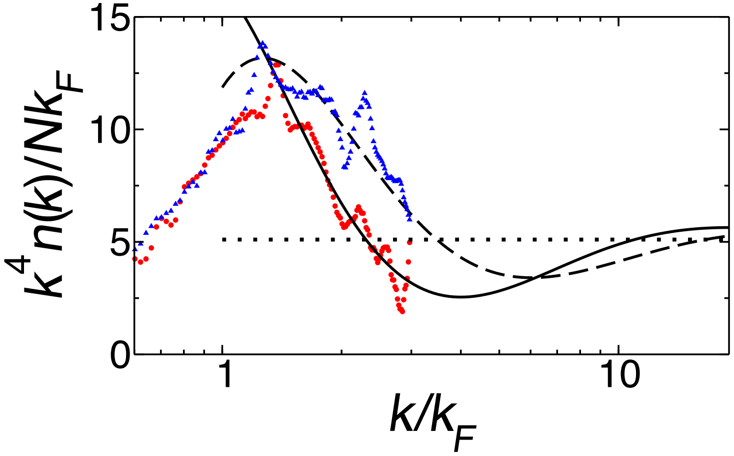
Summary

- **Universal relations** for strongly interacting atoms
- **OPE** is powerful

  Many-body physics controlled by few-body physics

- **Contacts** are central quantities
  - C_2 for Fermi gas with 2 spin states
  - C_2 and C_3 for Bose gas, Fermi gas with 2>spin states, and etc.
- **Efimov effect** is a key ingredient

  to understand **unitary Bose gas!**



Nov 14, 2013