Toward a self-consistent and unitary nuclear reaction network

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Outline

- **Overview: LANL light nuclear reaction program**
- **Unitarity: SBBN & beyond, reaction networks, R-matrix, ^{17}O,^{9}B examples**
- **Recent related development: EFT ⟷ R-matrix ($a_c \to 0 \quad \forall c$)**
- **Future work & conclusion**

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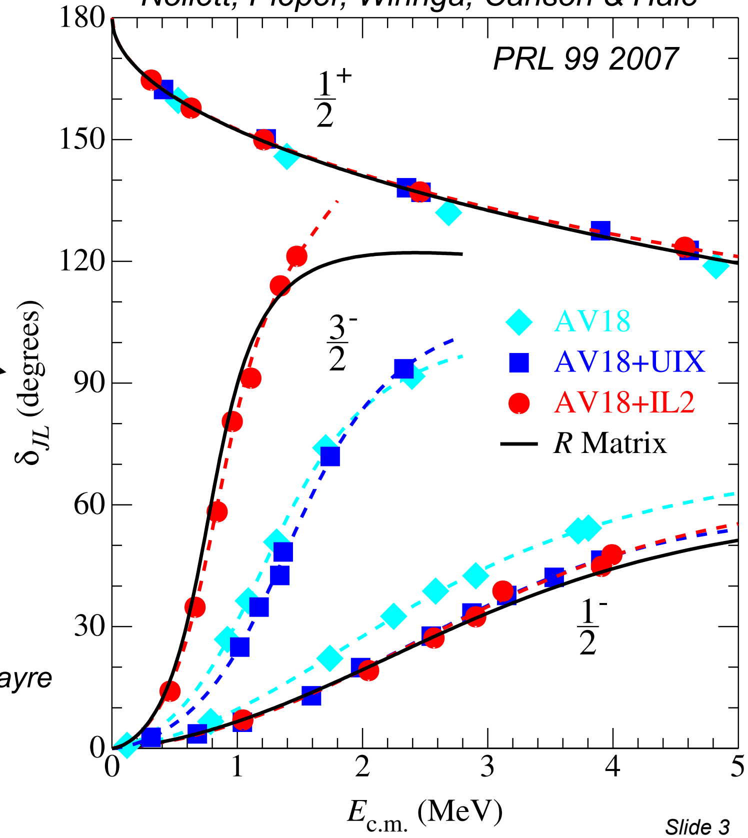
Light nuclear reaction program @ LANL

- **Motivation**
  - →Data sets: σ, σ(θ), $A_i(\theta)$, $C_{i,j}$, $K_i^{j'}$, Σ(γ),…→ T matrix → resonance spectrum
  - →***Unitary*** parametrization of compound nuclear system
  - →Applications: astrophysical, nuclear security, inertial confinement fusion, criticality safety, charge-particle transport, nuclear data (ENDF, ENSDF)
- **Ab initio**
  - →Variational MC; Green's function MC
  - →GFMC [PRL **99,** 022502 (2007)]
    - n-^{4}He phase shifts
    - comparison GFMC/R-matrix
  - →challenge: multichannel
    - *eg*. nα→nα, nα→dt & dt→dt
- **Phenomenology**
  - →R matrix (2→2 body scatt/reacs)
  - →3–body: isobaric models, sequential decay
    - *R-Matrix description of particle energy spectra produced by low-energy T + T reactions; w/G. Hale, C. Brune (OU), D. Sayre & J. Caggiano (LLNL)*

Nollett, Pieper, Wiringa, Carlson & Hale



EDA Analyses of Light Systems

A	System	Channels	Energy Range (MeV)
2	N-N	p+p; n+p, γ+d	0-30 0-40
3	N-d	p+d; n+d	0-4
4	^{4}H ^{4}Li	n+t p+^{3}He	0-20
	^{4}He	p+t n+^{3}He d+d	0-11 0-10 0-10
5	^{5}He	n+α d+t $^{5}He+\gamma$	0-28 0-10
	^{5}Li	p+α d+^{3}He	0-24 0-1.4

Analyses of Light Systems, Cont.

A	System (Channels)
6	^{6}He (^{5}He+n, t+t); ^{6}Li (d+^{4}He, t+^{3}He); ^{6}Be (^{5}Li+p, ^{3}He+^{3}He)
7	^{7}Li (t+^{4}He, n+^{6}Li); ^{7}Be (γ+^{7}Be, ^{3}He+^{4}He, p+^{6}Li)
8	^{8}Be (^{4}He+^{4}He, p+^{7}Li, n+^{7}Be, p+^{7}Li*, n+^{7}Be*, d+^{6}Li)
9	^{9}Be (^{8}Be+n, d+^{7}Li, t+^{6}Li); ^{9}B (γ+^{9}B, ^{8}Be+p, d+^{7}Be, ^{3}He+^{6}Li)
10	^{10}Be (n+^{9}Be, ^{6}He+α, ^{8}Be+nn, t+^{7}Li); ^{10}B (α+^{6}Li, p+^{9}Be, ^{3}He+^{7}Li)
11	^{11}B (α+^{7}Li, α+^{7}Li*, ^{8}Be+t, n+^{10}B); ^{11}C (α+^{7}Be, p+^{10}B)
12	^{12}C (^{8}Be+α, p+^{11}B)
13	^{13}C (n+^{12}C, n+^{12}C*)
14	^{14}C (n+^{13}C)
15	^{15}N (p+^{14}C, n+^{14}N, α+^{11}B)
16	^{16}O (γ+^{16}O, α+^{12}C)
17	^{17}O (n+^{16}O, α+^{13}C)
18	^{18}Ne (p+^{17}F, p+^{17}F*, α+^{14}O)

26 tabulated analyses

LANL R-matrix light element IAEA standards: status

n+p: no new work since 2008; cross sections in pretty good shape below 30 MeV; main need is extension to higher energies (150-200 MeV), with associated covariances.

n+^{3}He: Some new work, especially for n+^{3}He capture; ^{3}He(n,n)^{3}He scattering data re-worked by Drosg and Lisowski – could be used in a new analysis of the ^{4}He system.

n+^{6}Li: Some new work on ^{7}Li system around 2008 included new LANSCE measurements of ^{6}Li(n,t)^{4}He differential cross section – was included in ENDF/B-VII.1 above 1 MeV. Cross sections should be re-visited below 1 MeV, although there may not be any new data.

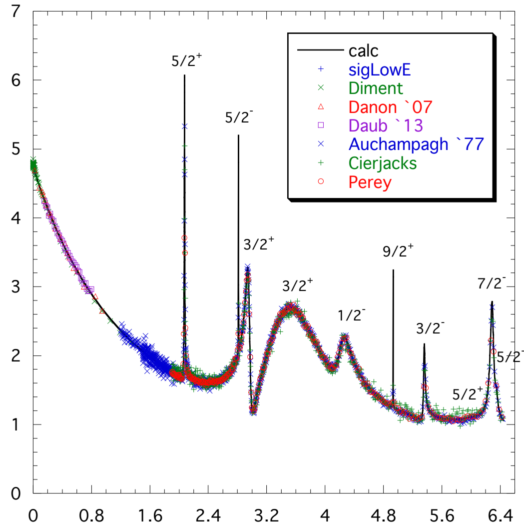
n+^{10}B: No new work since last standards evaluation. New data from Geel, Ohio U. [including first measurement of the (n,p) cross section]. **R**-matrix analysis for ^{11}B system should be extended above 1 MeV.

n+^{12,13}C: Considerable new work in the last couple of years. New data for ^{12}C(n,n'γ) from Geel, Los Alamos changed the (n,n') cross sections. Isotopic evaluations combined to make more accurate standard evaluation for C-0.

Notation: X(a,b)Y
X-target Y-recoil
a-projectile b-detected

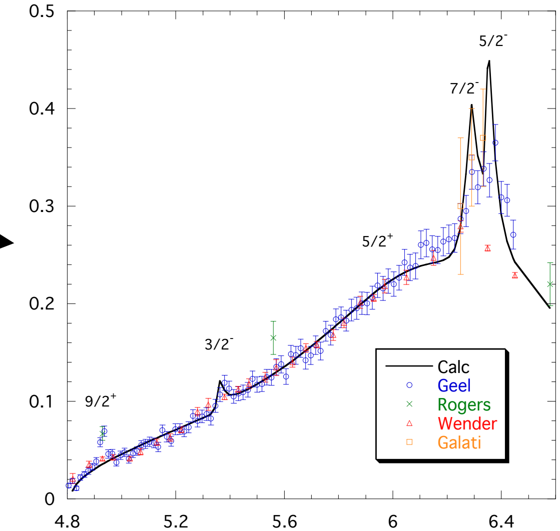
$^{13,14}C$ system analyses: σ_T (b) vs. E_n (MeV)

n+^{12}C Total Cross Section

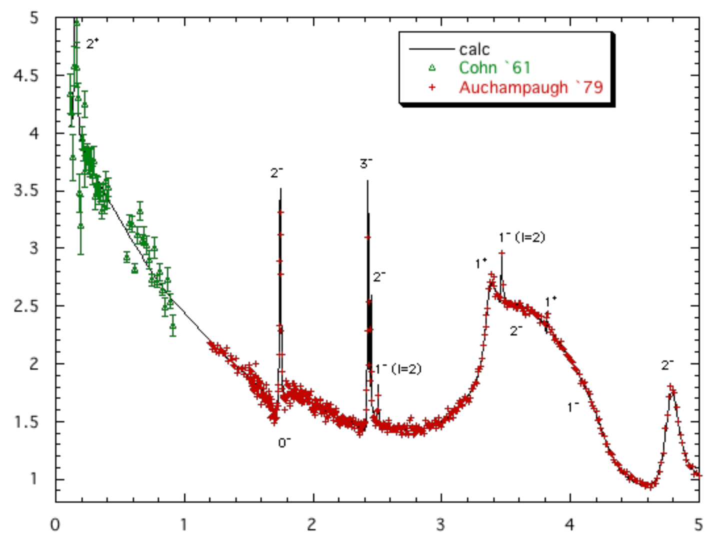


Two-channel analysis

^{12}C(n,n') Cross Section



n+^{13}C Total Cross Section



Single-channel analysis

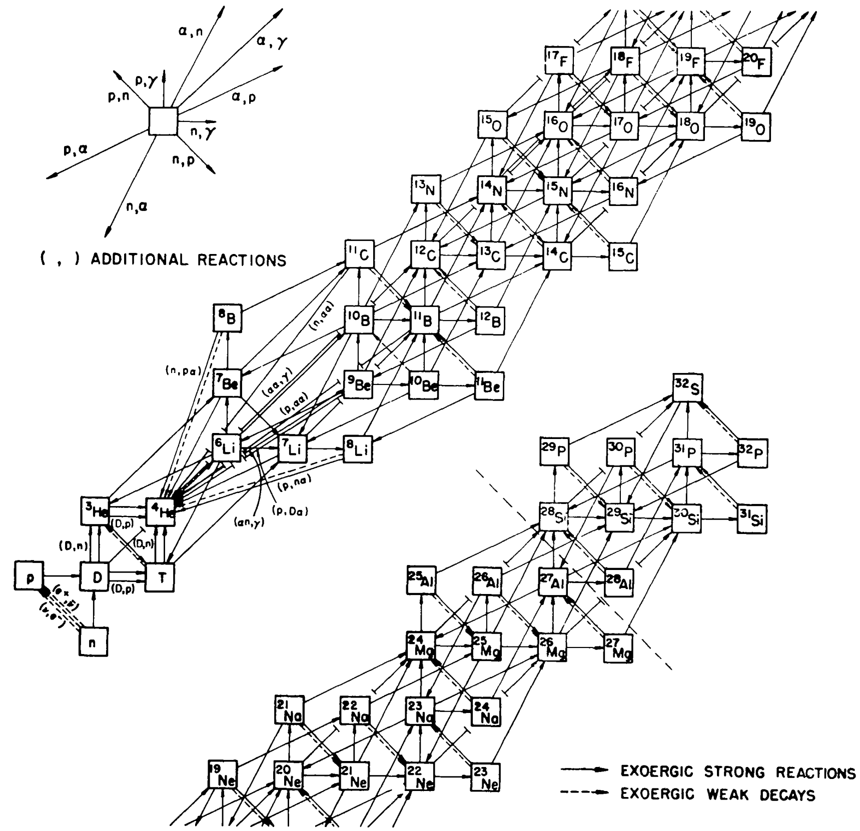
Analyses by GMH/MWP

Unitary, self-consistent primordial nucleosynthesis

- **State of standard big-bang nucleosynthesis (BBN)**
  - →d & ^{4}He abundances: signature success cosmology+nucl astro+astroparticle
    - but there's at least one Lithium (^{7}Li) Problem [^{6}Li too? See: *Lind et.al. 2013*]
  - →coming *precision* observations of d, ^{4}He, η, N_{eff} demand new BBN capabilities
  - →resolution of ^{7}Li problem:
    - observational/stellar astrophysics?
    - ^{7}Li controversial anomaly: nuclear physics solution?
    - new physics?
- **Advance BBN as a tool for precision cosmology**
  - →incorporate unitarity into strong & electroweak interactions (next slide)
  - →couple unitary reaction network (URN) to full Boltzmann transport code
    - neutrino energy distribution function evolution/transport code
    - fully coupled to nuclear reaction network
    - calculate light primordial element abundance for non-standard BBN
      - active-sterile ν mixing
      - massive particle out-of-equilibrium decays→energetic active SM particles
  - →Produce tools/codes for nuc-astro-particle community: test new physics w/BBN
    - existing codes are based on Wagoner's (1969) code

Nuclear reaction network

- **Single-process (non-unitary) analysis**
  - → $\sigma_{\alpha\beta}(E) \pm \delta\sigma_{\alpha\beta}(E)$ from expt
  - → fit form (non-res+narrow res) to $\sigma_{\alpha\beta}(E)$
  - → compute $\langle \sigma v \rangle (T)$ → reactivity → network
  - → NB: norm. systematics can be large
    - ^{17}O case in 2 slides
- **Multi-channel (unitary) analysis**
  - → Construct unitary parametrization
    - R-matrix (Wigner-Eisenbud '47)
  - → simultaneous fit of unpolarized/pol'd scatt/reac data → determine *T*(or *S*)matrix
  - → determines a unitary reaction network (URN) for analyzed compound systems



Wagoner ApJSuppl '69

Formal unitarity: consequences

$$\left.\begin{aligned} \delta_{fi} &= \textstyle\sum_n S^\dagger_{fn} S_{ni} \\ S_{fi} &= \delta_{fi} + 2i\rho_f T_{fi} \\ \rho_n &= \delta(H_0 - E_n) \end{aligned}\right\}$$

$$\boxed{T_{fi} - T^\dagger_{fi} = 2i \sum_n T^\dagger_{fn} \rho_n T_{ni}}$$

NB: unitarity implies optical theorem $\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im } f(0)$; but *not just* the O.T.

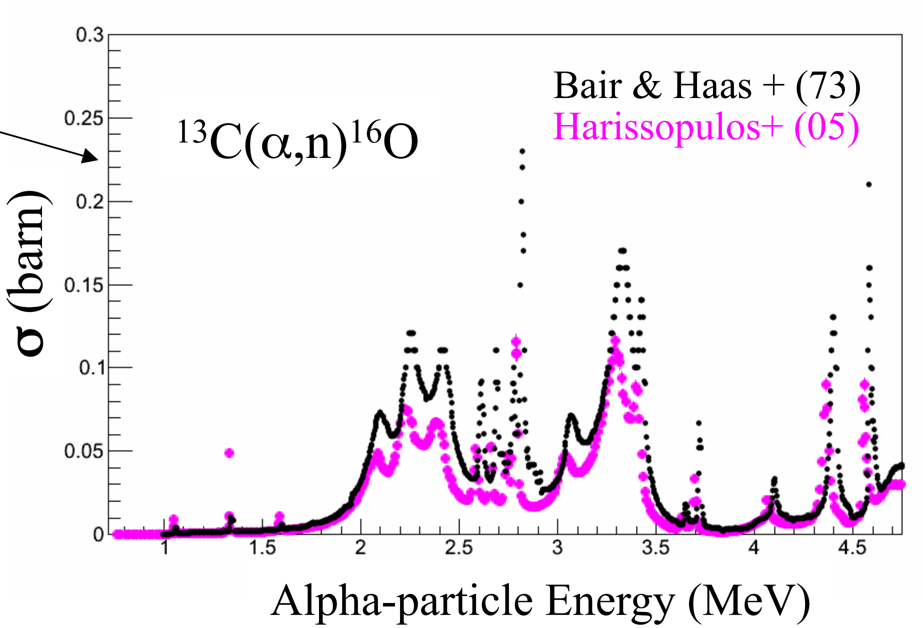
- **Implications of unitarity constraint on transition matrix**

1. Doesn't uniquely determine T_{ij}; highly restrictive, however
   Elastic: $\text{Im } T_{11} = -\rho_1, \; E < E_2$ (assuming T & P invariance)
   Multichannel: $\text{Im } \mathbf{T} = -\boldsymbol{\rho}$
2. Unitarity violating transformations
   - cannot scale ***any*** set: $T_{ij} \to \alpha_{ij} T_{ij} \qquad \alpha_{ij} \in \mathbb{R}$
   - cannot rotate ***any*** set: $T_{ij} \to e^{i\theta_{ij}} T_{ij} \qquad \theta_{ij} \in \mathbb{R}$

   ★ consequence of linear 'LHS' ∝ quadratic 'RHS'
3. Unitary parametrizations of data provide constraints that experiment may violate
   ★ *normalization*, in particular → $\boxed{\text{Observable} \propto \text{KF } |T_{fi}|^2}$
   ★ next slide: ^{17}O compound system

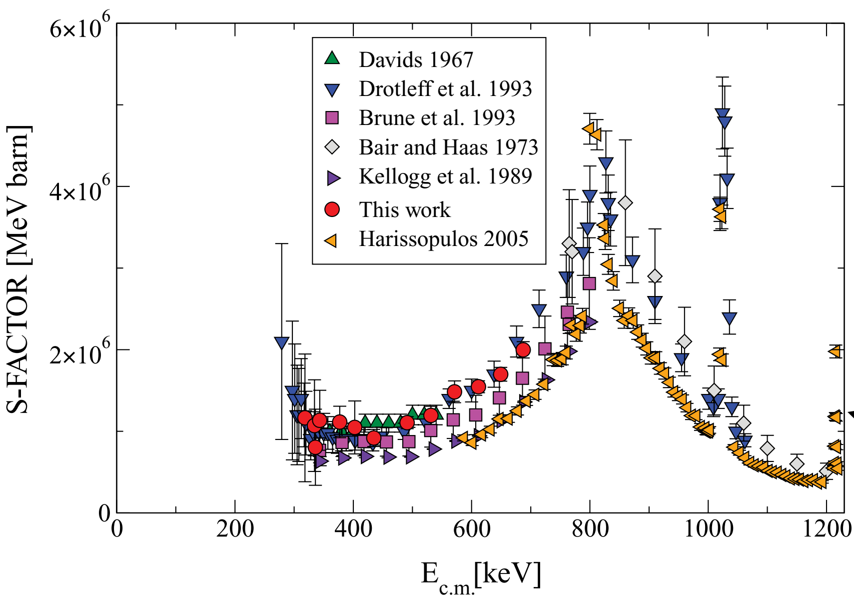
^{17}O compound system: experimental status

Recent Bochum (Harissopulos '05) measurement $^{13}C(\alpha,n)^{16}O$ vs. older ORNL (Bair & Haas '73)



Credit: S. Kunieda

Heil et.al. PRC **78** 025803 ('08)



- Bochum data 2/3*B&H(73)
- Karlsruhe (Heil '08) consistent w/ORNL

Tempting to conclude that B&H73 was right all along!

^{17}O compound system: experimental status

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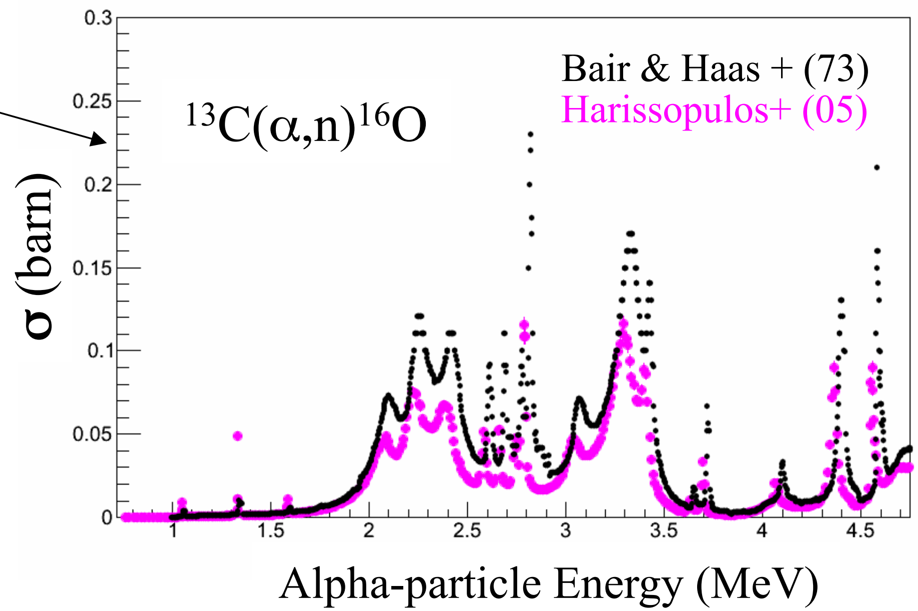
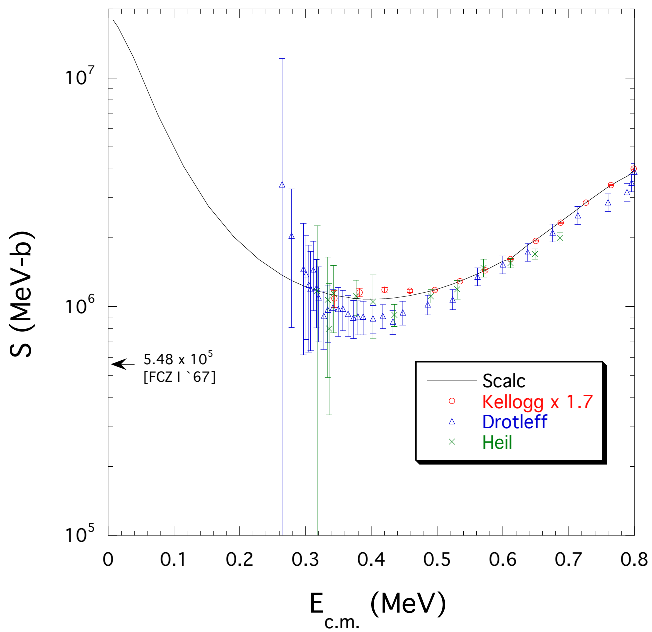


Credit: S. Kunieda

Hale (2013, unpublished)

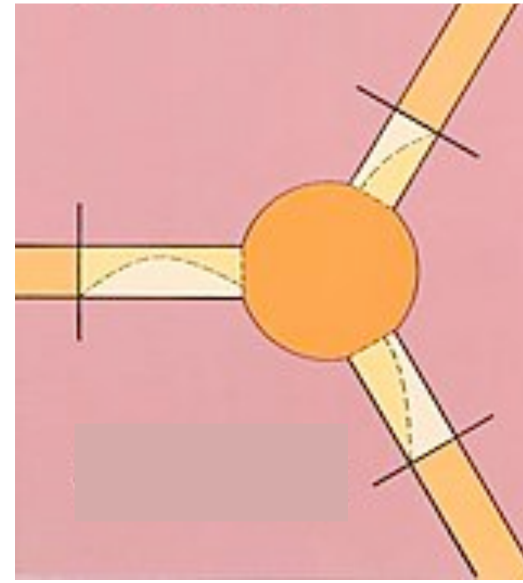


- Subthreshold ½$^+$
  - deep min in σ_T
  - $S(0) >> S_{FCZ67}(0)$

Tempting to conclude that B&H73 was right all along!

Basics of R-matrix

- **Assumptions** (cf. Lane & Thomas RMP '58)
  - a) Non-relativistic QM (L&T58); LANL-EDA uses rel. kin.
  - b) Two-body channels only ('c'); aux. spectra code
  - c) Conservation of N, Z
  - d) Finite radius a_c beyond $V_{pol}\approx 0$; sharp boundaries
- **Separated pairs, "channels"**
  - → A nucleons → (A_1,A_2)
  - → $c=\{\alpha s_1 m_1 s_2 m_2\} \to \{\alpha(s_1 s_2) s m_s \ell m_\ell\} \to \{\alpha(s_1 s_2) s\ell, JM\}$
  - → Assume $a_c = a_\alpha$ ➔ many c have same channel in configuration space
- **Channel surface**
  - → Consider configuration space of 3A dimensions
  - → Set of points: $\cup_c r_{\alpha(c)} = a_{\alpha(c)}$
  - → Surfaces coincide but assumed to have negl. prob.
  - → Channels are cylinders normal to channel surf.



Example: ^{8}Be compound system

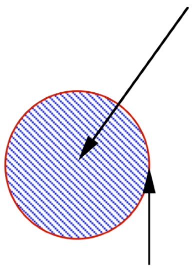
$$\mathrm{Li}^7+p \to \mathrm{Be}^{8*} \to \begin{cases} \mathrm{Li}^7+p \text{ (elastic scattering)} \\ \mathrm{Li}^{7*}+p' \text{ (inelastic scattering)} \\ \mathrm{Be}^7+n \\ \mathrm{Li}^6+d \\ \mathrm{He}^4+\mathrm{He}^4 \\ \mathrm{Be}^8+\text{photon, etc. (capture)} \end{cases}$$

R-matrix formalism

INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$$\underbrace{H + \mathcal{L}_B}$$

compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space



$$|\psi^+) = (H + \mathcal{L}_B - E)^{-1}\mathcal{L}_B|\psi^+)$$

SURFACE

$$\mathcal{L}_B = \sum_c |c)(c|\left(\frac{\partial}{\partial r_c} r_c - B_c\right),$$

$$(\mathbf{r}_c|c) = \frac{\hbar}{\sqrt{2\mu_c a_c}} \frac{\delta(r_c - a_c)}{r_c} \left[(\phi_{s_1}^{\mu_1} \otimes \phi_{s_2}^{\mu_2})_s^{\mu} \otimes Y_l^m(\hat{\mathbf{r}}_c)\right]_J^M$$

$$R_{c'c} = (c'|(H + \mathcal{L}_B - E)^{-1}|c) = \sum_\lambda \frac{(c'|\lambda)(\lambda|c)}{E_\lambda - E}$$

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle = -I_{c'}(r_{c'})\delta_{c'c} + O_{c'}(r_{c'})S_{c'c}$$

Measurements

$$\mathbf{T} = \rho^{1/2}\mathbf{O}^{-1}\mathbf{R_L}\mathbf{O}^{-1}\rho^{1/2} - \mathbf{F}\mathbf{O}^{-1}$$
$$\mathbf{S} = 1 + 2\pi i\mathbf{T}$$
$$\mathbf{R}_L = [\mathbf{R}_B^{-1} - \mathbf{L} + \mathbf{B}]^{-1}$$

Bloch operator $\mathcal{L}_B = \sum_c |c)(c|\left[\frac{\partial}{\partial r_c} r_c - B_c\right]$ ensures Hermiticity of Hamiltonian restricted to internal region

- R-matrix theory: unitary, multichannel parametrization of (not just resonance) data
- Interior/Exterior regions
  - → Interior: strong interactions
  - → Exterior: Coulomb/non-polarizing interactions
  - → Channel surface

$$\mathcal{S}_c : r_c = a_c \qquad \mathcal{S} = \sum_c \mathcal{S}_c$$

- R-matrix elements
  - → Projections on channel surface functions $(\mathbf{r}_c|c)$ of Green's function

$$G_B = [H + \mathcal{L}_B - E]^{-1}$$

  - → Boundary conditions

$$B_c = \frac{1}{u_c(a_c)} \frac{du_c}{dr_c}\bigg|_{r_c = a_c}$$

Electromagnetic channels [after Newton & Hale]

- **One-photon sector of Fock space**
  - → Photon 'wave function'

$$\mathbf{A}_{\mathbf{k}}(\mathbf{r}) = \left(\frac{2}{\pi\hbar c}\right)^{1/2} \sum_{jm} i^j \sum_{\lambda',\lambda=e,m,0} \mathbf{Y}^{(\lambda')}_{jm}(\hat{\mathbf{r}}) u^j_{\lambda'\lambda}(r) \mathbf{Y}^{(\lambda)}_{jm}(\hat{\mathbf{k}}) \cdot \chi$$

  - → Radial part

$$u^j_{ee} = -[f'_j(\rho) + t^j_{ee} h^{+\prime}_j(\rho)] \qquad u^j_{0e} = -\frac{\sqrt{j(j+1)}}{\rho}[f_j(\rho) + t^j_{e0} h^+_j(\rho)]$$

$$u^j_{mm} = \quad [f_j(\rho) + t^j_{mm} h^+_j(\rho)] \qquad u^j_{0m} = u^j_{me} = u^j_{em} = 0$$

  - → Photon channel surface functions

$$(\mathbf{r}_c|c) = \left(\frac{\hbar c}{2\rho_\gamma}\right)^{1/2} \frac{\delta(r_\gamma - a_\gamma)}{r_\gamma} \left[\phi_{s\nu} \otimes \mathbf{Y}^{(e,m)}_{jm}(\hat{\mathbf{r}}_\gamma)\right]_{JM}$$

    - Photon 'mass': $\hbar k_\gamma / c$

  - → R-matrix definition preserved

$$(c'|\psi) = \sum_c R^B_{c'c}(c|\frac{\partial}{\partial r_c} r_c - B_c|\psi)$$

$$\mathbf{T} = \rho^{1/2}\mathbf{O}^{-1}\mathbf{R_L}\mathbf{O}^{-1}\rho^{1/2} - \mathbf{F}\mathbf{O}^{-1}$$

$$\mathbf{R}_L = [\mathbf{R}_B^{-1} - \mathbf{L} + \mathbf{B}]^{-1}$$

$$\mathbf{L} = \rho\mathbf{O}'\mathbf{O}^{-1}$$

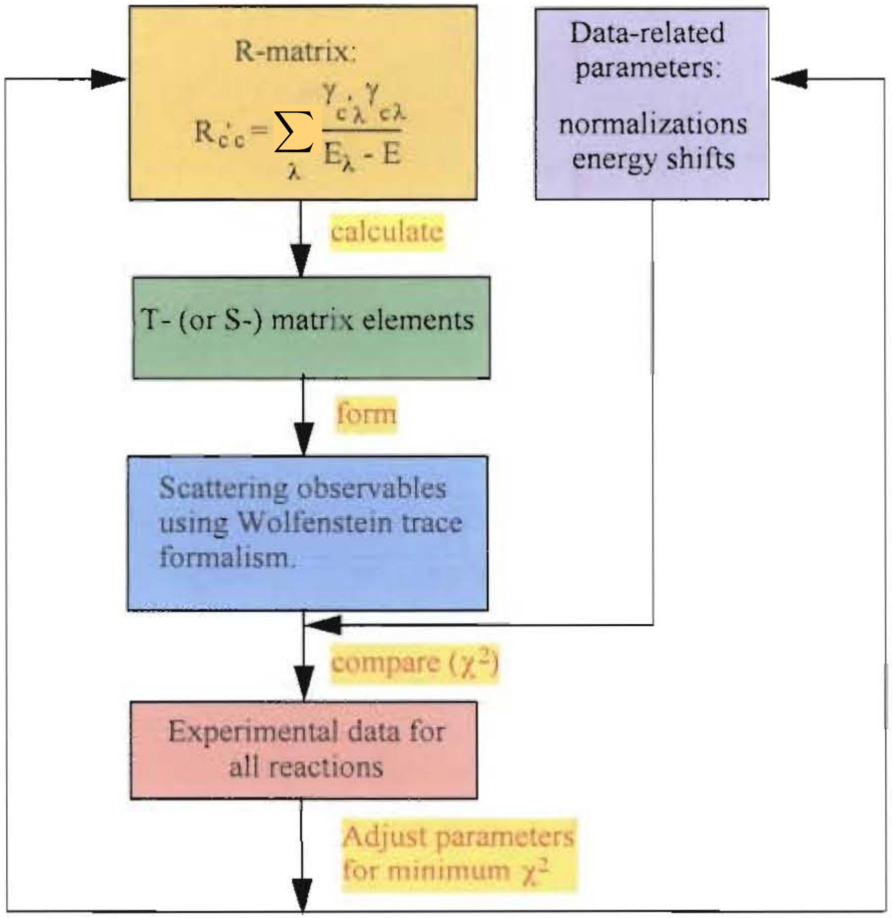
$$F = \mathrm{Im}\ \mathbf{O}$$

Implementation in EDA

- **EDA = Energy Dependent Analysis**
  - → Adjust E_λ & $\gamma_{c\lambda}$
- **Any number of two-body channels**
  - → Arbitrary spins, masses, charges (incl. mass zero)
- **Scattering observables**
  - → Wolfenstein trace formalism
- **Data**
  - → Normalization
  - → Energy shifts
  - → Energy resolution/spread
- **Fit (rank-1 var. metric) solution**

$$\chi^2_{EDA} = \sum_i \left[\frac{nX_i(\mathbf{p}) - R_i}{\delta R_i}\right]^2 + \left[\frac{nS-1}{\delta S/S}\right]^2$$

- ***Covariance determined***



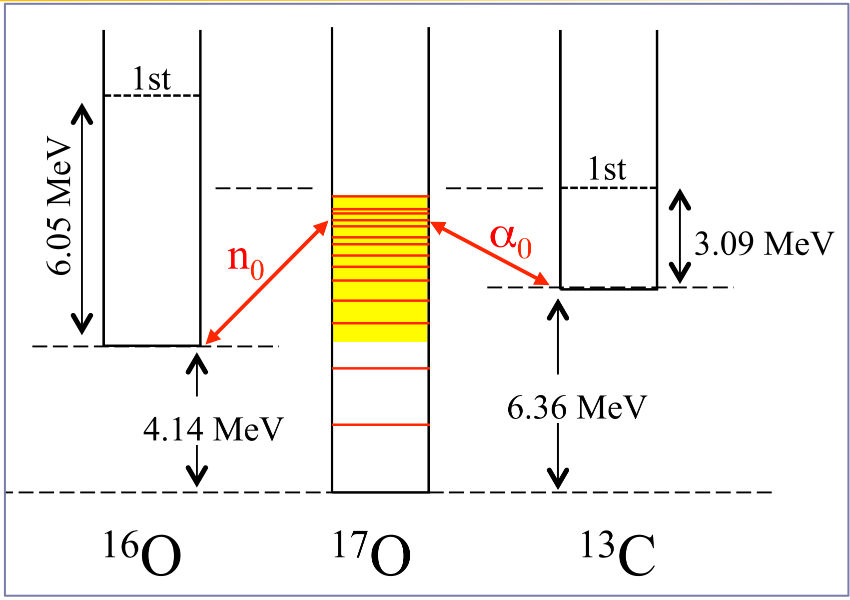
^{17}O analysis configuration

Channel	a_c (fm)	l_{max}
$n+^{16}O$	4.3	4
$\alpha+^{13}C$	5.4	5

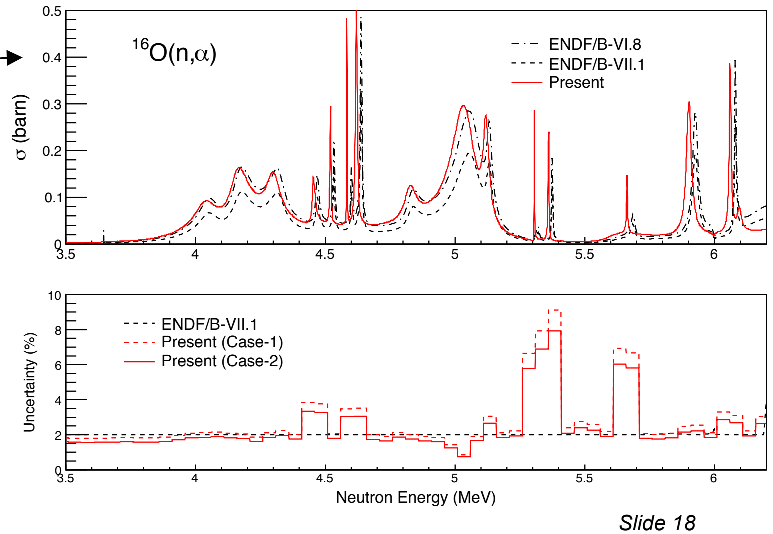
Reaction	Energies (MeV)	# data points	Data types
$^{16}O(n,n)^{16}O$	$E_n = 0 – 7$	2718	σ_T, $\sigma(\theta)$, $P_n(\theta)$
$^{16}O(n,\alpha)^{13}C$	$E_n = 2.35 – 5$	850	σ_{int}, $\sigma(\theta)$, $A_n(\theta)$
$^{13}C(\alpha,n)^{16}O$	$E_\alpha = 0 – 5.4$	874	σ_{int}
$^{13}C(\alpha,\alpha)^{13}C$	$E_\alpha = 2 – 5.7$	1296	$\sigma(\theta)$
total		5738	8

R-matrix analyses support B&H73/Heil08

- **LANL R-matrix fit to Bair & Haas '73**
  - → two-channel fit: (^{16}O,n) & (^{13}C,α)
    - $\ell_n = 0, \ldots, 4; \quad \ell_\alpha = 0, \ldots, 5$
  - → data included: $\sigma_T(E)$
    - ^{16}O(n,n), ^{16}O(n,α), ^{13}C(α,n)
      - $\sigma_T, \sigma(\theta), P_n(\theta), \sigma_{\rm int}$
    - χ^2 minimization: normalizations float
  - → Test Hariss05 data
    - remove B&H73/Heil08 data
    - fix Hariss05 norm to unity
      - unable to obtain fit with realistic χ^2 (< 2.0)
      - due to tension with σ_T
    - now allow Hariss05 norm to float
      - requires scale factor of ~1.5, consistent with B&H73
- **Kunieda/Kawano analysis [ND2013]**
  - → similar to LANL R-matrix(EDA)/ENDF/B-VI.8
  - → with independent R-matrix code
  - → KK give uncertainty analysis: see ND2013 proceedings in *Nucl. Data Sh.*
  - → ***Right to conclude B&H73 data correct on the basis of unitarity.***

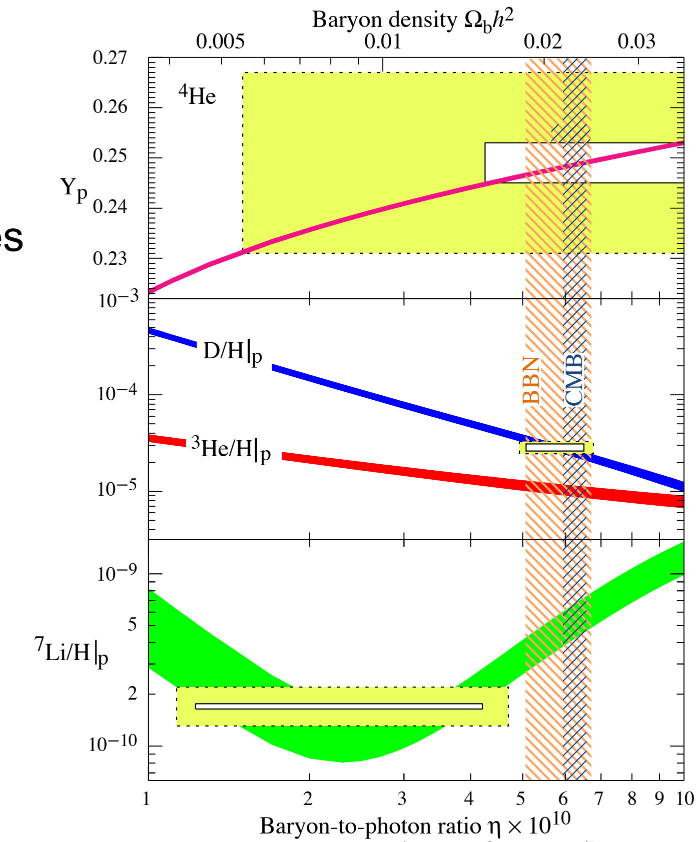


Credit: S. Kunieda



A nuclear physics solution to the BBN ^{7}Li problem?

- **Primordial nucleosynthesis**
  - → Probes physics of early universe
  - → Big-bang nucleosynthesis: D, ^{4}He, ^{7}Li abundances
  - → D, ^{4}He abundances agree with theo/expl uncertainties
  - → At η_{wmap} (CMB) $^{7}Li/H|_{BBN} \sim (2.2-4.2) * {}^{7}Li/H|_{halo^*}$
  - → Discrepancy ~ 4.5−5.5σ ⟶ the “Li problem”



- **Resonant destruction ^{7}Li**
  - → Prod. mass 7 “well understood”; destruction not
  - → Cyburt & Pospelov *arXiv:0906.4373; IJMPE, 21(2012)*
    - $^{7}Be(d,p)\alpha\alpha$ & $^{7}Be(d,\gamma)^{9}B$ resonant enhancement
    - Identify ^{9}B $E_{5/2+} \approx 16.7$ MeV $\approx E_{thr}(d+{}^{7}Be)+200$ keV
      - *Near threshold*
    - $(E_r,\Gamma_d) \approx (170-220, 10-40)$ keV solve Li problem
  - → Chakraborty, Fields & Olive *PRD83, 063006 (2011)*
    - More general approach: A=8,9,10 & 11
    - Identify as possibly important: ^{9}B, ^{10}B, ^{10}C
  - → ‘Large’ widths
    - Both conclude “large channel radius” required

NB: both approaches assume validity of TUNL-NDG tables

^{9}B analysis: included data

- **^{6}Li+^{3}He elastic** *Buzhinski et.al., Izv. Rossiiskoi Akademii Nauk, Ser.Fiz., Vol.43, p.158 (1979)*
  - → Differential cross section
  - → 1.30 MeV < E(^{3}He) < 1.97 MeV
- **^{6}Li+^{3}He → p+^{8}Be*** *Elwyn et.al., Phys. Rev. C 22, 1406 (1980)*
  - → Integrated cross section
  - → Quasi-two-body, excited-state, summed final channel
  - → 0.66 MeV < E(^{3}He) < 5.00 MeV
- **^{6}Li+^{3}He → d+^{7}Be** *D.W. Barr & J.S. Gilmore, unpublished (1965)*
  - → Integrated cross section
  - → 0.42 MeV < E(^{3}He) < 4.94 MeV
- **^{6}Li+^{3}He → γ+^{9}B** *Aleksic & Popic, Fizika 10, 273-278 (1978)*
  - → Integrated cross section
  - → 0.7 MeV < E(^{3}He) < 0.825 MeV
  - → New to ^{9}B analysis
- **New evaluation**
  - → Separate ^{8}Be* states
    - 2^+@200 keV [16.9 MeV], 1^+@650 keV [17.6 MeV], 1^+@1.1 MeV[18.2 MeV]
  - → n+^{8}B: E_{thresh}(^{3}He) = 3 MeV
  - → Simultaneous analysis with ^{9}Be mirror system

Data accessed via EXFOR/CSISRS database (C4 format)

R-matrix configuration in EDA code

- Hadronic channels (in blue, not included)

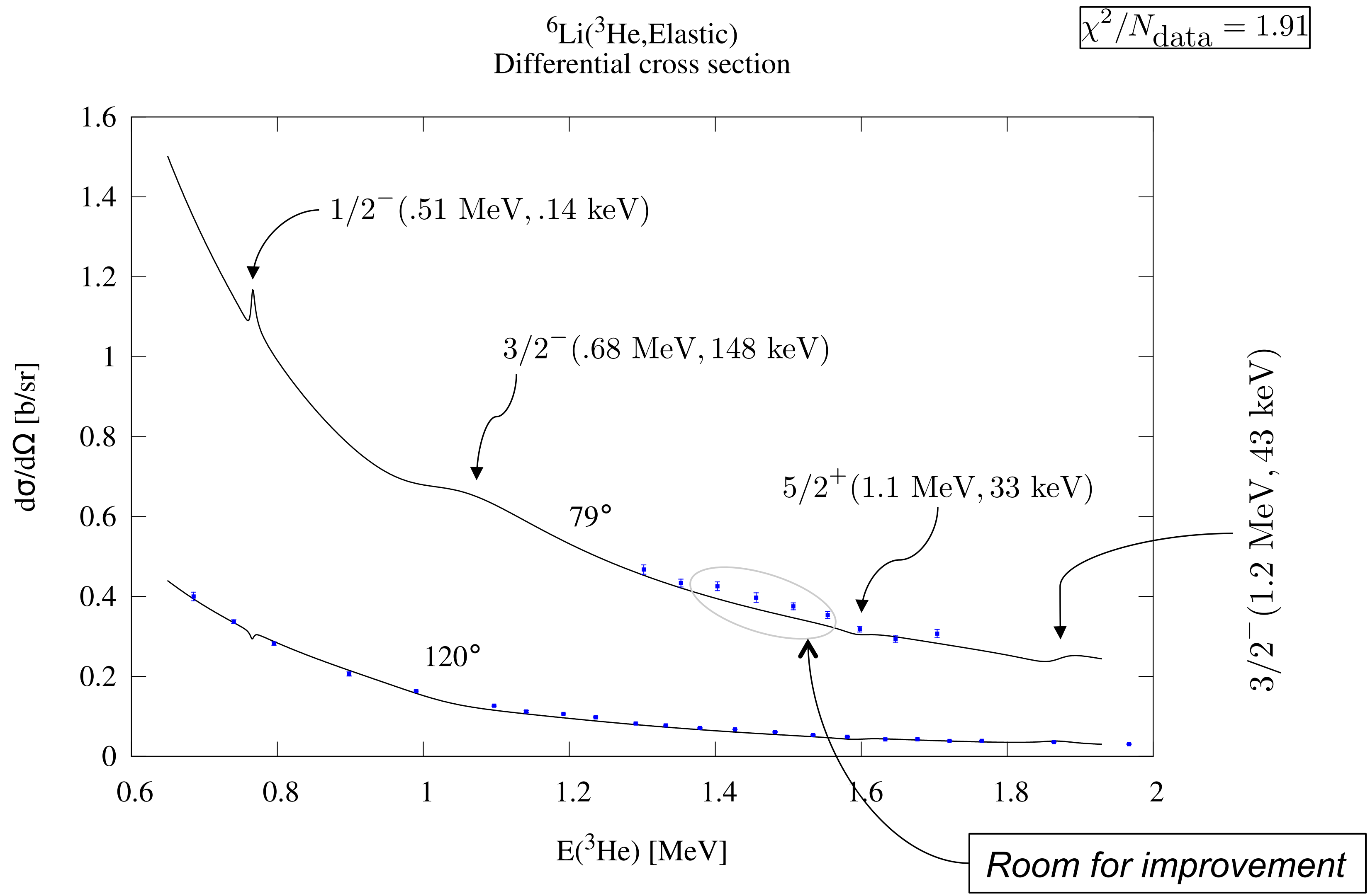
$A_1A_2^\pi$	^{3}He^{6}Li$^+$(1)		p^8Be^{*+}(2)		d^7Be$^-$(3)		
ℓ \ S	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
0	$^4S_{3/2}$	$^2S_{1/2}$	$^6S_{5/2}$	$^4S_{3/2}$	$^6S_{5/2}$	$^4S_{3/2}$	$^2S_{1/2}$
1	$^4P_{5/2,3/2,1/2}$	$^2P_{3/2,1/2}$	$^6P_{7/2,5/2,3/2}$	$^4P_{5/2,3/2,1/2}$	$^6P_{7/2,5/2,3/2}$	$^4P_{5/2,3/2,1/2}$	$^2P_{3/2,1/2}$
2	$^4D_{7/2,5/2,3/2,1/2}$	$^2D_{5/2,3/2}$	$^6D_{9/2,7/2,5/2,3/2,1/2}$	$^4D_{7/2,5/2,3/2,1/2}$	$^6D_{9/2,7/2,5/2,3/2,1/2}$	$^4D_{7/2,5/2,3/2,1/2}$	$^2D_{5/2,3/2}$
E_{thr}(CM, MeV)	16.6		16.7				16.5

Electromagnetic channel: $\gamma + {}^9B \rightarrow E_1^{3/2}, M_1^{5/2}, M_1^{3/2}, M_1^{1/2}, E_1^{5/2}, E_1^{1/2}$

<u>Full model space</u>: *state number; channel pair; LS; J; channel radius [fm]*

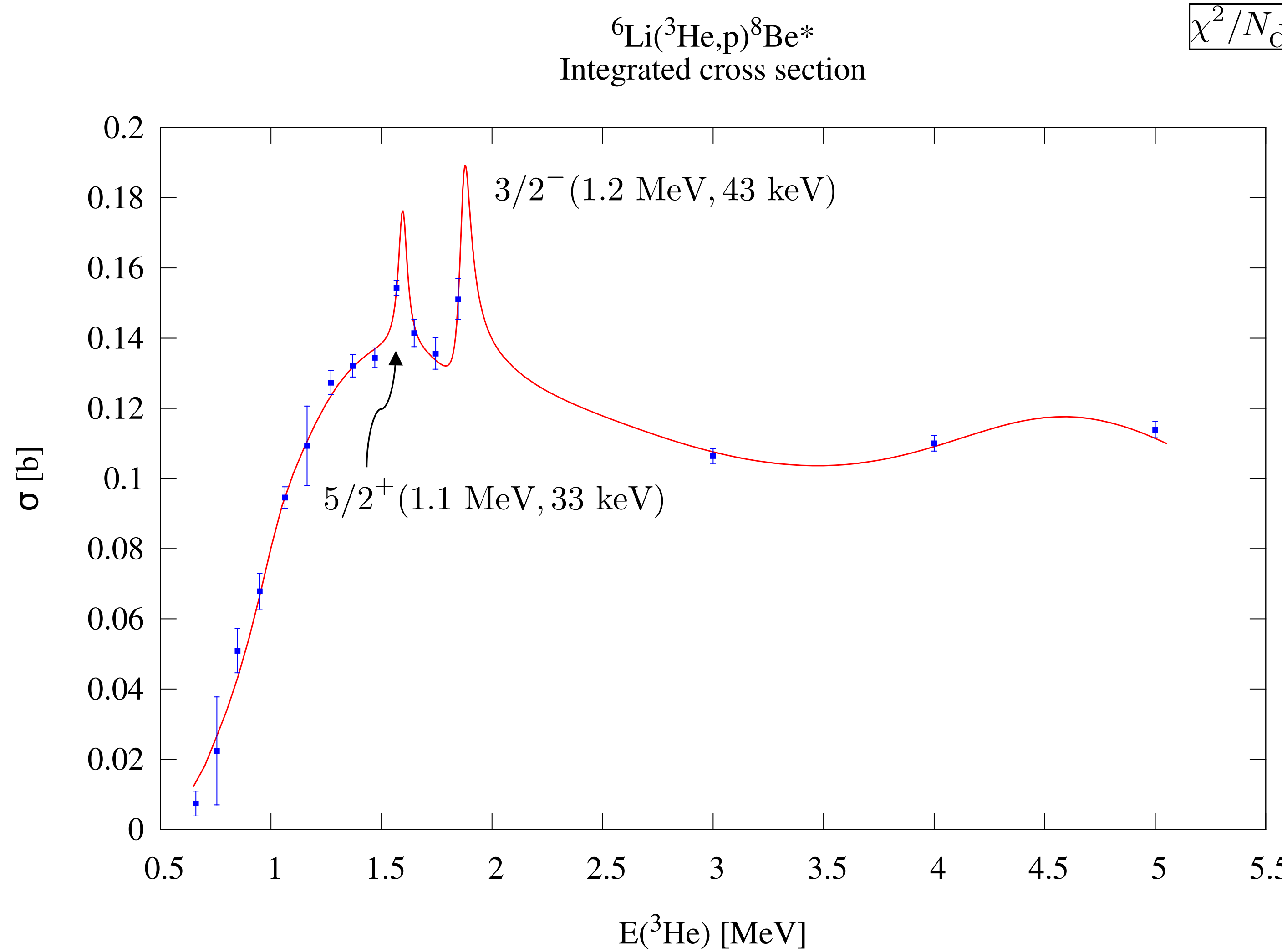
```
 1   1 4s 3/2    7.50000000f      20   1 4p 1/2    7.50000000f
 2   1 4d 3/2    7.50000000f      21   1 2p 1/2    7.50000000f
 3   1 2d 3/2    7.50000000f      22   2 4p 1/2    5.50000000f
 4   2 4s 3/2    5.50000000f      23   3 2s 1/2    7.00000000f
 5   3 6p 3/2    7.00000000f      24   4 M1 1/2   50.00000000f
 6   3 4p 3/2    7.00000000f      25   1 4d 7/2    7.50000000f
 7   3 2p 3/2    7.00000000f      26   3 6p 7/2    7.00000000f
 8   4 E1 3/2   50.00000000f      27   1 4d 5/2    7.50000000f
 9   1 4p 5/2    7.50000000f      28   1 2d 5/2    7.50000000f
10   2 6p 5/2    5.50000000f      29   2 6s 5/2    5.50000000f
11   2 4p 5/2    5.50000000f      30   3 6p 5/2    7.00000000f
12   3 6s 5/2    7.00000000f      31   3 4p 5/2    7.00000000f
13   4 M1 5/2   50.00000000f      32   4 E1 5/2   50.00000000f
14   1 4p 3/2    7.50000000f      33   1 4d 1/2    7.50000000f
15   1 2p 3/2    7.50000000f      34   1 2s 1/2    7.50000000f
16   2 6p 3/2    5.50000000f      35   3 4p 1/2    7.00000000f
17   2 4p 3/2    5.50000000f      36   3 2p 1/2    7.00000000f
18   3 4s 3/2    7.00000000f      37   4 E1 1/2   50.00000000f
19   4 M1 3/2   50.00000000f      38   2 6p 7/2    5.50000000f
```

Observable fit: $^3He+^6Li$ elastic DCS

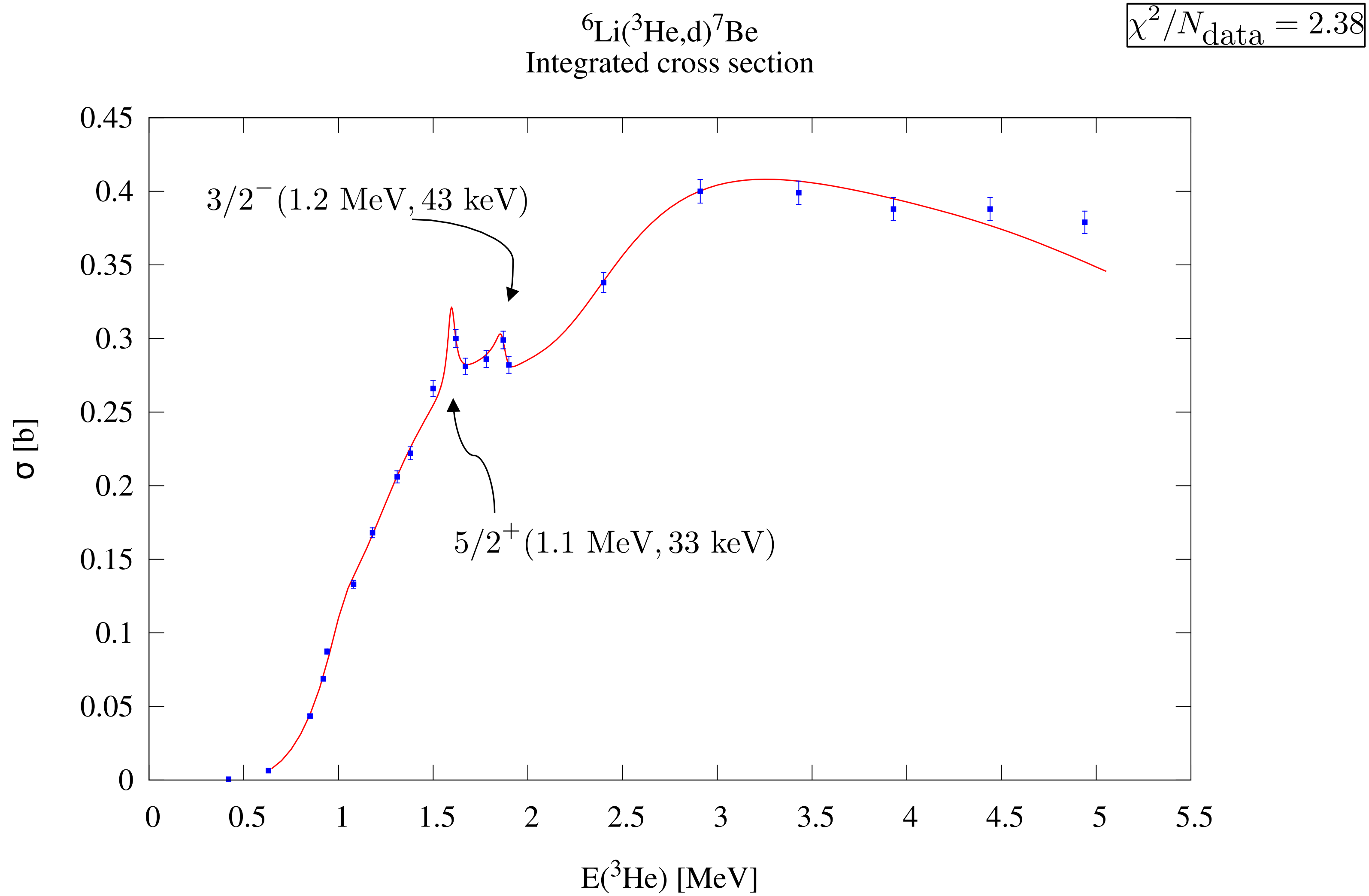


Observable fit: ^{6}Li(^{3}He,p)^{8}Be* integrated x-sec

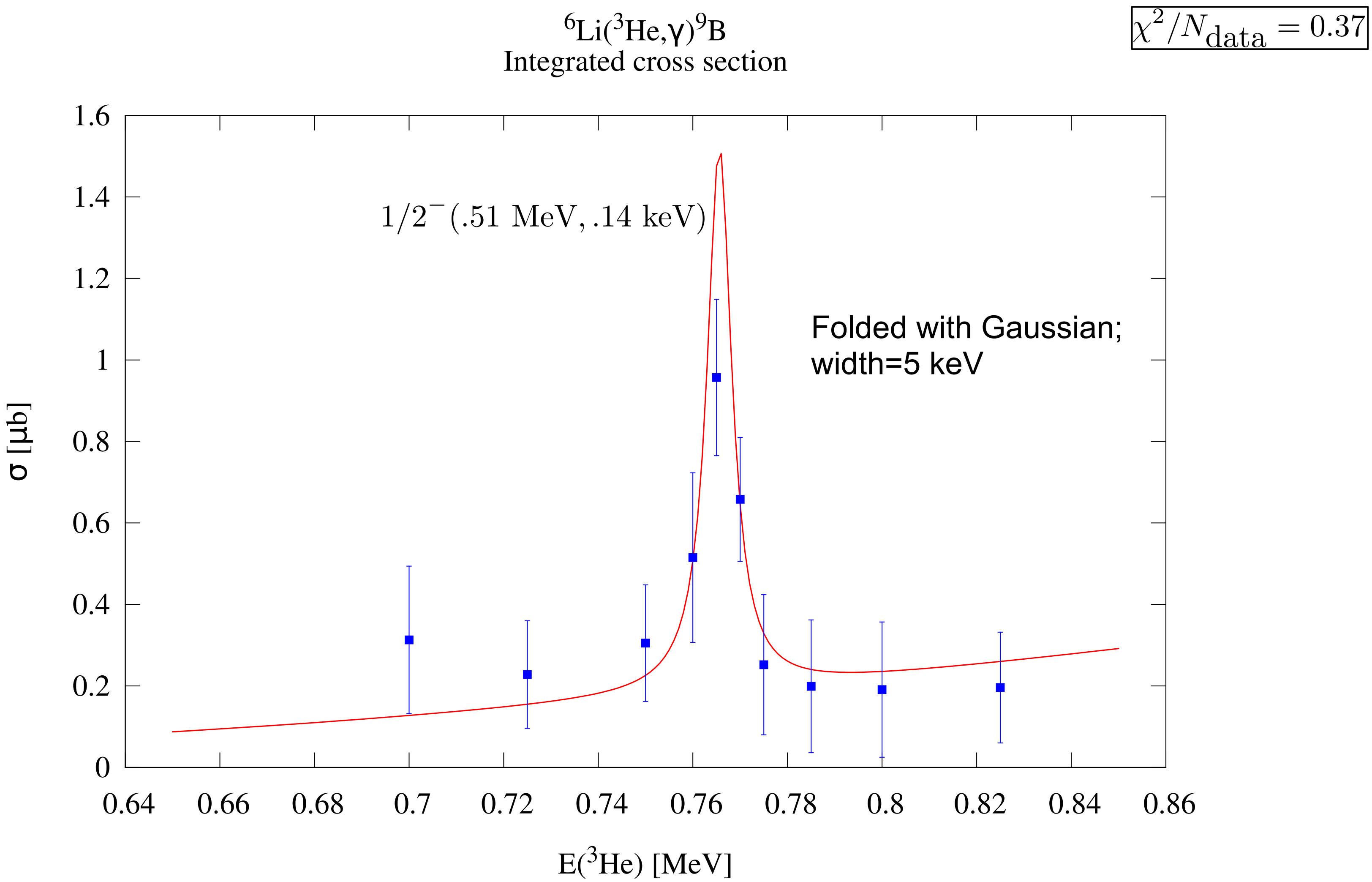
$\chi^2/N_{\mathrm{data}} = 0.55$



Observable fit: $^{6}Li(^{3}He,d)^{7}Be$ integrated x-sec



Observable fit: $^6Li(^3He,\gamma)^9B$ integrated x-sec



Analysis result: resonance structure

Ex(MeV)	Jpi	Gamma(keV)	Er(MeV)	ImEr(MeV)	E(3He)	Strength
16.46539	1/2-	768.46	-.1369	-0.3842	-0.2054	0.06 weak
17.11317	1/2-	0.14	0.5109	-0.6771E-04	0.7664	1.00 strong
17.20115	5/2-	871.63	0.5989	-0.4358	0.8984	0.40 weak
17.28086	3/2-	147.78	0.6785	-0.0739	1.0178	0.77 strong
17.66538	5/2+	33.33	1.0631	-0.0167	1.5947	0.98 strong
17.83619	7/2+	2036.21	1.2339	-1.0181	1.8509	0.15 weak
17.84773	3/2-	42.52	1.2454	-0.0213	1.8681	0.97 strong
18.04821	3/2+	767.11	1.4459	-0.3836	2.1689	0.54 weak
18.42292	1/2+	5446.32	1.8206	-2.7232	2.7309	0.03 weak
18.67716	1/2-	10278.41	2.0749	-5.1392	3.1124	0.15 weak
19.60923	3/2-	1478.22	3.0069	-0.7391	4.5104	0.52 weak

S-matrix pole & residue *Hale, Brown, Jarmie PRL 59 '87*

$$\mathcal{E}_{\lambda'\lambda} = E_\lambda \delta_{\lambda'\lambda} - \sum_c \gamma_{c\lambda'} [L_c(E) - B_c] \gamma_{c\lambda}$$

$$E_0 = E_r - i\Gamma/2 \qquad \text{residue: } i\rho_0\rho_0^T$$

$$\text{strength} = \frac{1}{\Gamma}\rho_0^\dagger \rho_0 = \frac{1}{\Gamma}\sum_c \Gamma_c$$

$$\rho_{0c} = \left(\frac{2k_{0c}a_c}{N}\right)^{1/2} \mathcal{O}_c^{-1}(k_{0c}a_c) \sum_\lambda \gamma_{c\lambda}(\lambda|\mu_0)$$

$$N = \sum_{\lambda'\lambda} (\lambda|\mu_0)(\lambda'|\mu_0) \left[\delta_{\lambda'\lambda} + \sum_c \gamma_{c\lambda'} \left.\frac{\partial L_c}{\partial E}\right|_{E=E_0} \gamma_{c\lambda}\right]$$

$$L_c = r_c \frac{\partial \mathcal{O}_c}{\partial r_c} \mathcal{O}_c^{-1}\Big|_{r_c=a_c}$$

NB: no strong resonance seen ~100 keV of ^{3}He+^{6}Li threshold

Analysis result: resonance structure

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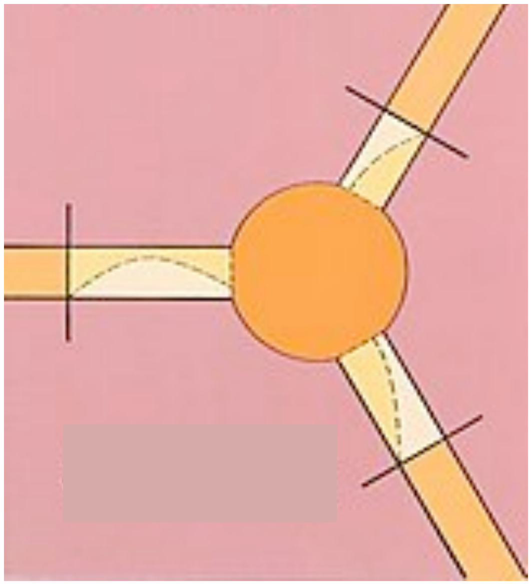
TUNL-NDG/ENSDF parameters

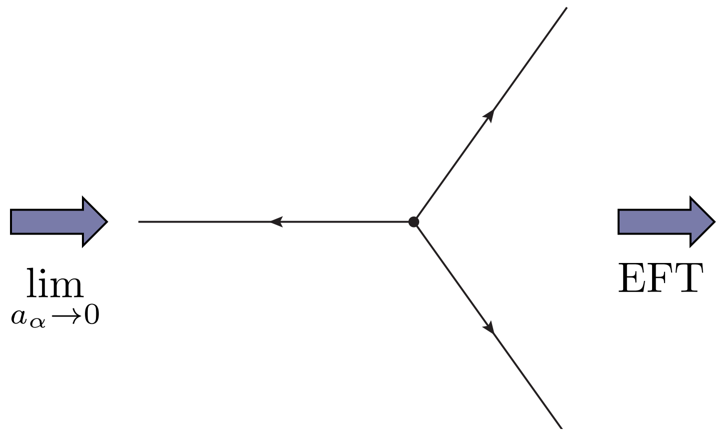
E_x [a] (MeV $\pm$ keV)	$J^\pi; T$	$\Gamma_{c.m.}$ (keV)	Decay
16.024 ± 25	$T = (\frac{1}{2})$	180 ± 16	
16.71 ± 100 [h]	$(\frac{5}{2}^+); (\frac{1}{2})$		
17.076 ± 4	$\frac{1}{2}^-; \frac{3}{2}$	22 ± 5	$(\gamma, {}^3He)$
17.190 ± 25		120 ± 40	p, d, ^3He
17.54 ± 100 [h,i]	$(\frac{7}{2}^+); (\frac{1}{2})$		
17.637 ± 10 [i]		71 ± 8	p, d, ^3He, α

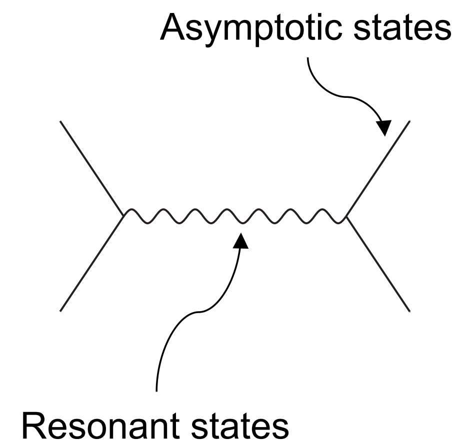
NB: no strong resonance seen ~100 keV of ^3He+^6Li threshold

The zero channel radius limit of R-matrix theory?!

Brown & Hale, Phys. Rev. C, arXiv:1308.0348
Hale, Brown & Paris, Phys. Rev. C, arXiv:1308:0349







- **'Conventional' R-matrix (left)**
  - → Consider the limit $a_\alpha \to 0$
- **Continued R-matrix theory (center)**
  - → Interior region: set of measure zero
- **Effective field theory* (right)**
  - → local Lagrangian (non-relativistic) field theory of stable and unstable particles

- **Next slides:**
  - → Consider $^1S_0$ *np* scattering
  - → study a_c dependence of R-matrix parameters
  - → Consider dt→nα R-matrix in zero channel radius limit
  - → Compare dt→nα R-matrix to EFT

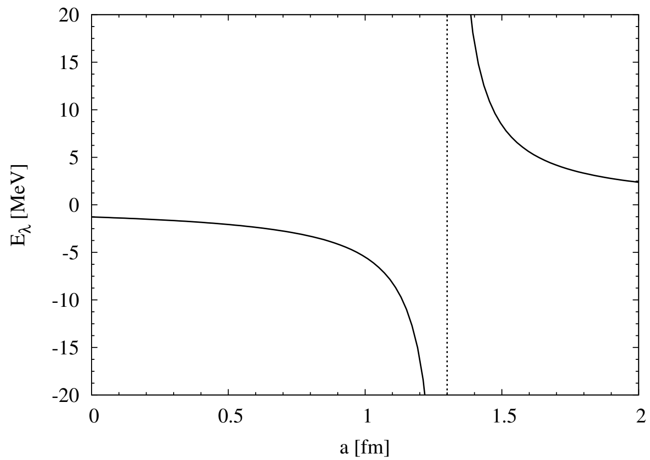
**Old fashioned EFT w/o power counting*

Zero channel radius limit: ${}^1S_0$ *np* scattering

$$S = e^{-2ika}\frac{1+ikaR}{1-ikaR} \qquad R = \frac{\gamma_\lambda^2(a)}{E_\lambda(a)-E} \qquad k\cot\delta = \frac{E_\lambda - E + kg^2\tan ka}{g^2-(E_\lambda-E)\frac{1}{k}\tan ka}$$

$$\rho = ka \qquad k=\sqrt{2\mu E} \qquad g^2 = a\gamma_\lambda^2$$

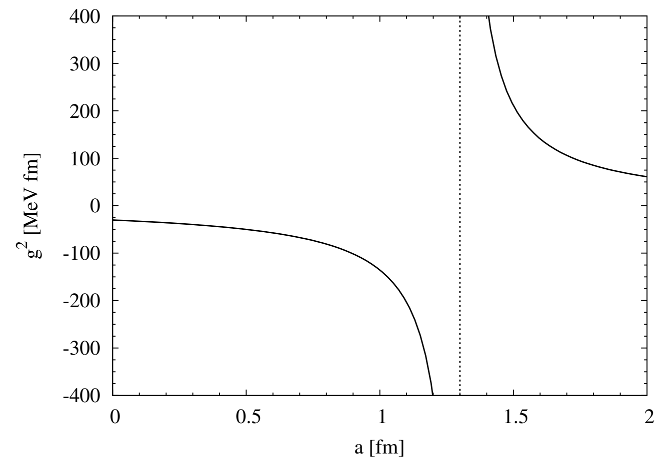
Effective range expansion: $a_0 = a - \frac{g^2}{E_\lambda}$ $\qquad r_0 = \frac{2a^3E_\lambda^2/3 - 2a^2E_\lambda g^2 + 2ag^4 - g^2\hbar^2/\mu}{(g^2-aE_\lambda)^2}$



$$E_\lambda(a) = \frac{\hbar^2(a_0-a)}{2\mu[r_0a_0^2/2 - a^3/3 - aa_0(a_0-a)]}$$

Pole position: $a_p = a_0 + \left\{a_0^3\left[\frac{3r_0}{2a_0}-1\right]\right\}^{1/3}$

$$g^2(a) = (a-a_0)E_\lambda(a) = -\frac{\hbar^2(a_0-a)^2}{2\mu[r_0a_0^2/2 - a^3/3 - aa_0(a_0-a)]}$$

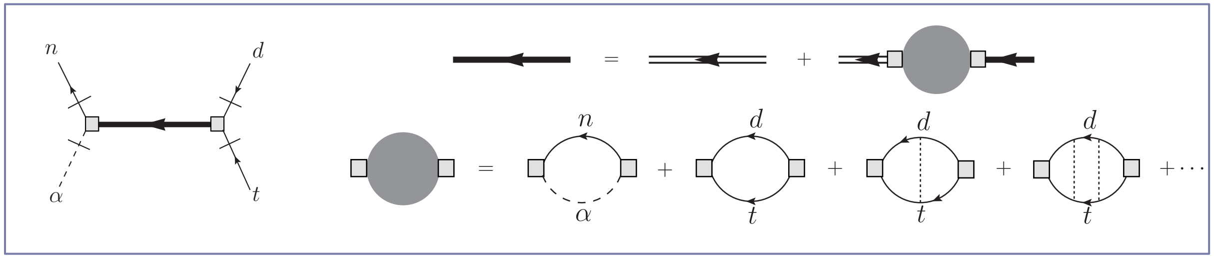


Effective field theory ⟺ R matrix: dt→nα

Exactly soluble EFT with 'wrong-sign' free Lagrangian and DOF:

Particle	Spin	Operators	Mass	Binding
Alpha	0^+	$\phi_\alpha^\dagger(\mathbf{r},t),\ \phi_\alpha(\mathbf{r},t)$	$m_\alpha = 2m_p + 2m_n$	ϵ_α
Deuteron	1^+	$\boldsymbol{\phi}_d^\dagger(\mathbf{r},t),\ \boldsymbol{\phi}_d(\mathbf{r},t)$	$m_d = m_p + m_n$	ϵ_d
Neutron	$\frac{1}{2}^+$	$\psi_n^\dagger(\mathbf{r},t),\ \psi_n(\mathbf{r},t)$	m_n	$\epsilon_n \equiv 0$
Triton	$\frac{1}{2}^+$	$\psi_t^\dagger(\mathbf{r},t),\ \psi_t(\mathbf{r},t)$	$m_t = m_p + 2m_n$	ϵ_t
^{5}He*	$\frac{3}{2}^+$	$\boldsymbol{\psi}_*^\dagger(\mathbf{r},t),\ \boldsymbol{\psi}_*(\mathbf{r},t)$	$m_* = 2m_p + 3m_n$	ϵ_*

$$\mathcal{L}_{\text{int}} = g_{dt}\left[\boldsymbol{\psi}_*^\dagger \cdot \boldsymbol{\phi}_d \psi_t + \psi_t^\dagger \boldsymbol{\phi}_d^\dagger \cdot \boldsymbol{\psi}_*\right] + g_{n\alpha}\left[\boldsymbol{\psi}_*^\dagger \cdot \boldsymbol{\Psi}_{n\alpha} + \boldsymbol{\Psi}_{n\alpha}^\dagger \cdot \boldsymbol{\psi}_*\right]$$



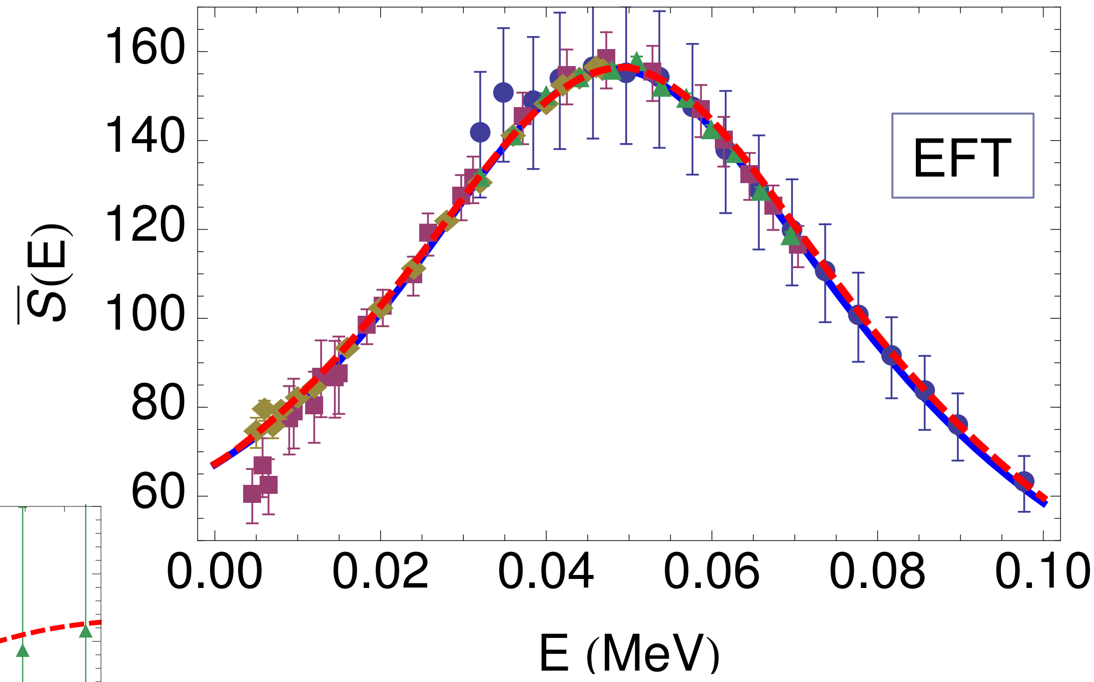
$$\sigma_{dt\to n\alpha} = \frac{8}{9} 4\pi m_{n\alpha} \frac{p_{n\alpha}^5}{v_{dt}} \frac{g_{dt}^2}{4\pi} \frac{g_{n\alpha}^2}{4\pi} \left|\psi_{\mathbf{p}_{dt}}^{(0)}(0)\right|^2 \Bigg| \left[\frac{p_{dt}^2}{2m_{dt}} - E_* - \frac{g_{dt}^2}{4\pi}\Delta(W)\right]^2 + \left[\frac{g_{dt}^2}{4\pi} 2m_{dt} p_{dt} \left|\psi_{\mathbf{p}_{dt}}^{(C)}(0)\right|^2 + \frac{g_{n\alpha}^2}{4\pi}\frac{2}{3} m_{n\alpha} p_{n\alpha}^5\right]^2 \Bigg|^{-2}$$

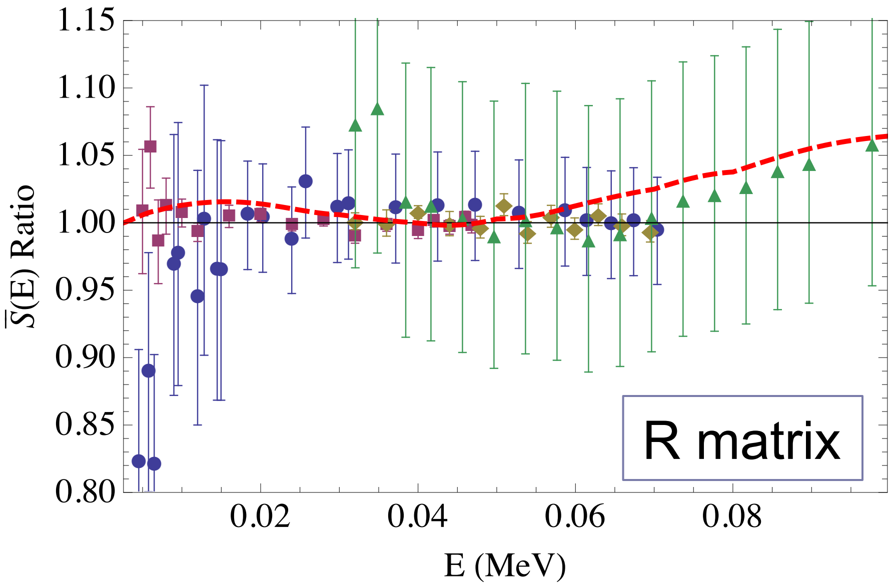
Identical to R-matrix (d,t)&(n,α) in the limit a_d, $a_n \to 0$

$$\gamma_d^2 = -\frac{g_d^2}{2\pi}\frac{\mu_d}{\hbar^2 a_d} \text{ and } \gamma_n^2 = -\frac{g_n^2}{6\pi}\frac{\mu_n}{\hbar^2 a_n^5}$$

Effective field theory ⟺ R matrix: dt→nα

- Dim'less astrophys factor vs E_d(CM)
- Solid-blue: 3 par EFT fit chi²/dof~0.8
- Dashed-red: Bosch & Hale (1992)
  - 2665 data/117 pars/chi²/dof~1.6





- Four parameter fit @ finite a
- Divide out Bosch-Hale92 fit
- Unphysical reduced widths $\gamma_c^2 < 0$ a<2 fm

Summary, findings & future work

Summary/findings

- Provided overview of current work in the LANL light nuclear reaction program
- Emphasize the utility of multichannel, unitary parametrization of light nuc data
  - ^{17}O norm issue: are Bair & Haas '73 data conclusive?
  - ^{9}B resonance spectrum: no resonances in ^{9}B that reside within ~200 (~100) keV of the d+^{7}Be (^{3}He+^{6}Li) threshold with 'large' widths 10—40 keV
  - Appears to rule out scenarios considered by *Cyburt & Pospelov (2009)* that low-lying, robust resonance in ^{9}B could explain the "Li problem"

Near-term, Future Work

- Complete ^{13,14}C analyses
- *NN* up to 200 MeV
- Improvements in the ^{9}B analysis: more channels; incorporate p+^{8}Be* angular data; proper treatment three-body final states
- Extend EFT—R-matrix approach to multichannel, multilevel problems

Supplementary material

Additional slides follow

Motivation

- **Cross section evaluation & resonance structure**
  - → *Nucl. Phys. A745, 155, 2004(2011)*

E_x [a] (MeV ± keV)	$J^\pi; T$	$\Gamma_{c.m.}$ (keV)	Decay
16.024 ± 25	$T = (\frac{1}{2})$	180 ± 16	
16.71 ± 100 [h]	$(\frac{5}{2}^+); (\frac{1}{2})$		
17.076 ± 4	$\frac{1}{2}^-; \frac{3}{2}$	22 ± 5	$(\gamma, {}^3He)$
17.190 ± 25		120 ± 40	$p, d, {}^3He$
17.54 ± 100 [h,i]	$(\frac{7}{2}^+); (\frac{1}{2})$		
17.637 ± 10 [i]		71 ± 8	$p, d, {}^3He, \alpha$

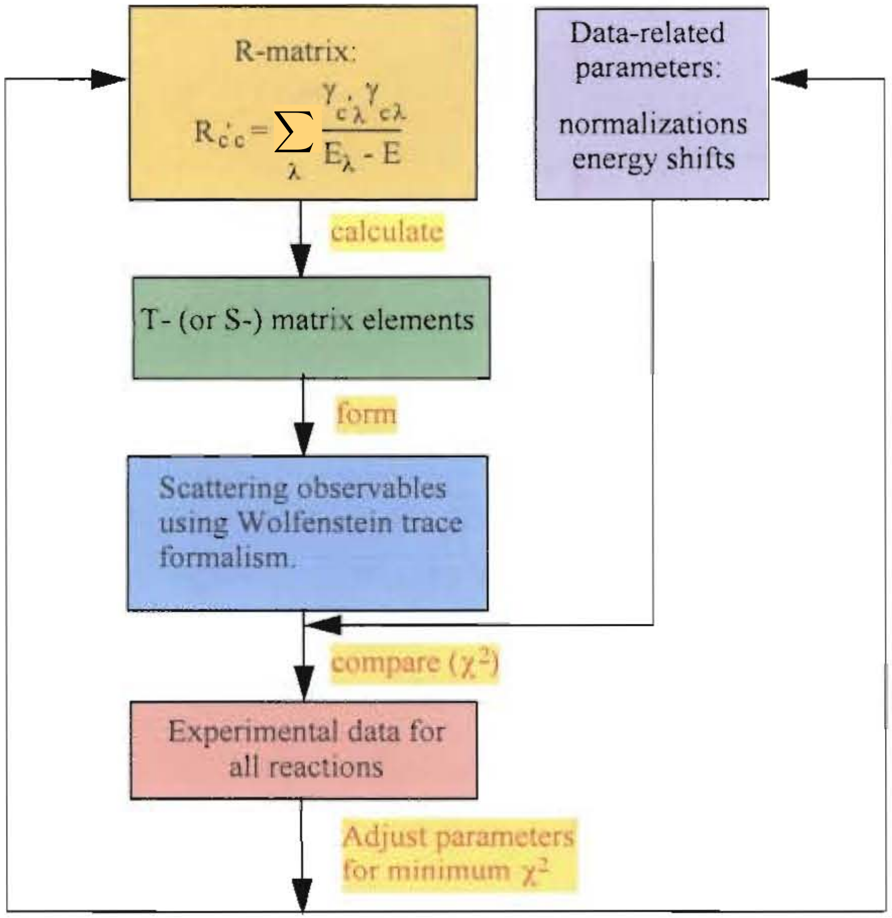
- **Astrophysical applications**
  - → Big bang nucleosynthesis
    - Nuclear physics solution to ^7Li predicted overproduction problem? *(cf. Hoyle)*
    - Details next slide.
- **Purpose within Los Alamos Nat. Lab programmatic**
  - → Continue the R-matrix program for various end-users
  - → Ongoing/upcoming analysis releases: ^{7}Be, ^{13}C *[G. Hale Tues. Session GA 2]*, ^{14}C, ^{17}O, …

Implementation in EDA

- **EDA = Energy Dependent Analysis**
  - → Adjust E_λ & $\gamma_{c\lambda}$
- **Any number of two-body channels**
  - → Arbitrary spins, masses, charges (incl. mass zero)
- **Scattering observables**
  - → Wolfenstein trace formalism
- **Data**
  - → Normalization
  - → Energy shifts
  - → Energy resolution/spread
- **Fit solution**

$$\chi^2_{EDA} = \sum_i \left[\frac{nX_i(\mathbf{p}) - R_i}{\delta R_i}\right]^2 + \left[\frac{nS - 1}{\delta S/S}\right]^2$$

- **Covariance determined**



Electromagnetic channels

- **One-photon sector of Fock space**
  - → Photon 'wave function'

$$\mathbf{A}_{\mathbf{k}}(\mathbf{r}) = \left(\frac{2}{\pi\hbar c}\right)^{1/2} \sum_{jm} i^j \sum_{\lambda',\lambda=e,m,0} \mathbf{Y}_{jm}^{(\lambda')}(\hat{\mathbf{r}}) u_{\lambda'\lambda}^j(r) \mathbf{Y}_{jm}^{(\lambda)}(\hat{\mathbf{k}}) \cdot \chi$$

  - → Radial part

$$u_{ee}^j = -[f_j'(\rho) + t_{ee}^j h_j^{+\prime}(\rho)] \qquad u_{0e}^j = -\frac{\sqrt{j(j+1)}}{\rho}[f_j(\rho) + t_{e0}^j h_j^+(\rho)]$$

$$u_{mm}^j = \quad [f_j(\rho) + t_{mm}^j h_j^+(\rho)] \qquad u_{0m}^j = u_{me}^j = u_{em}^j = 0$$

  - → Photon channel surface functions

$$(\mathbf{r}_c|c) = \left(\frac{\hbar c}{2\rho_\gamma}\right)^{1/2} \frac{\delta(r_\gamma - a_\gamma)}{r_\gamma} \left[\phi_{s\nu} \otimes \mathbf{Y}_{jm}^{(e,m)}(\hat{\mathbf{r}}_\gamma)\right]_{JM}$$

    - Photon 'mass': $\hbar k_\gamma / c$

  - → R-matrix definition preserved

$$(c'|\psi) = \sum_c R_{c'c}^B (c|\frac{\partial}{\partial r_c} r_c - B_c|\psi)$$

$$\mathbf{T} = \rho^{1/2}\mathbf{O}^{-1}\mathbf{R_L}\mathbf{O}^{-1}\rho^{1/2} - \mathbf{F}\mathbf{O}^{-1}$$

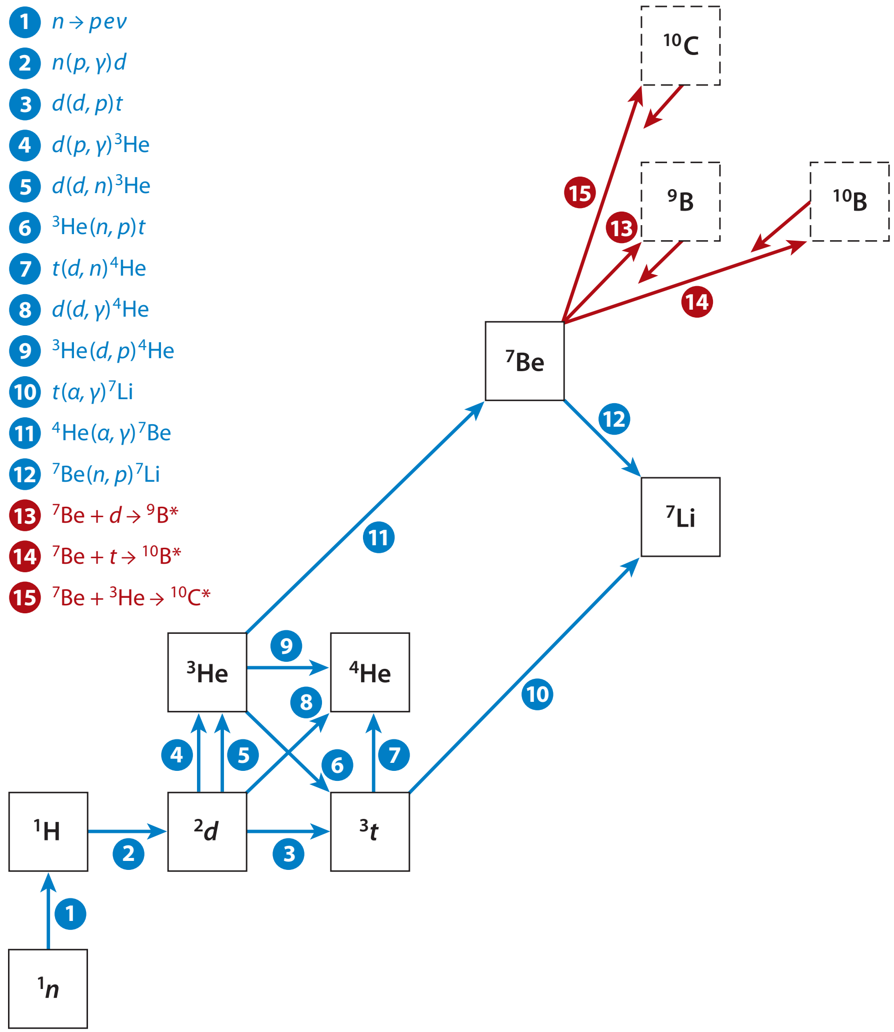
$$\mathbf{R}_L = [\mathbf{R}_B^{-1} - \mathbf{L} + \mathbf{B}]^{-1}$$

$$\mathbf{L} = \rho\mathbf{O}'\mathbf{O}^{-1}$$

$$F = \mathrm{Im}\ \mathbf{O}$$

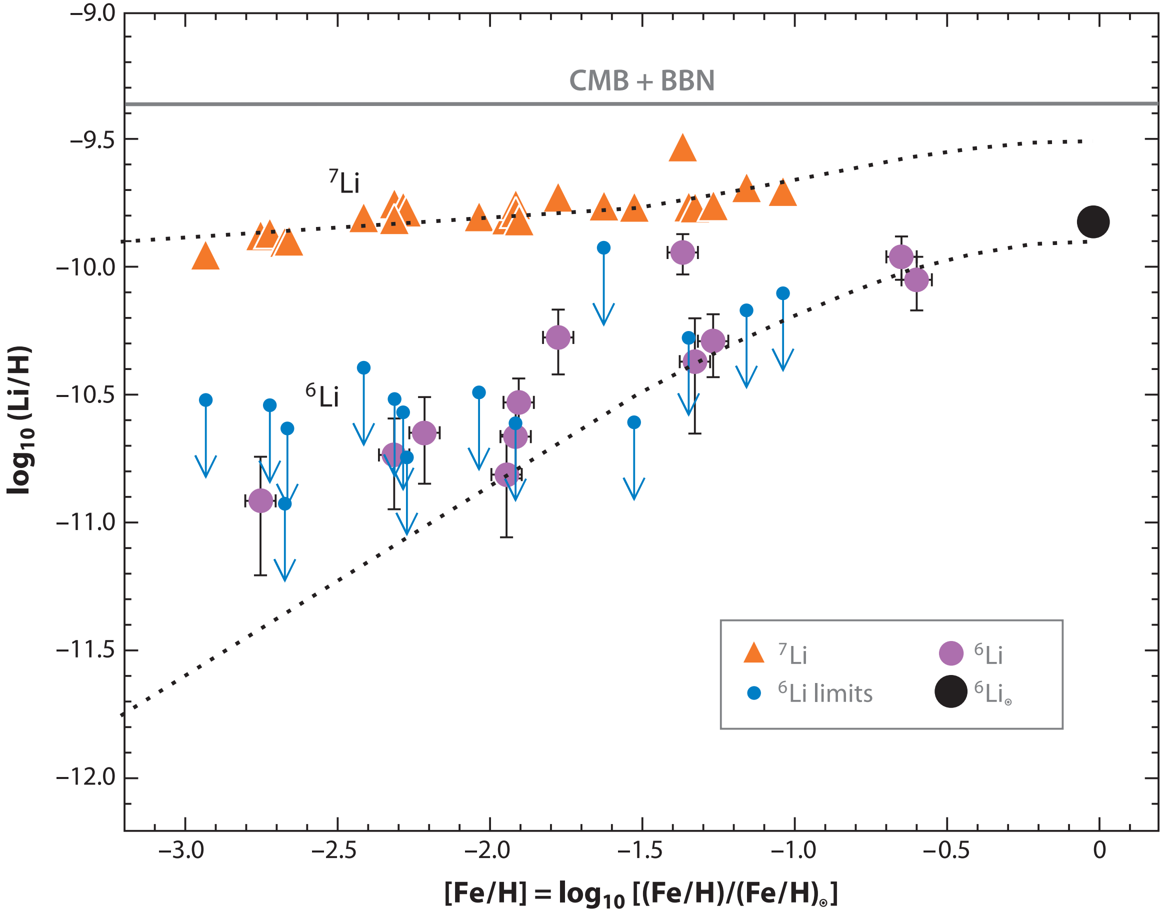
BBN reaction network (simplified)

- **Fields** *Annu. Rev. Nucl. Part. Sci. 2011. 61:47–68*



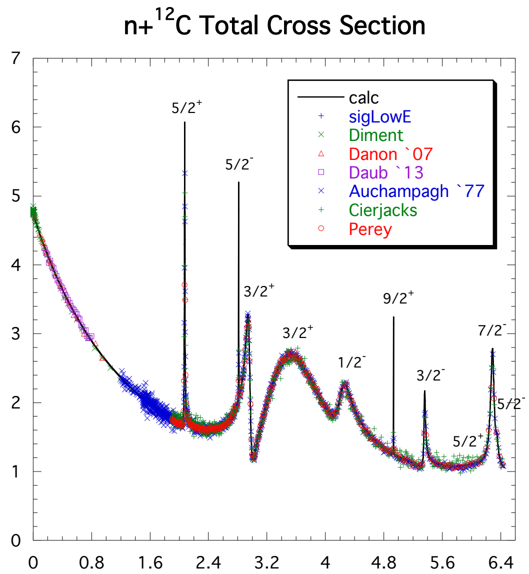
Spite Plateau

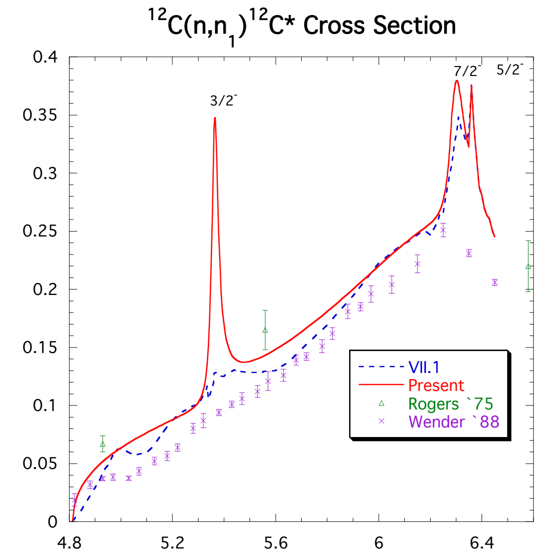
- **Measurement of primordial 7Li from low-metallicity halo dwarf stars**

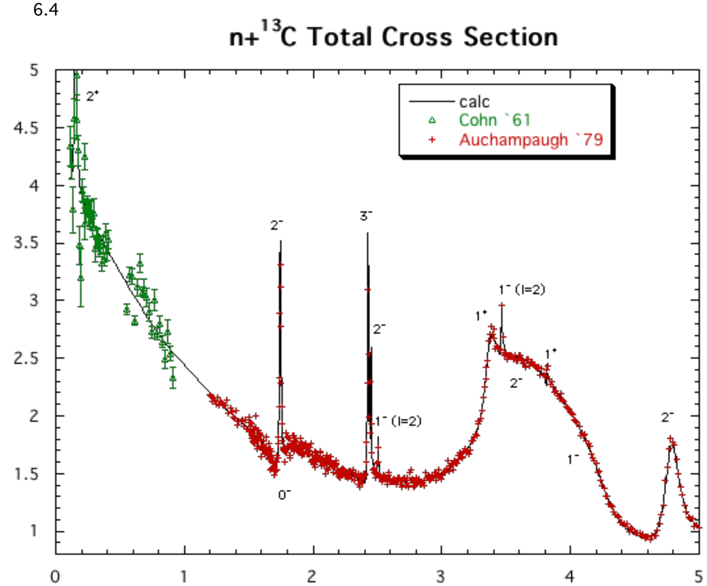


Asplund M, et al. Astrophys. J. 644:229 (2006)

^{13,14}C system analyses: σ_T (b) vs. E_n (MeV)







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