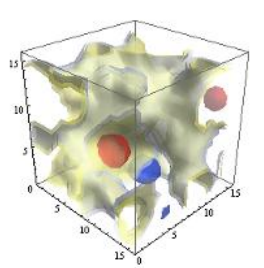
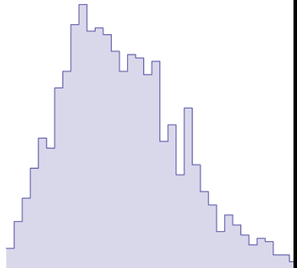
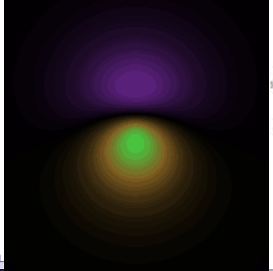
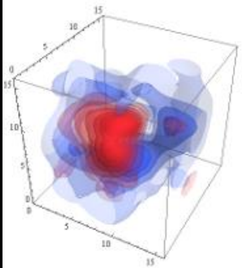
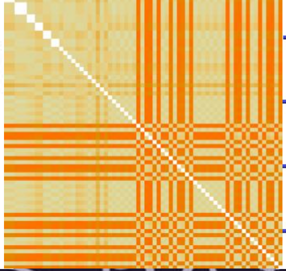
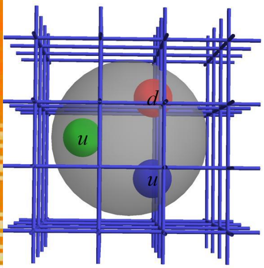


Parton Distribution Functions on the Lattice

Huey-Wen Lin
University of Washington

Outline

§ Introduction to PDFs

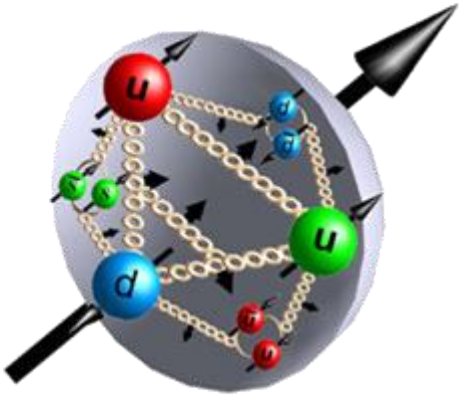
- A brief overview on global analysis

§ Lattice QCD

- Difficulties: why seek a new idea?

§ New Approach on the Lattice

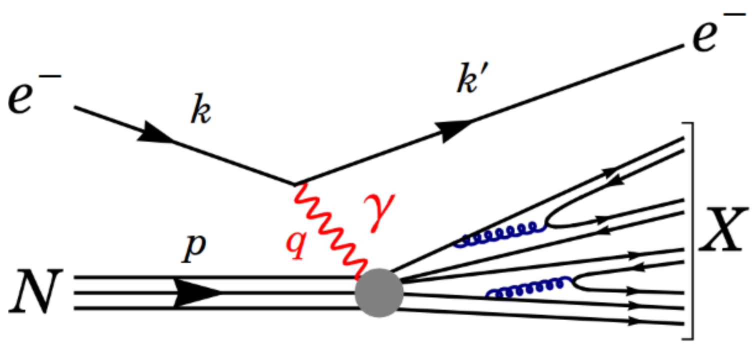
- Preliminary results on nucleon quark, helicity and transversity distributions



Parton Distribution Functions

§ Structure functions studied through scattering processes

- Deep inelastic scattering beginning in 1960s at SLAC
- Depend on energy scale (Q^2) and quark momentum fraction (x)

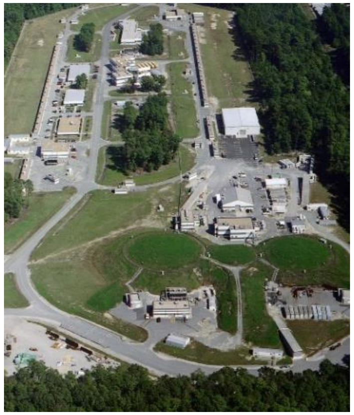


§ "Parton"

- 1969 by Feynman: pointlike constituents inside hadron → now known to be quarks and gluons

§ Still limited knowledge

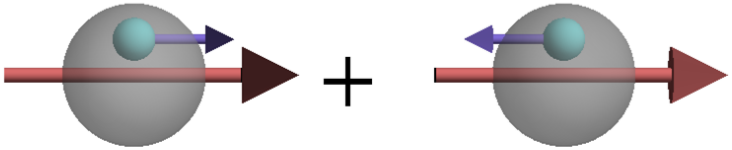
- Many ongoing/planned experiments (EIC, LHeC, ...)



Parton Distribution Functions

§ Quark distribution

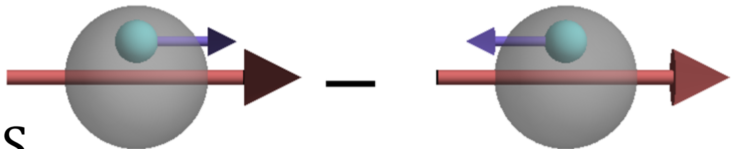
- Processes: DIS (F_2, σ), Drell-Yan, W-asymmetry, Z-rapidity, (γ+) jet, …
- Experiment: BCDMS, NMC, SLAC, JLab, HERA, E866, CDF, DØ,…



spin-averaged/unpolarized

§ Helicity distribution

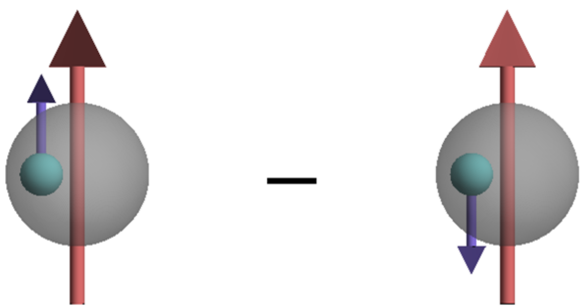
- Processes: polarized DIS, semi-inclusive DIS, photo- and electroproduction of hadrons and charm, *pp* collisions
- Experiment: EMC, HERMES, Hall A, CLAS, COMPASS, STAR, PHENIX, …



spin-dep./long. polarized

§ Transversity distribution

- Process: single-spin asymmetry in SIDIS, …
- Experiment: HERMES, COMPASS, Belle…

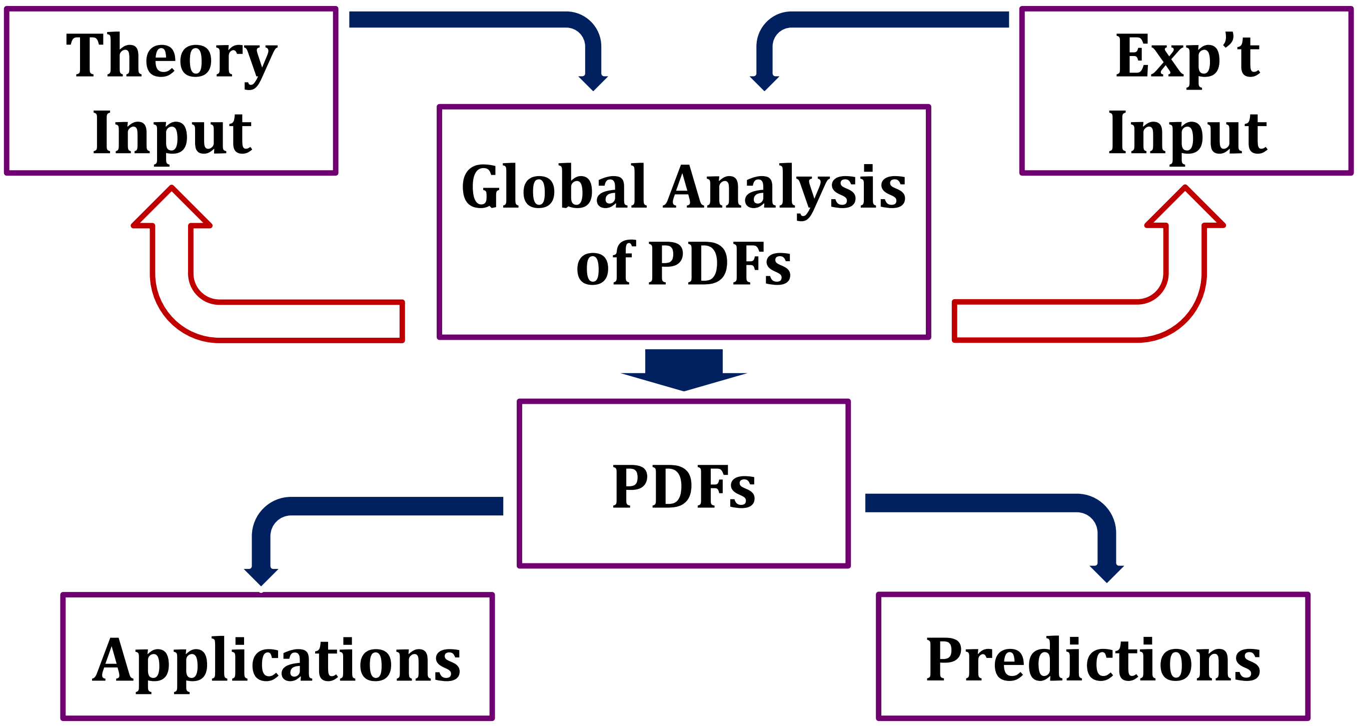


transversely polarized

Global Analysis

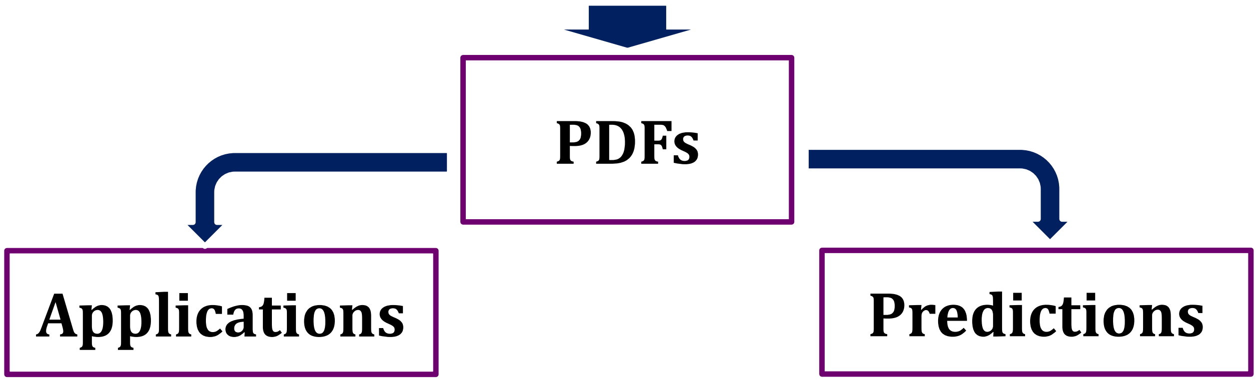
§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets



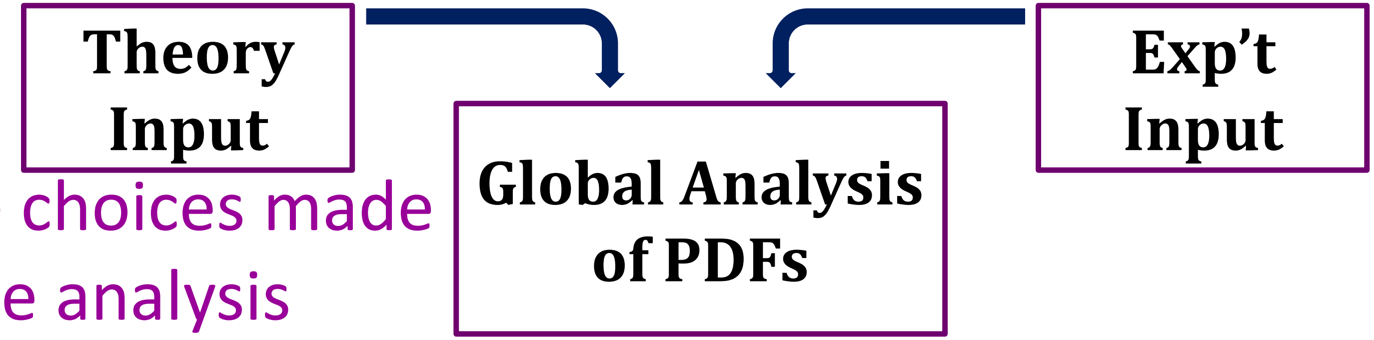
Global Analysis

- Experiments cover diverse kinematics of parton variables
  - Global analysis takes advantage of all data sets



- Important fundamental QCD property
  - Exploration of the valence and sea-quark content of the nucleon
- Important for BSM searches
  - Provides SM cross-section prediction for LHC new-physics search
  - IceCube PeV neutrinos can be explained by PDF uncertainties
  - Proton weak charge (medium-modification effects)

Global Analysis

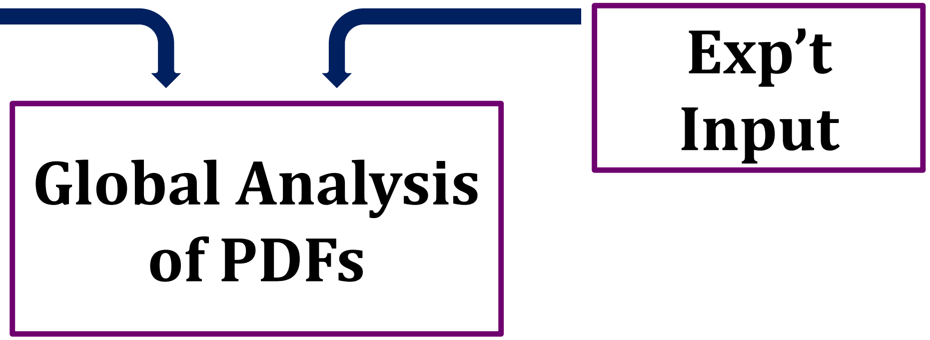


- Some choices made for the analysis
  - Choice of data sets and kinematic cuts
  - Strong coupling constant $\alpha_s(M_Z)$
  - Uncertainties in perturbation theory
    (depends on process whether LO, NLO or NNLO is known)
  - Evolution of PDFs to different scales
  - Parametrization assumptions

$$f(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x) \qquad P(x) = \frac{1+a_3 x + a_4 x^2}{e^{a_3 x}(1+e^{a_4} x)^{a_5}}$$

Discrepancies appear when data is scarce

Global Analysis



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  - Evolution of PDFs to different scales
  - Parametrization assumptions
- Sum rules to constrains the fit
  - Quark number, momentum, g_A, SU(3) flavor symmetry…
- Assumptions imposed where theory and exp’t are lacking
  - Charge symmetry, (anti-)strange, “sea” (antiquark) distribution…

Global Analysis

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

- § Some choices made for the analysis
  - Choice of data sets and kinematic cuts
  - Strong coupling constant $\alpha_s(M_Z)$
  - Uncertainties in perturbation theory
    (depends on process whether LO, NLO or NNLO is kn
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- § Sum rules to constrains the fit
  - Quark number, momentum, g_A, SU(3) flavor s
- § Assumptions imposed where theory and
  - Charge symmetry, (anti-)strange, "sea" (antiquark) distribution…

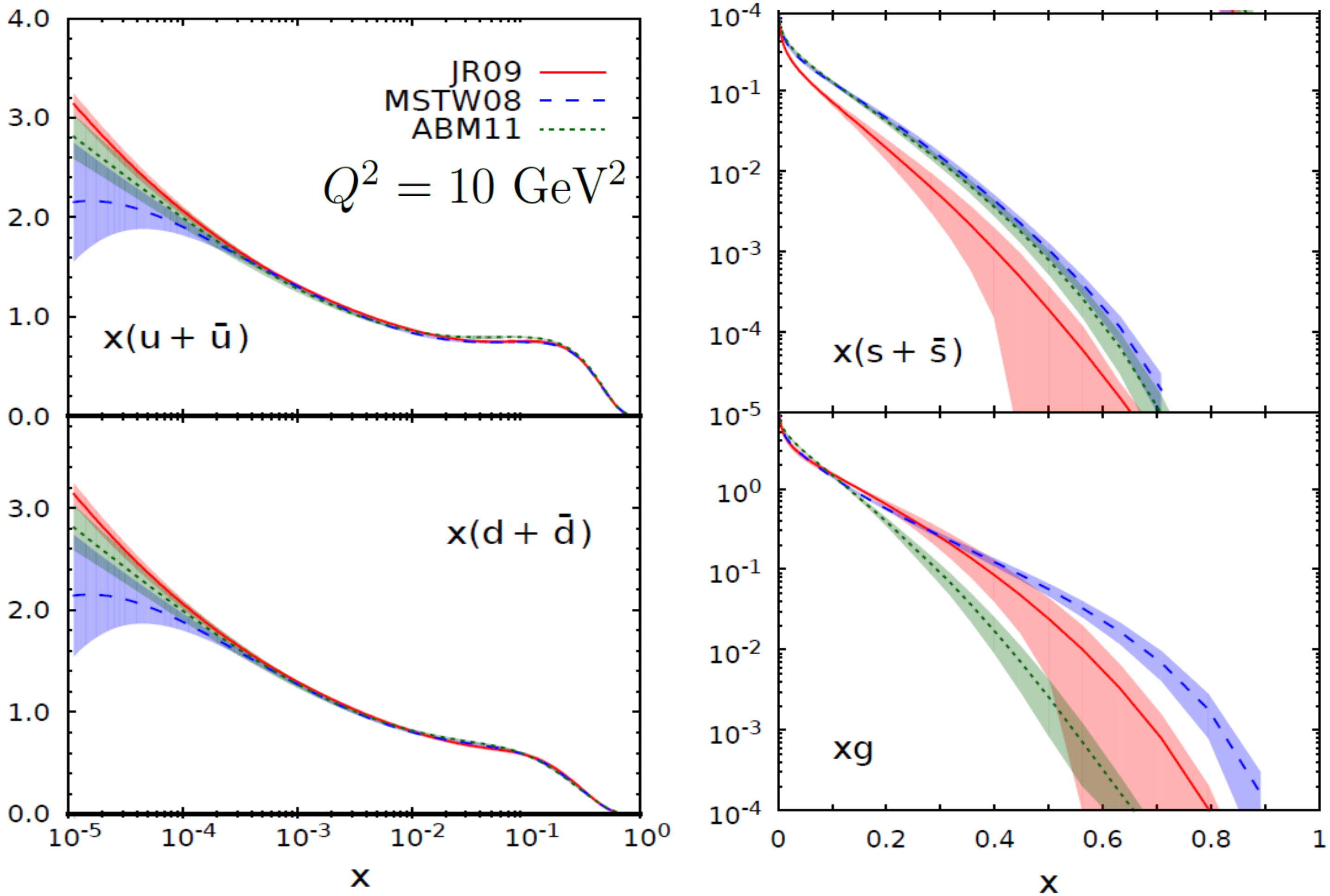
For example,
$s+\bar{s}=\kappa(\bar{u}+\bar{d})$
or symmetric sea in helicity

Global Momentum Analysis

§ Many groups have tackled the analysis

- CTEQ, MSTW, ABM, JR, NNPDF, etc.

Jimenez-Delgado, Melnitchouk, Owens, J.Phys. G40 (2013) 09310



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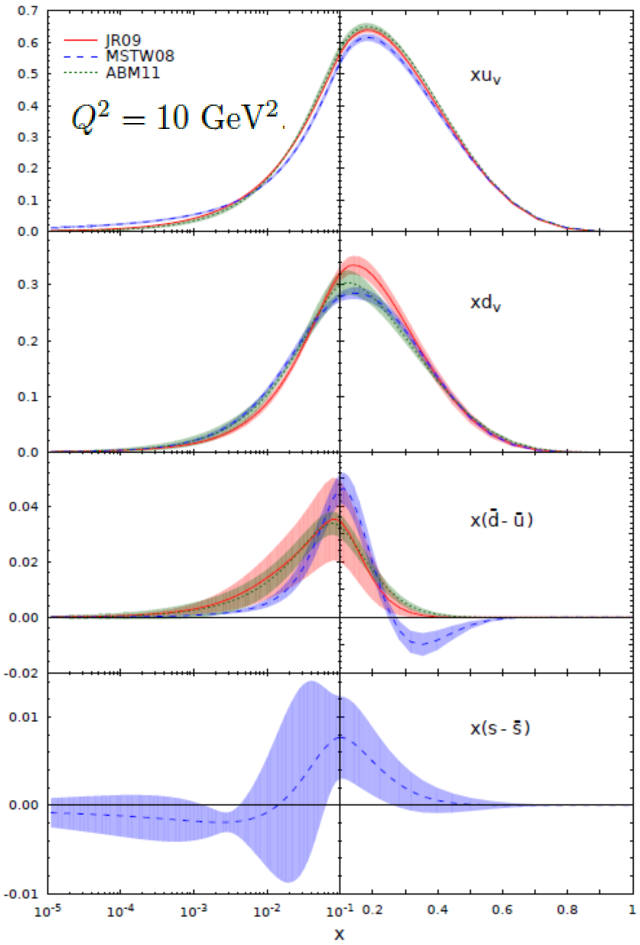
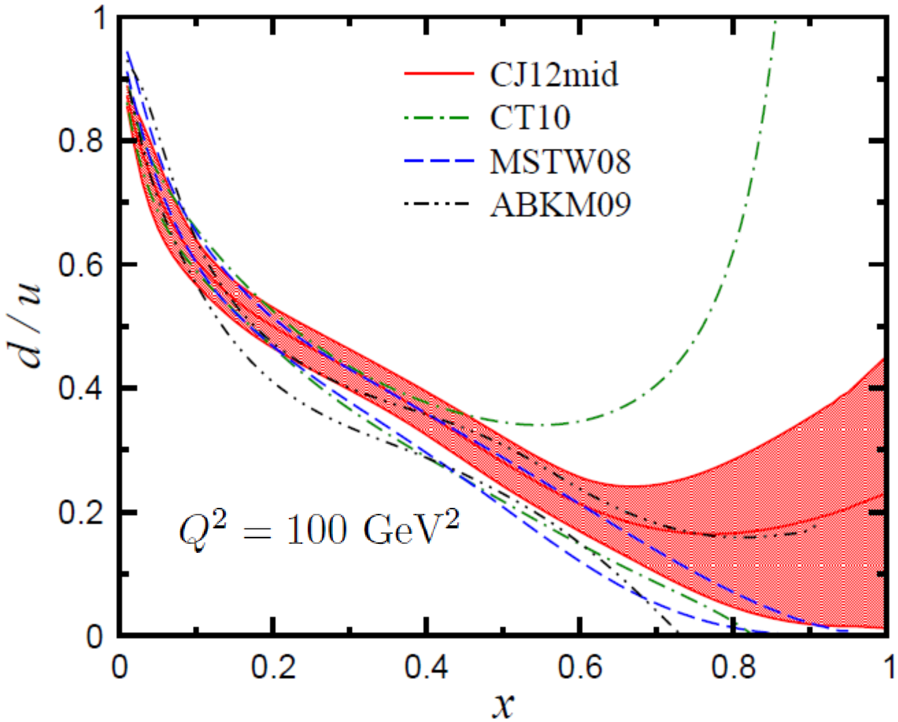
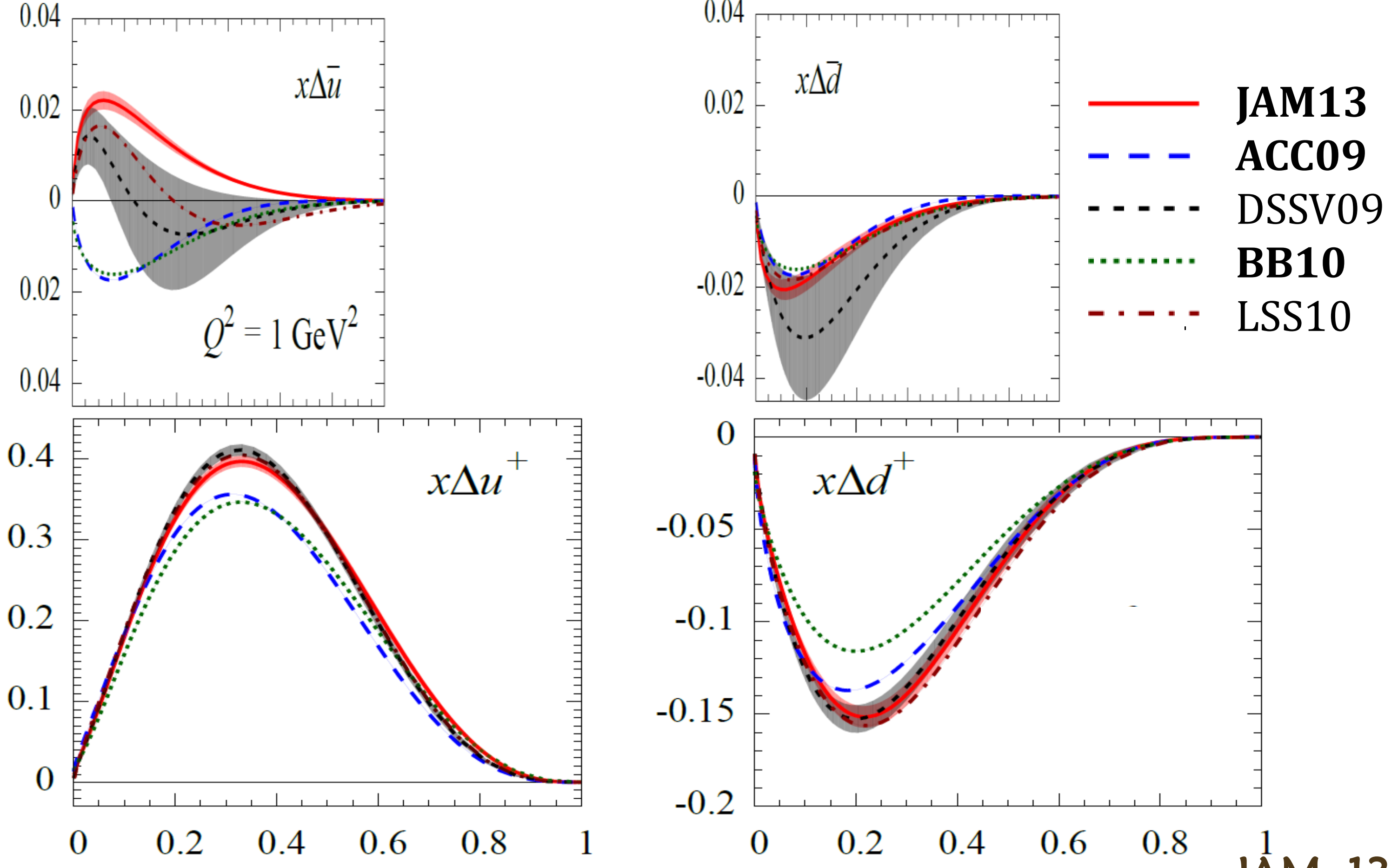




Global Helicity Analysis

§ Many groups have tackled the analysis

- DSSV, ACC, BB, LSS, JAM, etc.

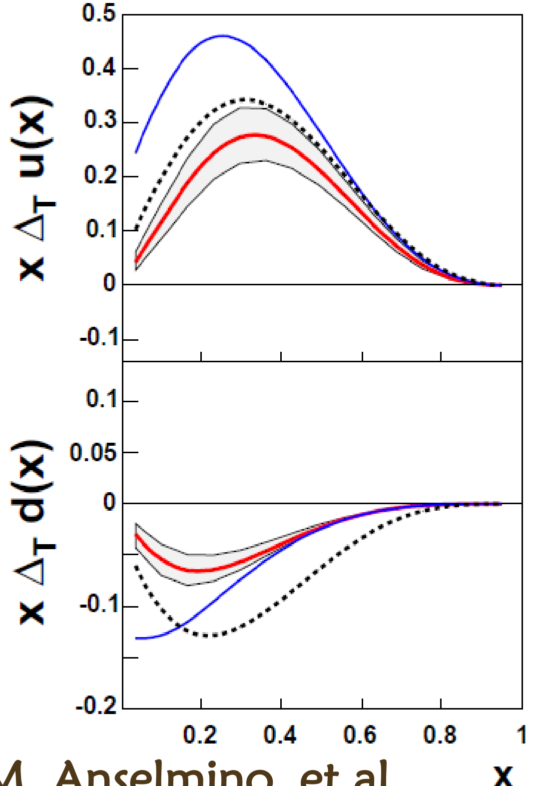


JAM, 1310.3734 [hep-ph]

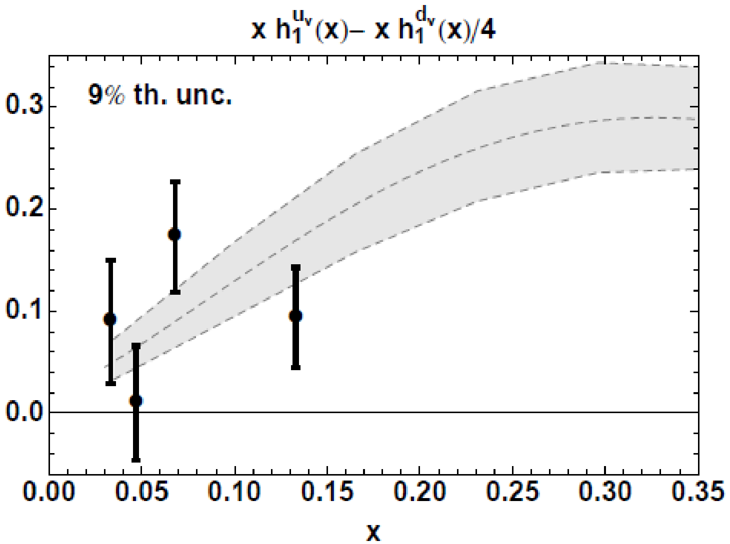
Transversity

§ There have only been 2 attempts (still very preliminary)

- Requires more theory input and experimental data
- More assumptions are made to extract the distribution



M. Anselmino, et al., Nucl.Phys.Proc.Suppl. 191, 98–107 (2009)



A. Bacchetta, A. Courtoy, and M. Radici, Phys.Rev.Lett. 107, 012001 (2011)

PDFs on the Lattice

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

- Ideal tool for studying nonperturbative hadron structure

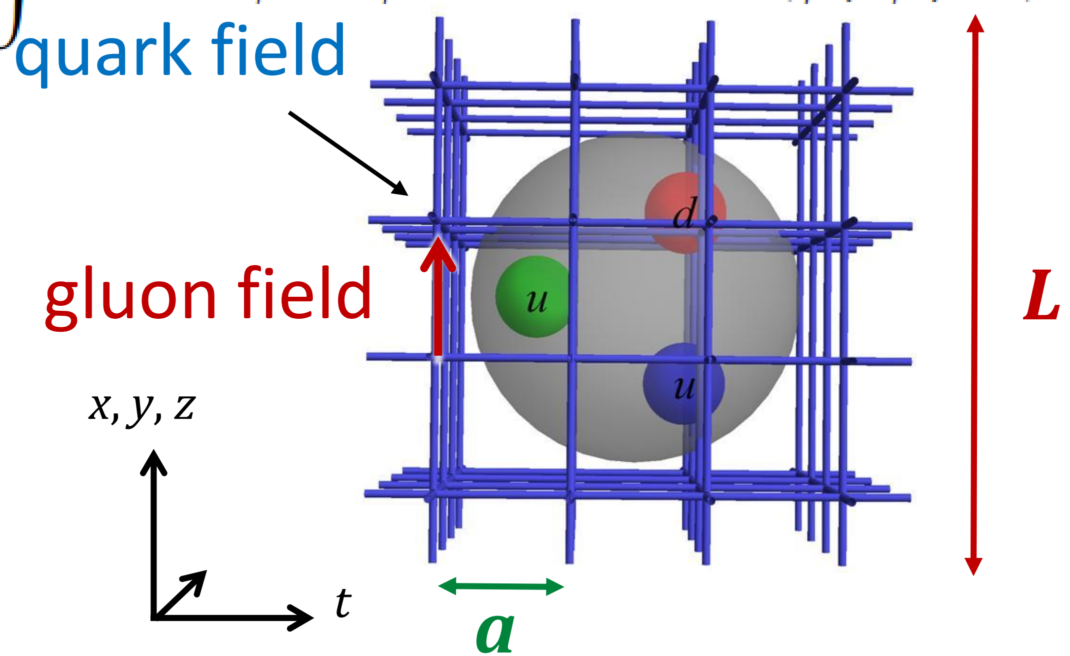
§ Physical observables are calculated from the path integral

$$\langle 0 \,|\, O(\overline{\psi}, \psi, A) \,|\, 0 \rangle = \frac{1}{Z} \int \mathcal{D}A\, \mathcal{D}\overline{\psi}\, \mathcal{D}\psi\, e^{iS(\overline{\psi}, \psi, A)}\, O(\overline{\psi}, \psi, A)$$

§ Lattice QCD

- Impose a UV cutoff
  discretize spacetime
- Impose an Infrared cutoff
  finite volume
- Wick rotate to Euclidean
- Use compact gauge group

$$U_\mu(x) = e^{i a g t_c A^c_\mu(x)}$$



PDFs on the Lattice

- Many lattice calculations of the moments of the PDFs

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx x^{n-1} q(x) = \int_0^1 dx x^{n-1}\left[q(x) - (-1)^{n-1}\overline{q}(x)\right]$$

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx x^{n-1} \Delta q(x) = \int_0^1 dx x^{n-1}\left[\Delta q(x) + (-1)^{n-1}\Delta\overline{q}(x)\right]$$

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx x^{n-1} \delta q(x) = \int_0^1 dx x^{n-1}\left[\delta q(x) - (-1)^{n-1}\delta\overline{q}(x)\right]$$

  - Limited to the lowest few moments
  - Might provide constraints on models or tests of experiment
- Also applies to GPDs: limited to 3rd moment
- Most progress made in quark contributions
  - Very costly to obtain useful gluon signal
  - Limited by available computational resources

$\langle x^n \rangle$ Moments

§ Leading moment ⟨*x*⟩, hypercubic decomposition

- $4_1 \otimes 4_1 = \quad 1_1 \oplus 3_1 \oplus 6_1 \oplus \quad 6_3$;

  O … ires $p \neq 0$)

- Both ope…

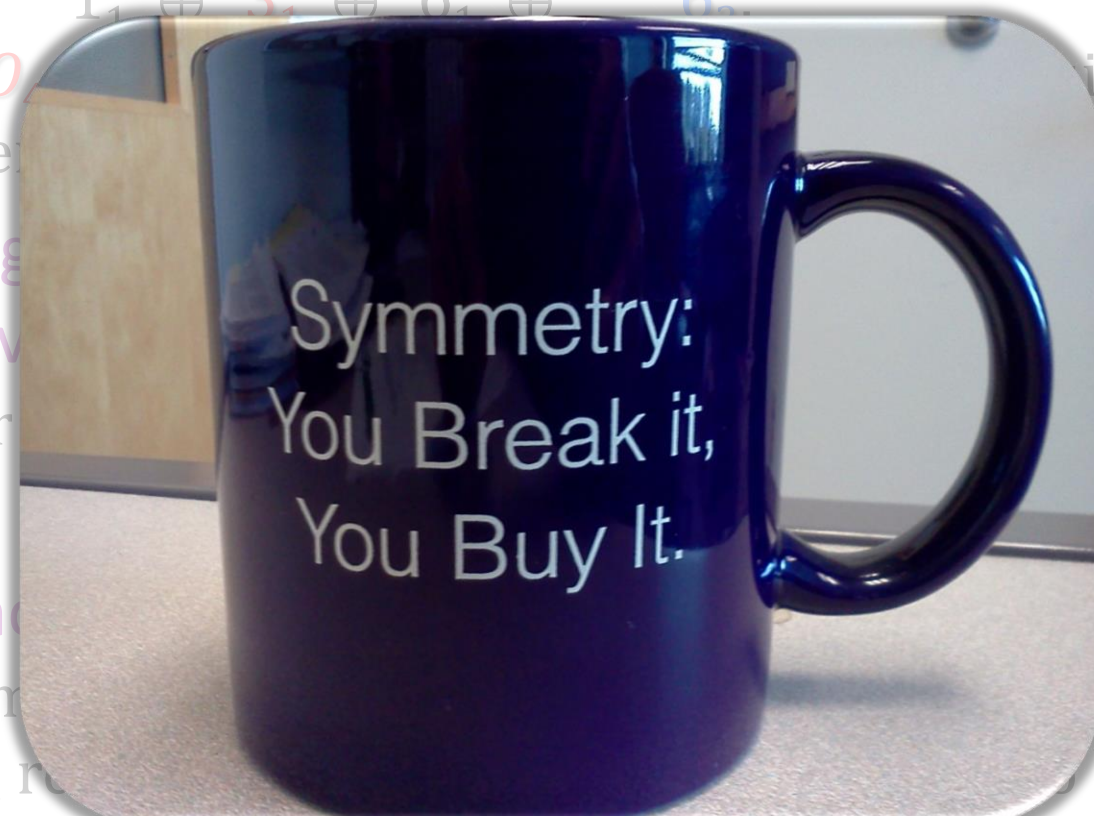
§ No mixing … mension

§ To improv…

- Consider … try:

  … $c_1 \bar{q}\, \sigma_{\mu\rho} \overleftrightarrow{D}_{[\nu} \overleftrightarrow{D}_{\rho]} q$

  … $\partial_\rho \left(\bar{q}\, \sigma_{\mu\rho} \overleftrightarrow{D}_\nu q \right)$



§ Higher m…

- 4_1: O_{111} m…
- 4_2: $O_{\{123\}}$ r… be nonzero
- 8_1: $O_{\{441\}} - (O_{\{221\}} + O_{\{331\}})/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops

$\langle x^n \rangle$ Moments

- Leading moment $\langle x \rangle$, hypercubic decomposition
  - $4_1 \otimes 4_1 = \quad 1_1 \oplus 3_1 \oplus 6_1 \oplus \quad 6_3$:
    $O_{44} - (O_{11}+O_{22}+O_{33})/3 \qquad O_{14}+O_{41}$, (requires $p \neq 0$)
  - Both operators go to same continuum limit
- No mixing with operators of same or lower dimension
- To improve to $O(a)$
  - Consider all irrelevant operators of same symmetry:

$$O_{\{\mu\nu\}} \longrightarrow (1 + a\, m_q\, c_0)\, O_{\{\mu\nu\}} + i\, a\, c_1\, \overline{q}\, \sigma_{\mu\rho}\, \overleftrightarrow{D}_{[\nu}\, \overleftrightarrow{D}_{\rho]}\, q + a\, c_2\, \overline{q}\, \overleftrightarrow{D}_{[\mu}\, \overleftrightarrow{D}_{\nu]}\, q + i\, a\, c_3\, \partial_\rho \left(\overline{q}\, \sigma_{\mu\rho}\, \overleftrightarrow{D}_\nu\, q\right)$$

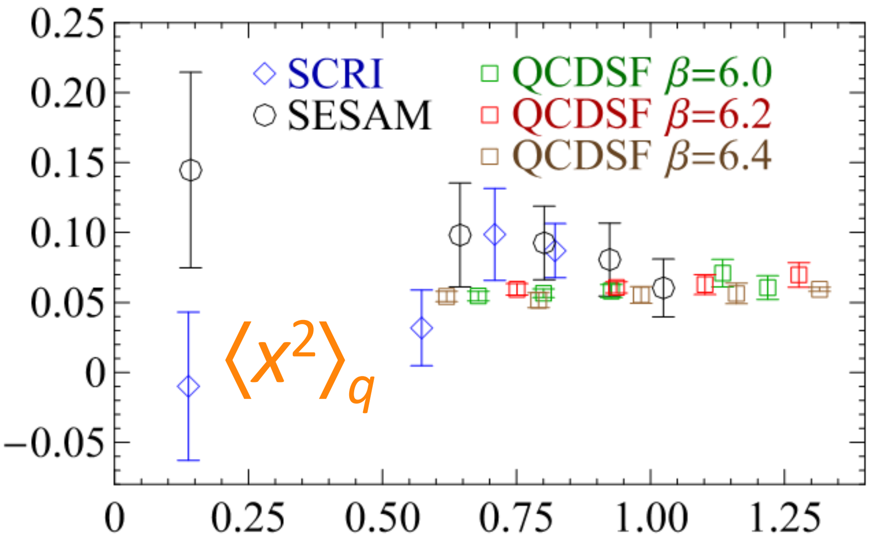
- Higher moments $\langle x^2 \rangle$
  - 4_1: O_{111} mixes with $\bar{q}\gamma_1 q$ with coefficient $\sim 1/a^2$
  - 4_2: $O_{\{123\}}$ requires all momentum components to be nonzero
  - 8_1: $O_{\{441\}} - (O_{\{221\}}+O_{\{331\}})/2$ mixes under renormalization
- For higher spin, all ops mix with lower-dimension ops

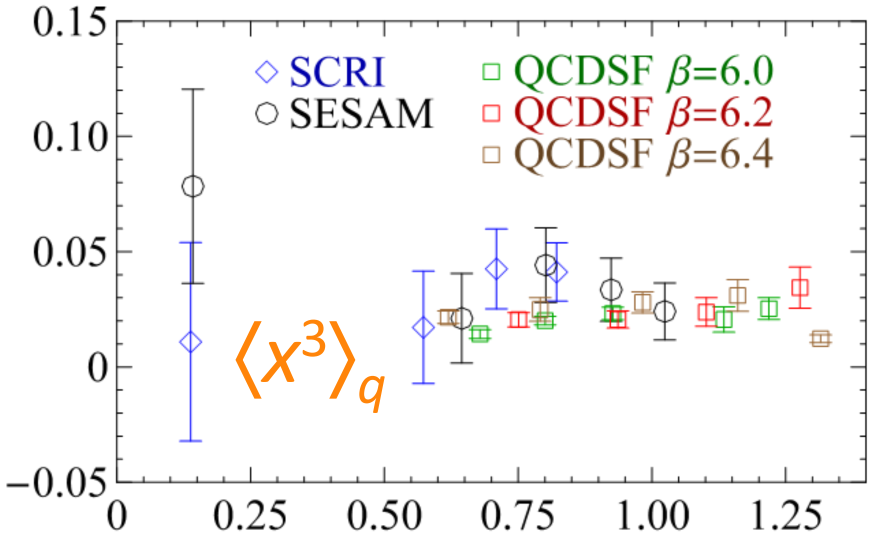
$\langle x^n \rangle$ Moments

- For higher spin, all ops mix with lower-dimension ops
  - Tricks: subtraction to remove divergent terms, heavy fields, four-point functions… None is practical enough
- Relative error grows in higher moments
  - Calculation would be costly

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f



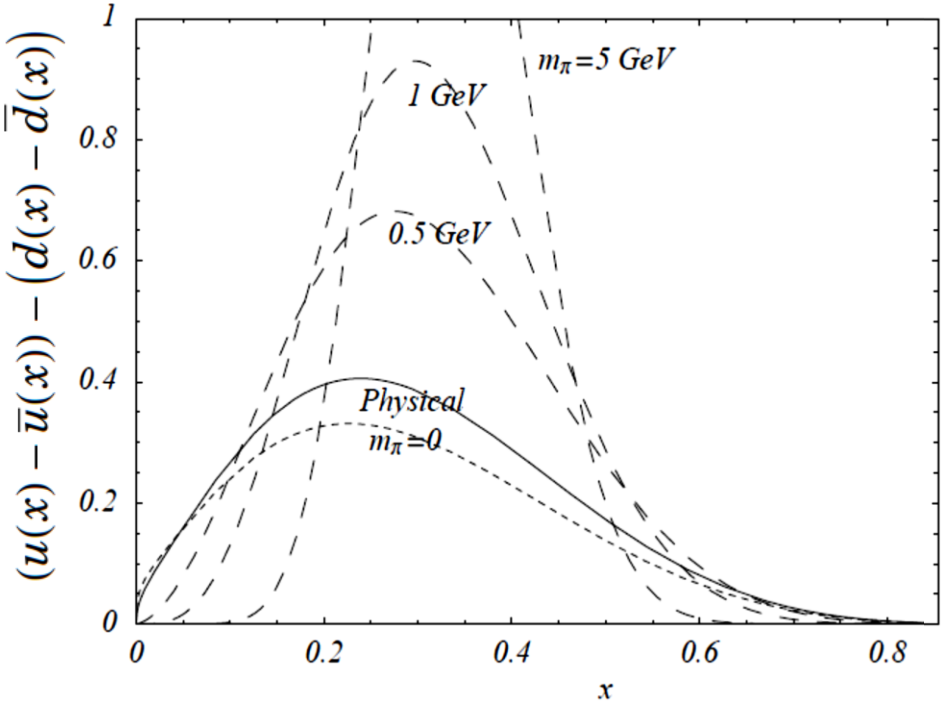


Limited Access

§ What can we learn about the *x*-distribution?

- Make an ansatz of some smooth form for the distribution and fix the parameters by matching to the lattice moments

$$xq(x) = a\,x^b(1-x)^c\left(1+\epsilon\sqrt{x}+\gamma x\right)$$



Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

W. Detmold et al, Eur.Phys.J.direct C3 (2001) 1–15

Quark Distribution

§ Lightcone nucleon quark distribution

- Transform lab coordinates to light-cone ones $x_\pm = z \pm t$

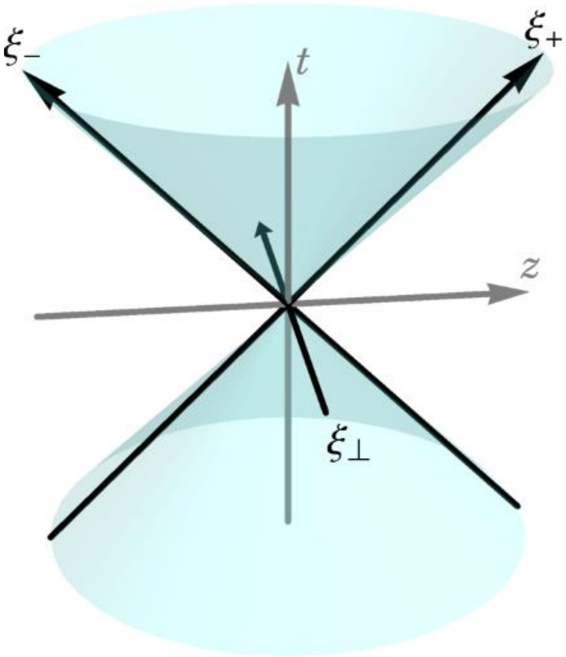
$$q(x,\mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left\langle P \left| \overline{\psi}(\xi_-)\gamma_+ \exp\left(-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right)\psi(0) \right| P \right\rangle$$

Renormalization scale μ

Lightcone coordinate $\xi_\pm = (t \pm z)/\sqrt{2}$

Nucleon momentum P_μ

Gluon potential A_+



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Lightcone coordinate $\xi_{\pm} = (t \pm z)/\sqrt{2}$

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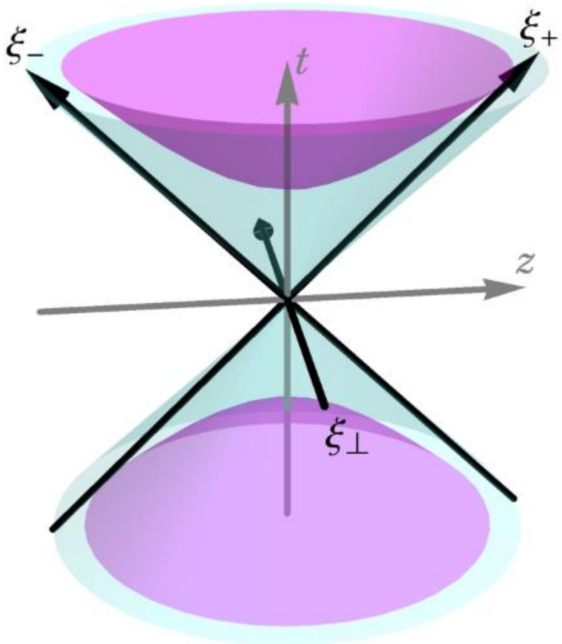

- Massive particles lie on hyperboloids invariant under Lorentz transformation

The Idea

§ Lightcone nucleon quark distribution

$$q(x,\,\mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left\langle P \,\middle|\, \overline{\psi}(\xi_-)\,\gamma_+ \exp\!\left(-ig\int_0^{\xi_-} d\eta_-\, A_+(\eta_-)\right)\psi(0) \,\middle|\, P \right\rangle$$

Renormalization scale μ

Lightcone coordinate $\xi_\pm = (t \pm z)/\sqrt{2}$

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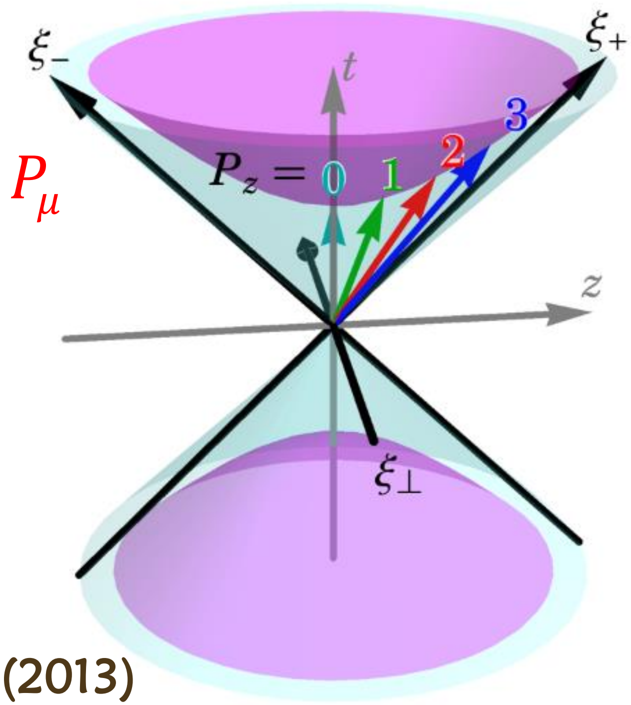
Gluon potential A_+



§ Approaching lightcone with large *P*

- Just another limit to take, like taking $a \to 0$

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

The Idea

§ Finite-momentum quark distribution

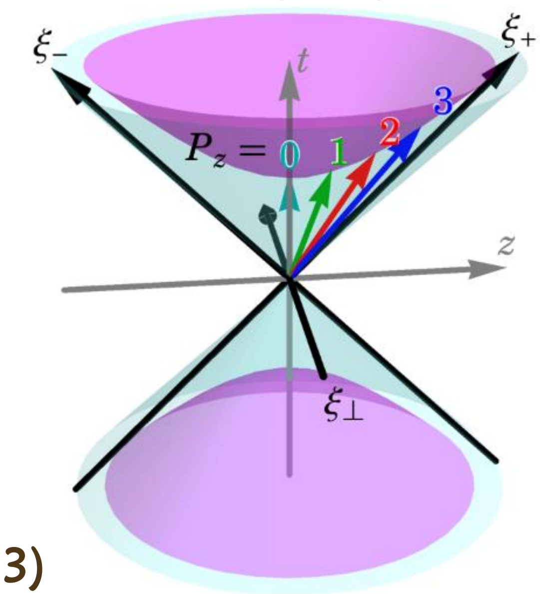
$$q(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\,\gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle + \mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/P_z^2,\, M_N^2/P_z^2\right)$$

$x = k_z/P_z$

Lattice z coordinate

Nucleon momentum $P_\mu = \{P_0, 0, 0, P_z\}$

Product of lattice gauge links



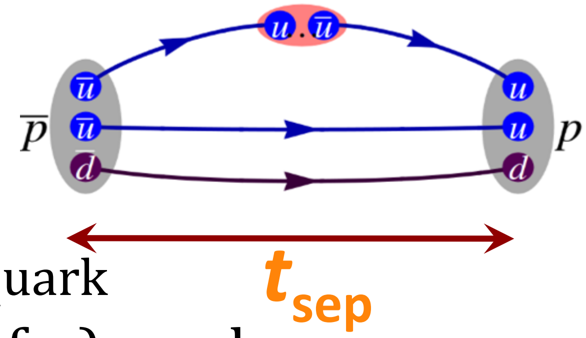
- In $P_z \longrightarrow \infty$ limit, parton distribution is recovered
- For finite P_z, corrections are needed

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

Some Lattice Details

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm** ($L \approx$ 2.88 fm)
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separation ($t_{sep} \approx$ 0.96 and 1.2 fm) used



§ Properties known on these lattices

- Lattice Z_Γ for bilinear operator ~ 1
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

Hyak @ UW

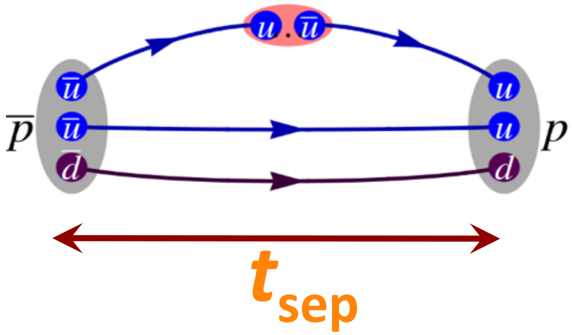
§ Feasible with today's computational resources!

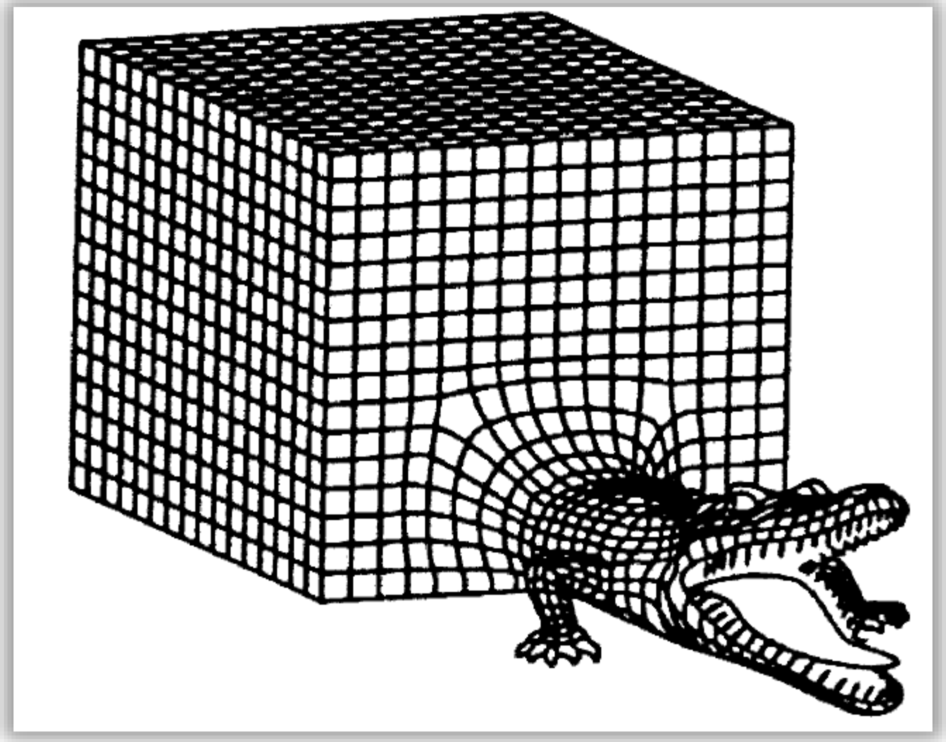
- 8/16 nodes on UW Hyak cluster

Warning!

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm** ($L \approx$ 2.88 fm)





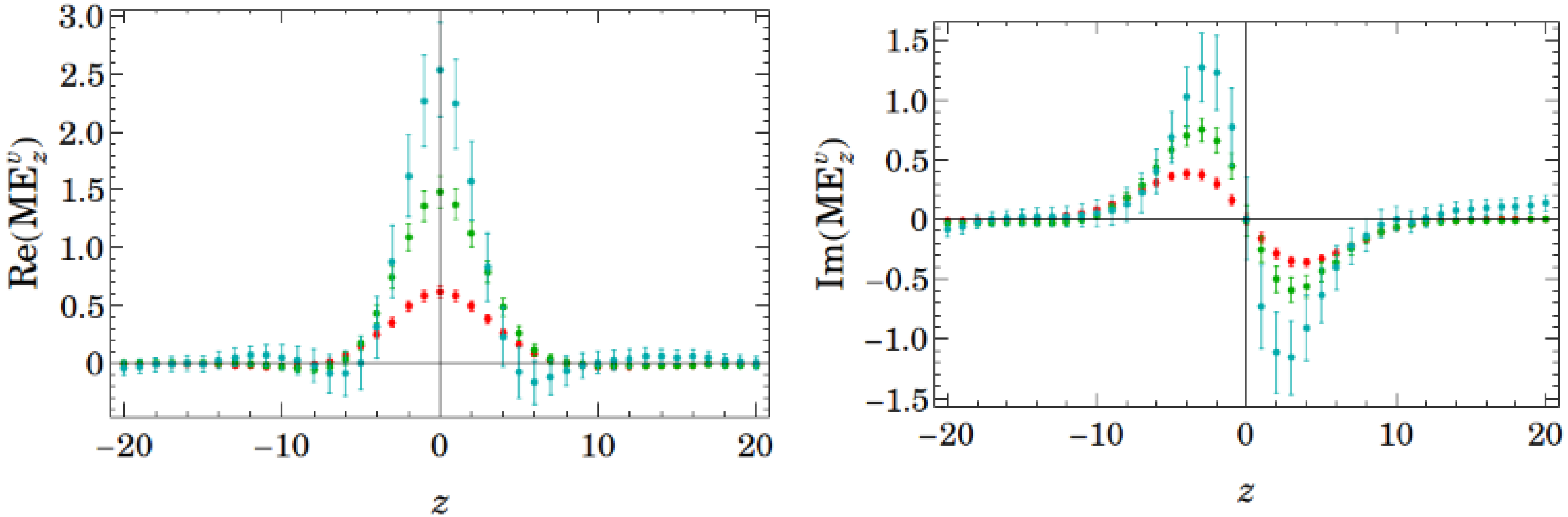
NO SYSTEMATICS YET!

Quark Distribution

§ Exploratory study

$$\left\langle P \,\middle|\, \overline{\psi}(z)\, \gamma_z \exp\!\left(-i g \int_0^z d z'\, A_z(z')\right) \psi(0) \,\middle|\, P \right\rangle$$

- How many links are needed?



- Lattice momenta discretized by finite size of volume

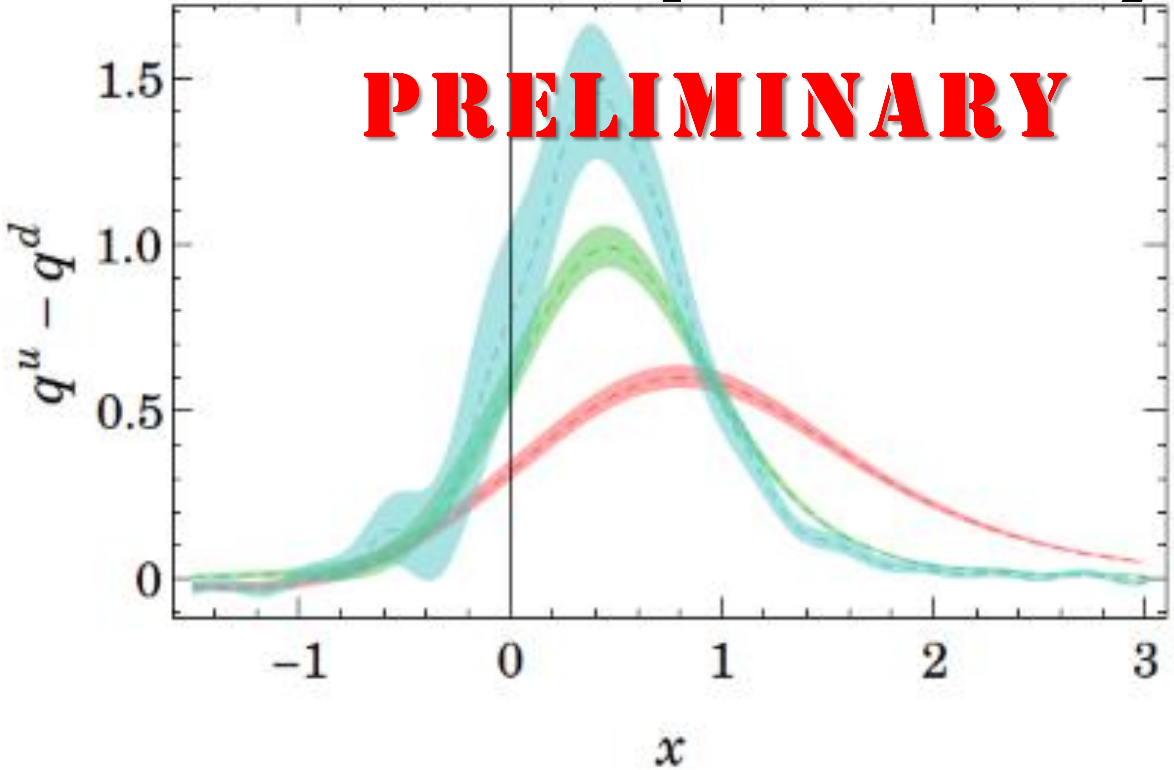
$$P_z \in \{1, 2, 3\}\ {}^{2\pi}/_{L}$$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$



Uncorrected bare lattice results

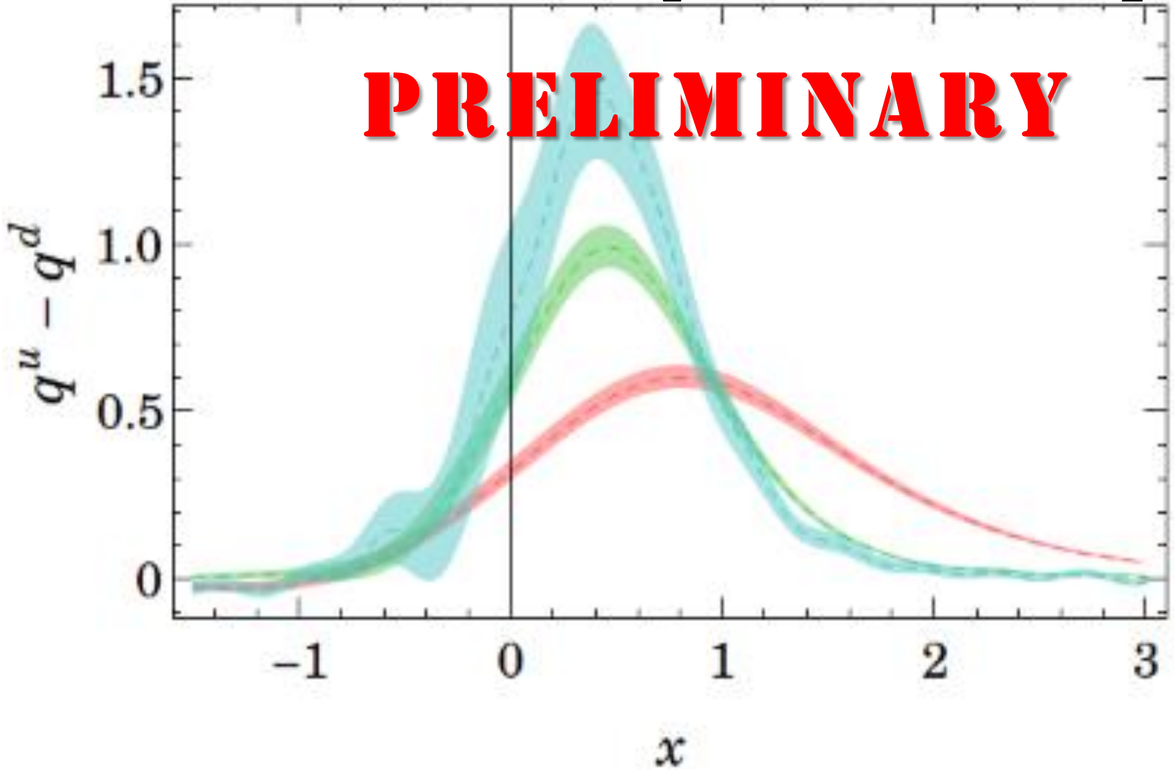
$x = k_z / P_z$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$P_z \in \{1, 2, 3\}\ {}^{2\pi}/_L$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections

Quark Distribution

§ Back to the continuum

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\,\mu) = q_{\mathrm{FP}}(x,\,\mu,\,P_z) + O\left(\Lambda_{\mathrm{QCD}}^2 / P_z^2\right) + O\left(M_N^2 / P_z^2\right) + O(\alpha_s)$$

What we want

What we calculate on the lattice

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$

Smaller P_z correction but complicated twist-4 operator (extrapolate it away)

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

Dominant correction (for nucleon); known scaling form

J.-W. Chen et al. (in preparation)

Finite $P_z \to \infty$

Estimate $O(20\%)$ effect

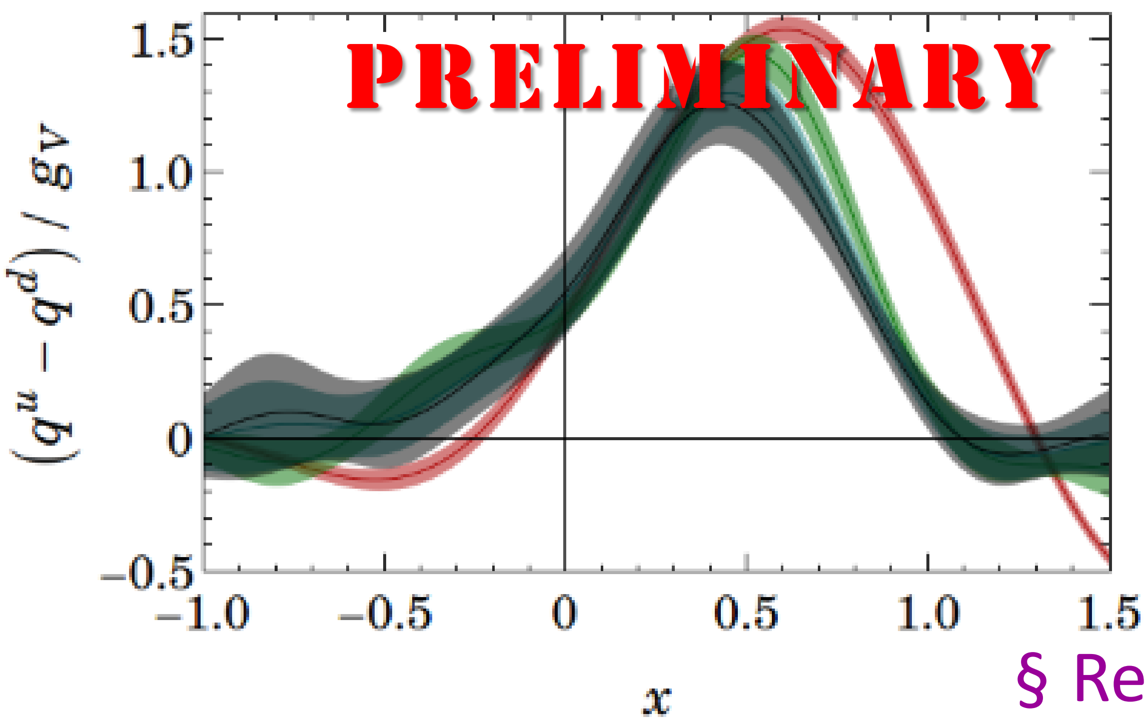
X. Xiong, X. Ji, J. Zhang, 1310.7471 [hep-ph]

Quarky Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\mathrm{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx\, q(x, \mu, P_z)}$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

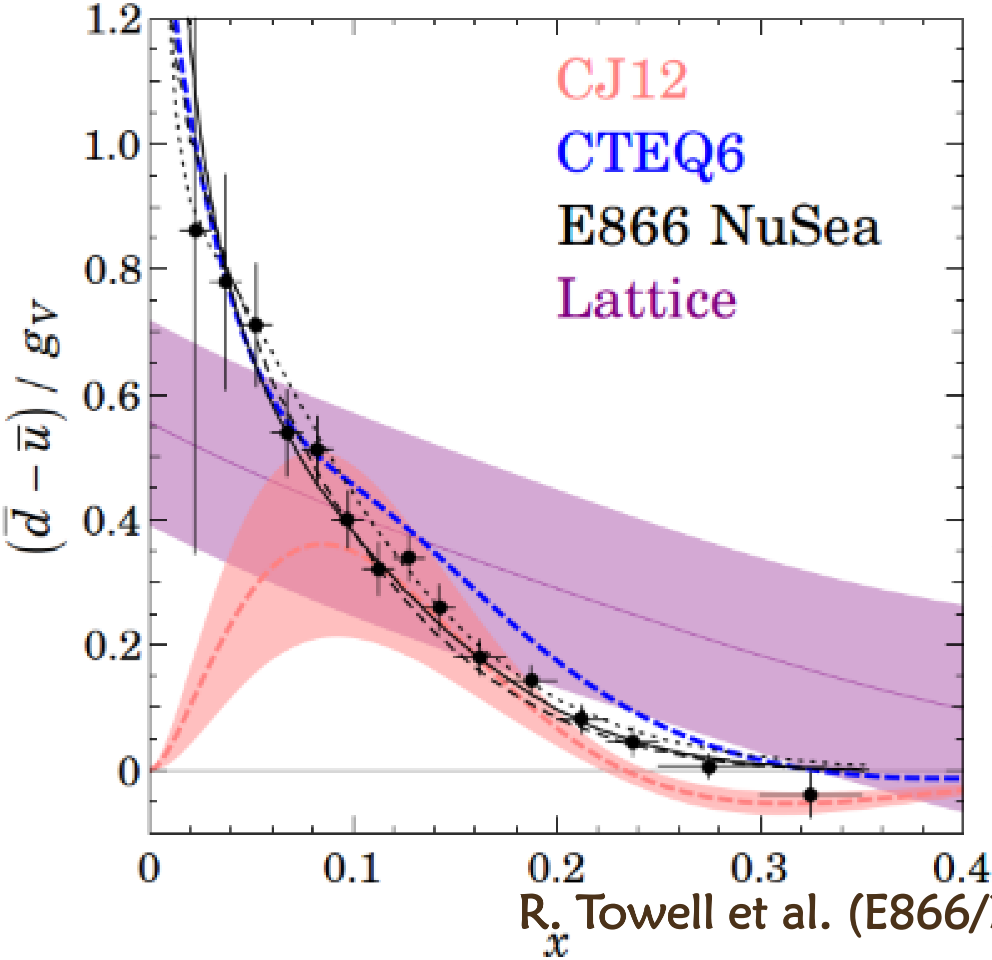
No significant finite-momentum effect seen for $P_z > 1$

§ Renormalization needed

Quark Distribution

§ Compare with experiments

$$\overline{q_f}(x) = -q_f(-x)$$



Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Future improvement to have larger lattice volume

$$\int dx\,\frac{\bar{u}(x)-\bar{d}(x)}{g_V} \approx -0.16(12)$$

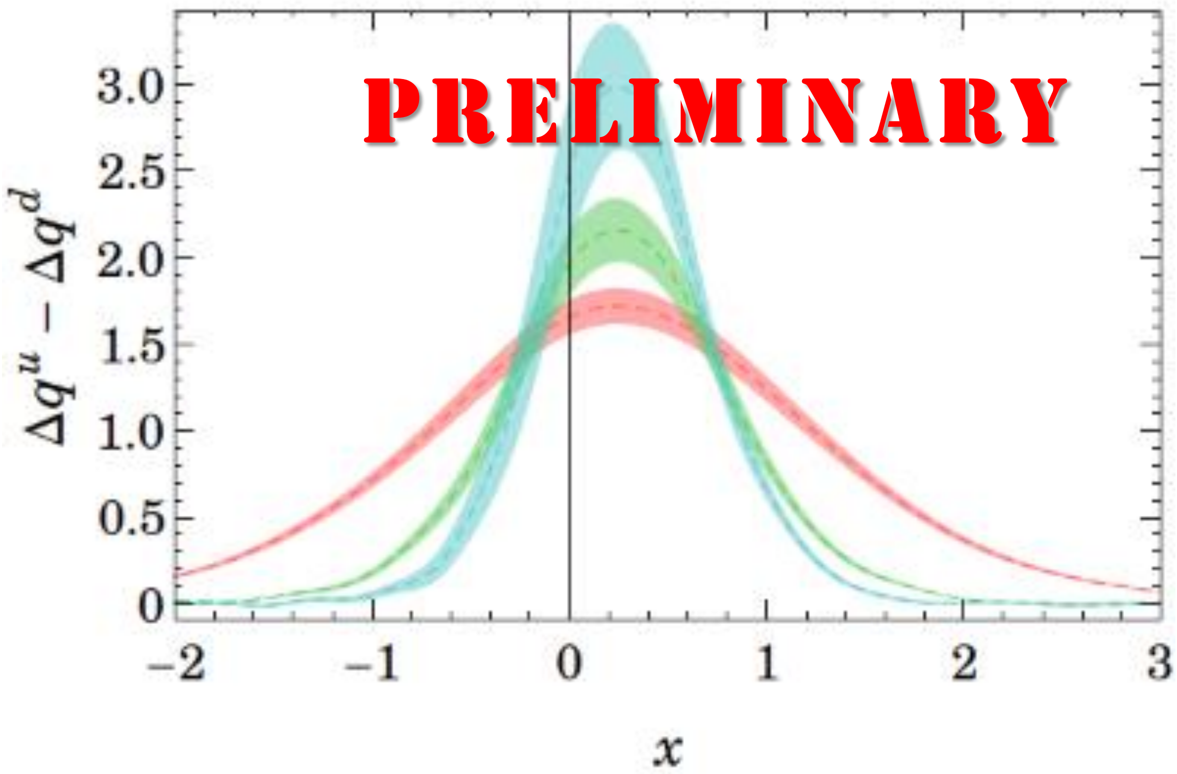
Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Helicity Distribution

§ Exploratory study

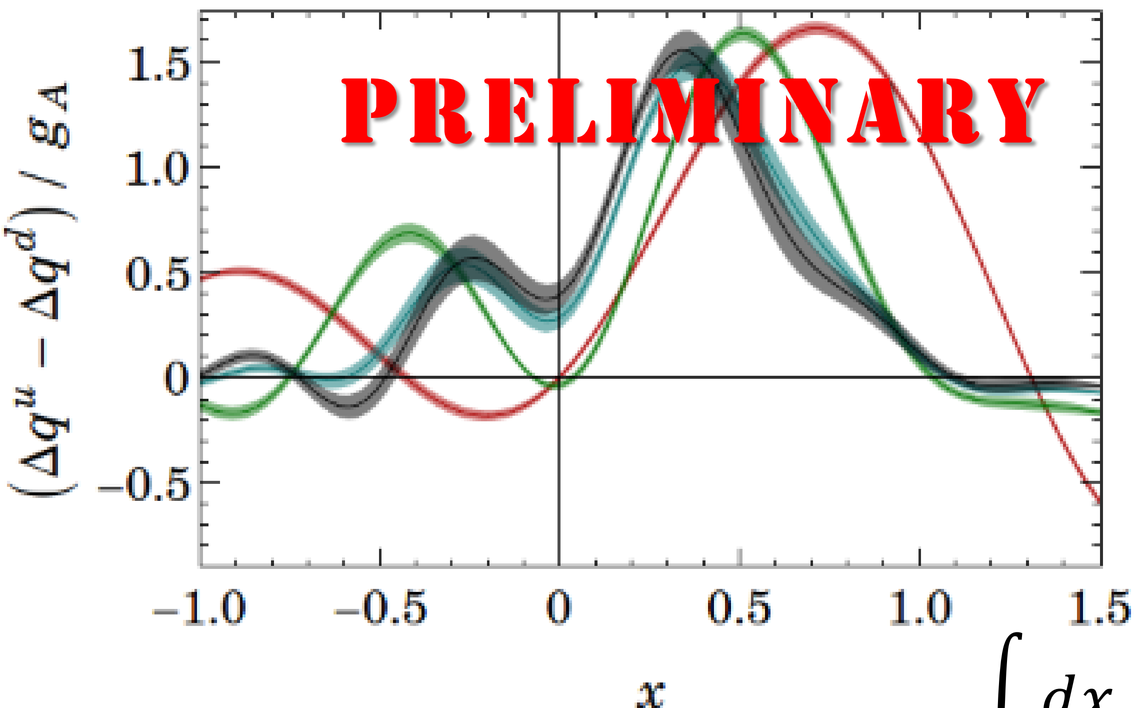
$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$



Helicity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

Larger $O(\Lambda_{QCD}^2/P_z^2)$ seen but well fit by extrapolation

$$\int dx\, \frac{\Delta\bar{u}(x) - \Delta\bar{d}(x)}{g_A} \approx 0.19(5)$$

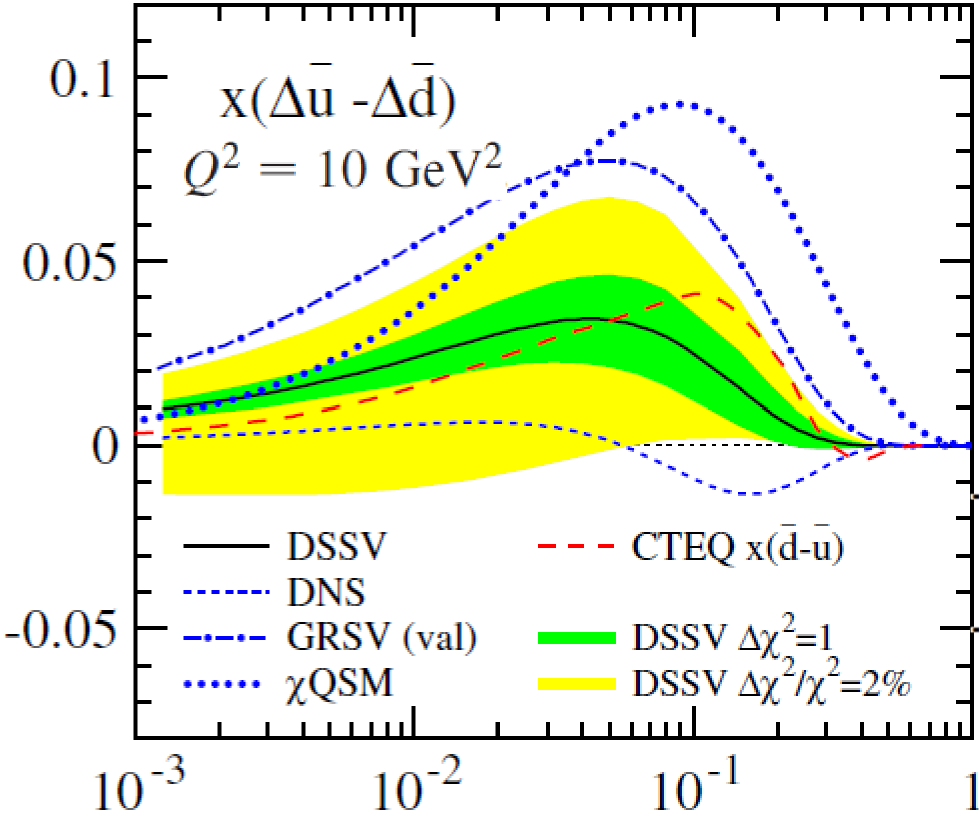
Helicity Distribution

§ Model: large-N_c predicts larger polarized antiquark asymmetry

chiral quark-soliton model $\int dx\,(\Delta\bar{u}(x) - \Delta\bar{d}(x)) \approx 0.31$

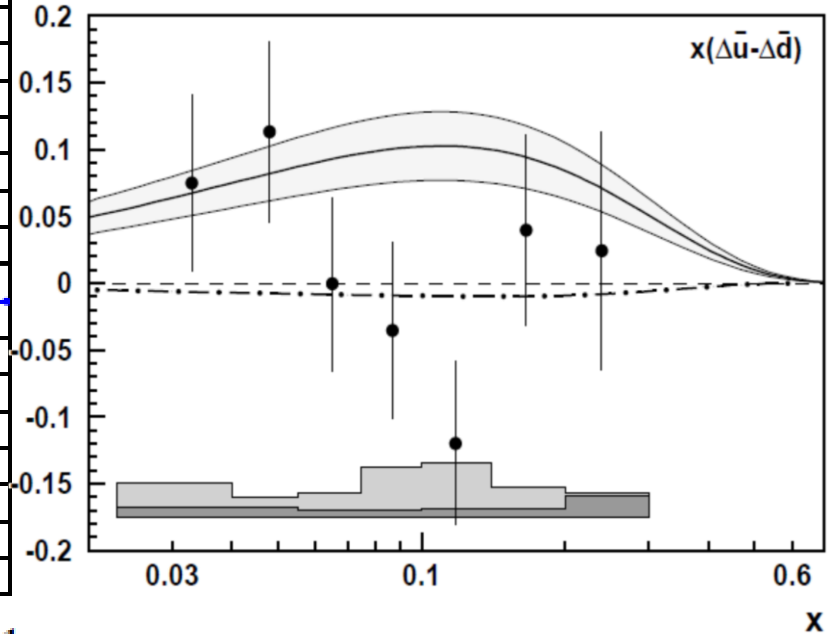
B. Dressler et al, hep-ph/9809487

§ Experimental comparison



D. De Florian et al., Phys.Rev. D80, (2009) 034030

A. Airapetian et al. (HERMES), Phys.Rev. D71, 012003 (2005)



Helicity Distribution

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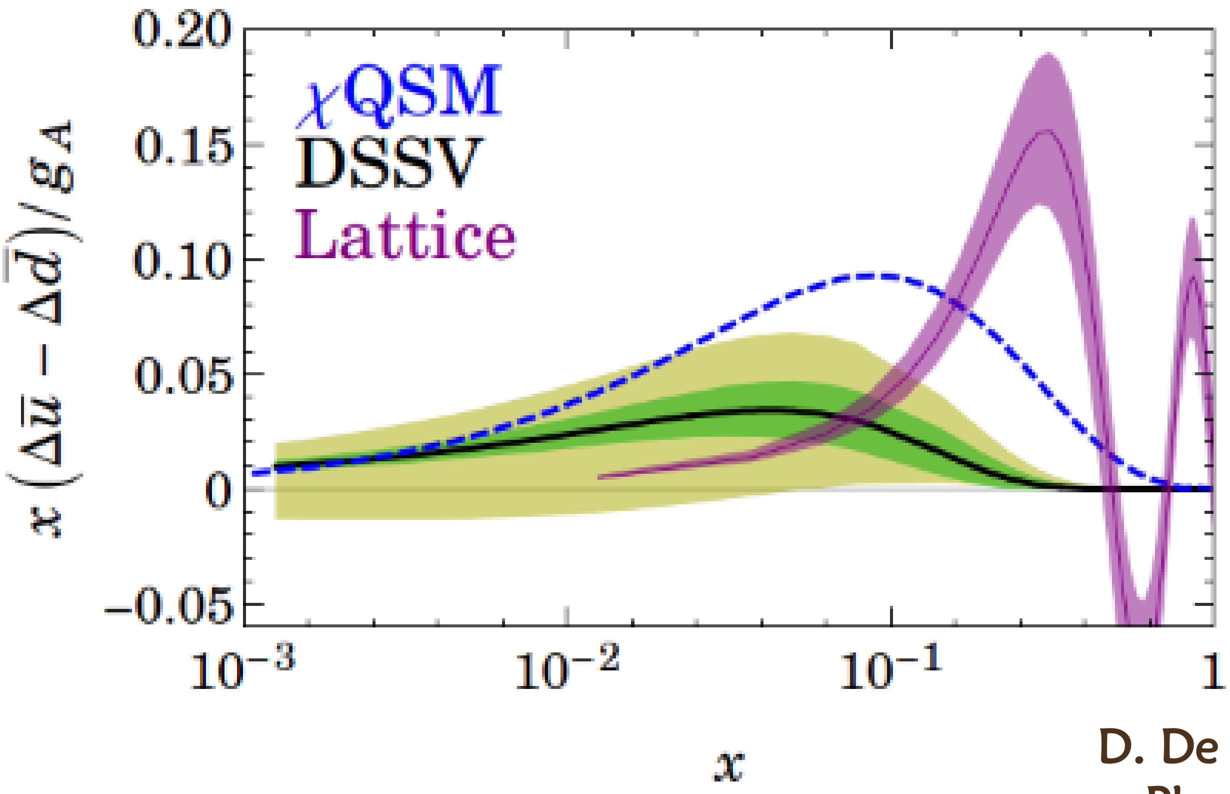
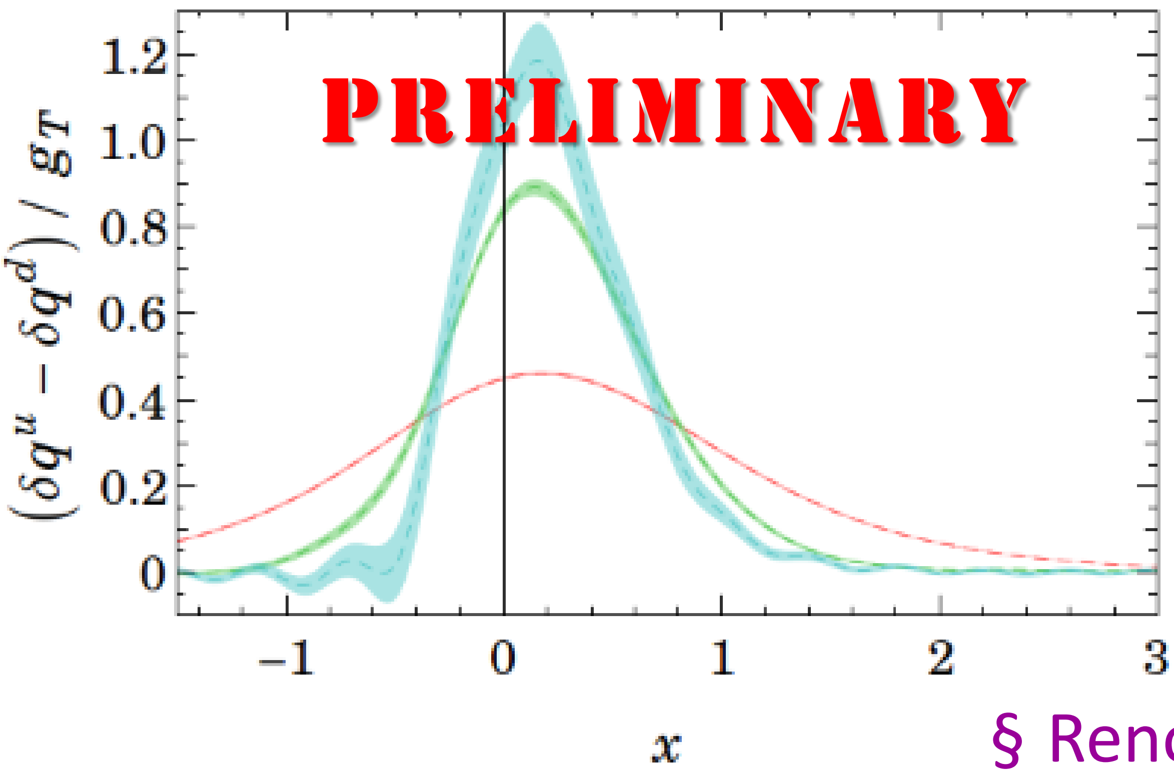


D. De Florian et al., Phys.Rev. D80, (2009) 034030

Transversity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \sigma_{xy} \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$



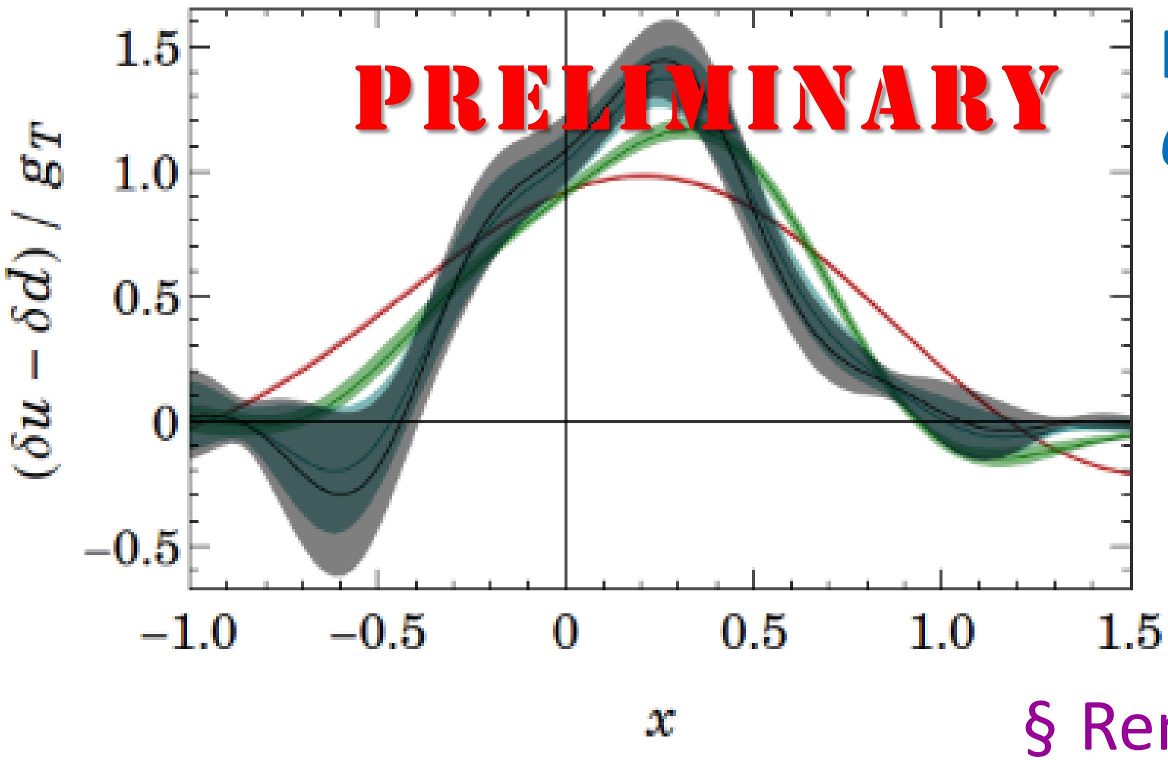
Uncorrected bare lattice results

§ Renormalization needed

Transversity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \sigma_{xy} \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

§ Renormalization needed

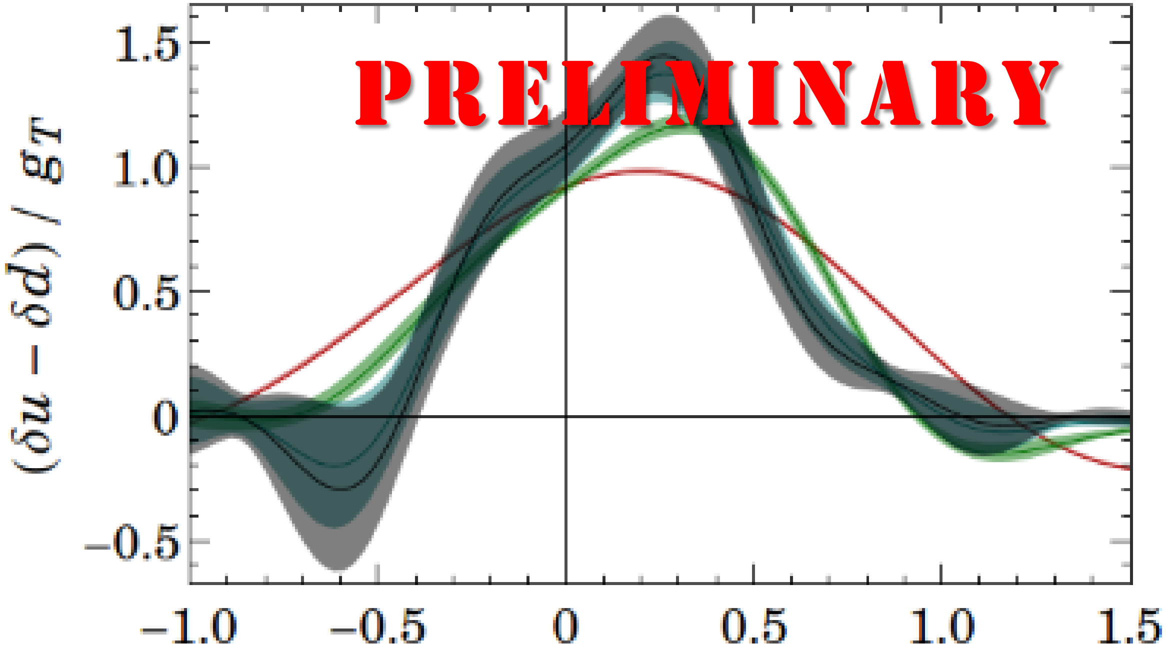
Transversity Distribution

§ Exploratory study

- We found $\delta\bar{u} < \delta\bar{d}$ with large sea asymmetry
- Chiral quark-soliton model

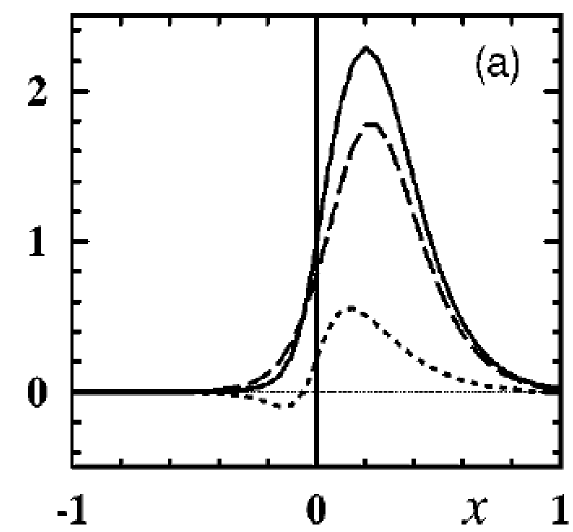
$$\int dx\, \frac{\delta\bar{u}(x) - \delta\bar{d}(x)}{g_T} \approx -0.23(9)$$

$$\int dx\, \left(\delta\bar{u}(x) - \delta\bar{d}(x)\right) \approx -0.082$$



$\delta\overline{q_f}(x) = -\delta q_f(-x)$

CQS model

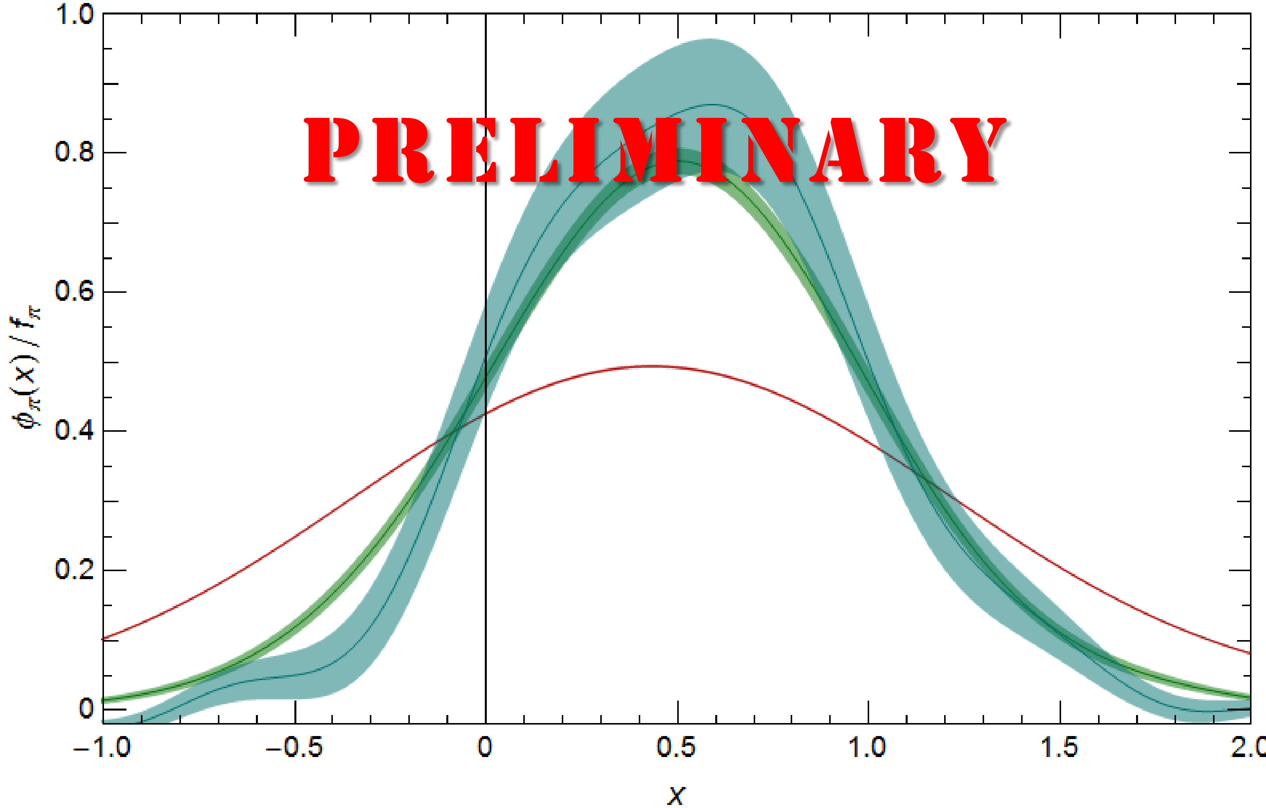


P. Schweitzer et al. PRD 64, 034013 (2001)

Pion Distribution Amplitude

§ Exploratory study

$$\int \frac{dz}{2\pi}\, e^{-izk_z} \left\langle 0 \middle| \bar{d}(z)\, \gamma_z\, \gamma_5 \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) u(0) \middle| \pi^+(P) \right\rangle$$



Only leading mass correction applied

Dominated by $O(\Lambda_{QCD}^2/P_z^2)$ errors

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary and Outlook

Exciting time for hadron structure on the lattice

- Overcoming longstanding obstacle to *x*-distribution
  - New idea by Ji for studying full *x* dependence of PDFs
  - Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass
- Hope this study motivates others to give Ji's method a try
- Caveats
  - Not a precision calculation *yet* , proper renormalization,…
  - Systematics due to large momenta (some ideas to improve it)
- Need improvement for large-momentum sources
  - Applications: large-*q* form factors, hadronic and flavor physics, …
- Many more quantities to study
  - strange/charm/beauty sea distributions, gluons, TMD…