PQCD factorization for heavy quarkonium production and fragmentation functions

Hong Zhang
Stony Brook University
Brookhaven National Laboratory

Based on Y.-Q. Ma, J.-W. Qiu and HZ, arXiv: 1311.7078
Y.-Q. Ma, J.-W. Qiu and HZ, arXiv: 1401:0524

Outline

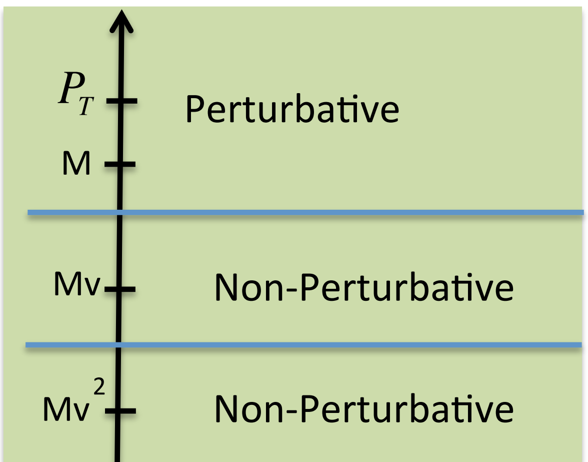
- ✧ Introduction
- ✧ PQCD factorization for heavy quarkonia production
- ✧ Apply NRQCD to calculate fragmentation functions
- ✧ Summary and outlook

Heavy quarkonium

- Effectively, a non-relativistic QCD bound state

  Charmonium: $v^2 \approx 0.3$ Bottomonium: $v^2 \approx 0.1$

- Multiple Scales



Detect full energy range of QCD

Hard—Production of $Q\bar{Q}$

Soft—Relative Momentum

Λ_{QCD}

Ultrasoft—Binding Energy

- Spectroscopic notation

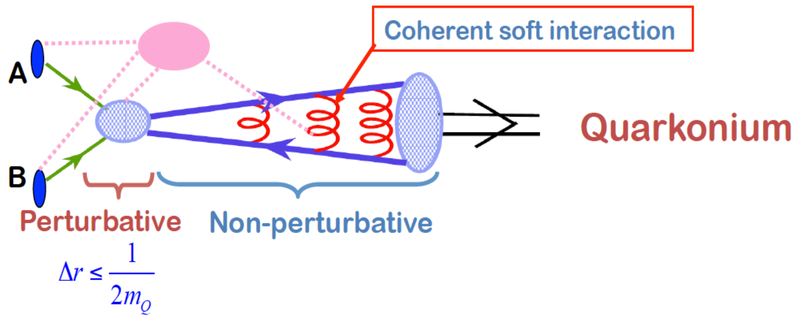
$c\bar{c}$: $\eta_C(^1S_0)$, $J/\psi(^3S_1)$, $\chi_{CJ}(^3P_J)$ … 2S, 2P…

$b\bar{b}$: …$\Upsilon(^3S_1)$, $\chi_{bJ}(^3P_J)$ … 2S, 2P…

$$^{2S+1}L_J$$

Historical Models: CSM

- **<u>Conjectured</u>** factorization form



$$\sigma_{AB\to J/\psi} = \sum_{\text{states}} \int d\Gamma_{Q\bar{Q}} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{Q\bar{Q}}} \right] F_{\text{states}(Q\bar{Q})\to J/\psi}(p_Q, p_{\bar{Q}}, P_{J/\psi})$$

production of heavy quark pair — different models have different assumptions

- Color Singlet Model (CSM): 1975-

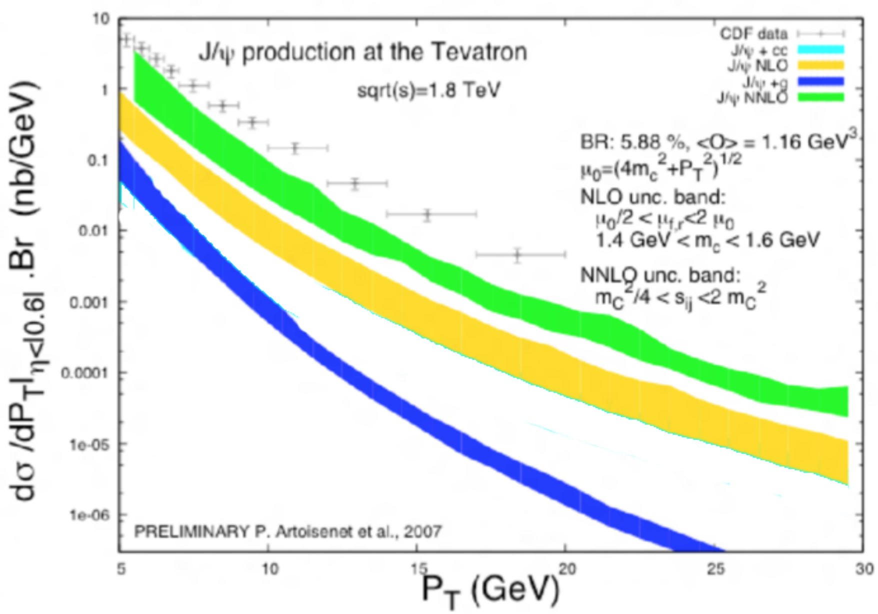
Assumes $Q\overline{Q}$ evolves into quarkonium as isolated particles, no interaction with environment.

⇒ $Q\overline{Q}$ must have same quantum number (color singlet, J^{PC}) as quarkonium.

Einhorn and Ellis (1975), Chang (1980), Berger and Jone (1981)

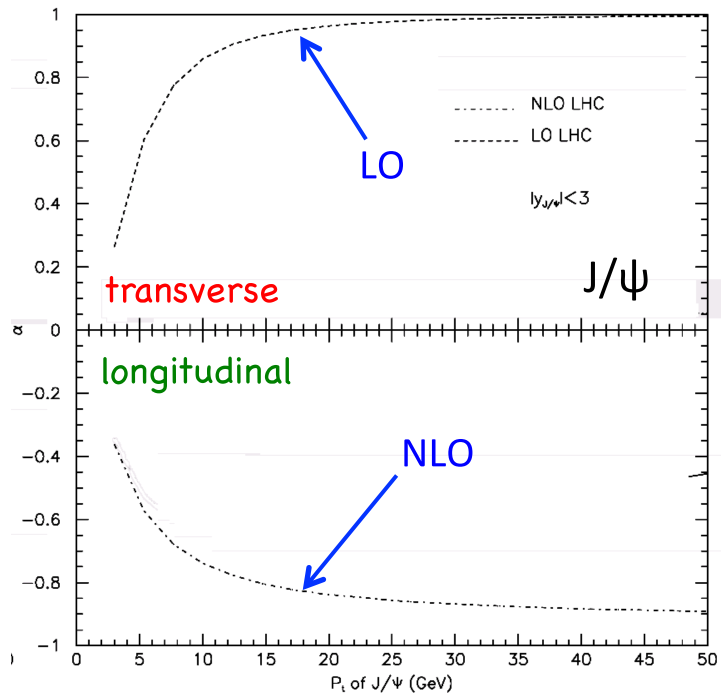
Historical Models: CSM

- Color Singlet Model (cont'd)
  - ✓ No free parameter
  - ✧ Large corrections from higher orders.
  - ✧ P-wave is IR-divergent—CSM is incomplete!



NLO is more than 10 times larger than LO!

F. Maltoni QWG 2007



NLO flips the polarization of LO!

B. Gong et, al. Phys. Rev. Lett, 100 (2008) 232001

Historical Models: CEM

- Color Evaporation Model (CEM): 1977-

Fritsch (1977), Halzen (1977), …

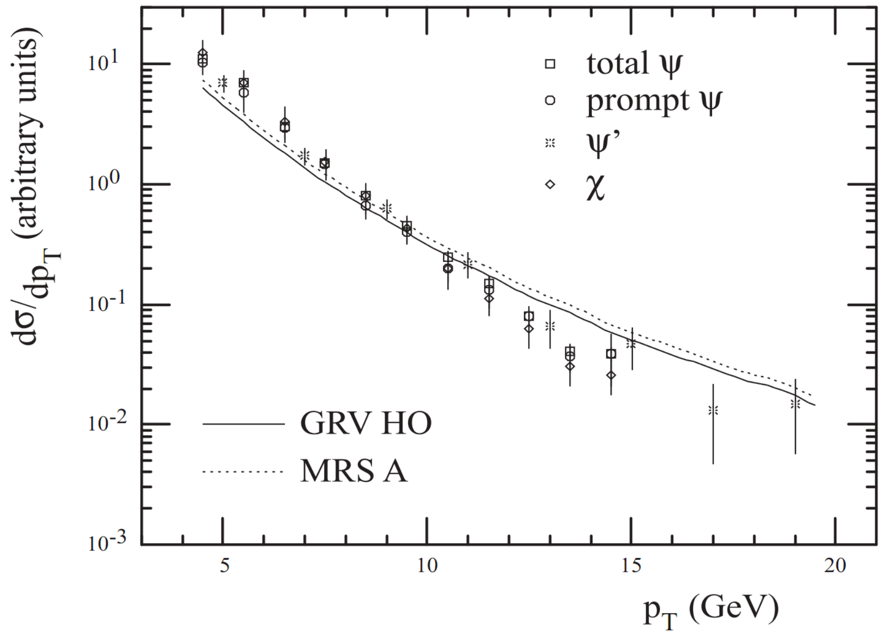
Invariant mass > open charm/bottom threshold → open charm/bottom

Otherwise → a fixed possibility to go to each state.

Predictive power: one parameter per quarkonium state.

✧ Predicts less suppression;

✧ Predicts $\sigma(A)/\sigma(B)$ is process-independent, which conflicts with data.



CEM shows less P_T suppression.

J. F. Amundson et al, Phys. Lett. B 390 (1997) 323

NRQCD: framework

- NRQCD Lagrangian

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L}_{\text{bilinear}} + \dots + \delta\mathcal{L}_{\text{4-fermion}}$$

Caswell, Lapage (1986)

$$\mathcal{L}_{\text{heavy}} = \psi^{+}\left(iD_t + \frac{D^2}{2M}\right)\psi + \chi^{+}\left(iD_t - \frac{D^2}{2M}\right)\chi$$

$$\delta\mathcal{L}_{\text{bilinear}} = \frac{c_1}{8M^3}\left[\psi^{+}(D^2)^2\psi - \chi^{+}(D^2)^2\chi\right]$$

Relativistic correction at $O(v^2)$

$$+\frac{c_2}{8M^2}\left[\psi^{+}(D\cdot gE - gE\cdot D)\psi + \chi^{+}(D\cdot gE - gE\cdot D)\chi\right]$$

$$+\frac{c_3}{8M^2}\left[\psi^{+}(iD\times gE - gE\times iD)\cdot\sigma\psi + \chi^{+}(iD\times gE - gE\times iD)\cdot\sigma\chi\right]$$

$$+\frac{c_4}{2M}\left[\psi^{+}(gB\cdot\sigma)\psi - \chi^{+}(gB\cdot\sigma)\chi\right]$$

$\delta\mathcal{L}_{\text{4-fermion}}$ non-renormalizable terms

- NRQCD factorization for quarkonium production

Bodwin, Braaten, Lepage (1995)

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{M^{d_n-4}}\langle 0|O_n^H|0\rangle$$

$$O_n^H = \chi^{+}\kappa_n\psi \sum_X |H+X\rangle\langle H+X|\, \psi^{+}\kappa'_n\chi$$

$Q\bar{Q}$ and H can have different quantum number. later transition effect is included in LDMEs.

Factorization is not proved!

NRQCD: Predictive Power

- Double Expansion of α_s and v
  - Short distance coefficient: calculated perturbatively
  - Non-perturbative matrix elements:

    color singlet ME: related to wave functions as CSM
    color octet ME: extracted from HQ production data

  e.g. J/ψ production to order O(v^4), three parameters: $\langle {}^1S_0^{(8)} \rangle$, $\langle {}^3S_1^{(8)} \rangle$ and $\langle {}^3P_0^{(8)} \rangle$

- CSM and CEM are special cases of NRQCD   Bodwin, Braaten, Lee, PRD (2005)

- P-wave divergence is cancelled by color octet contribution

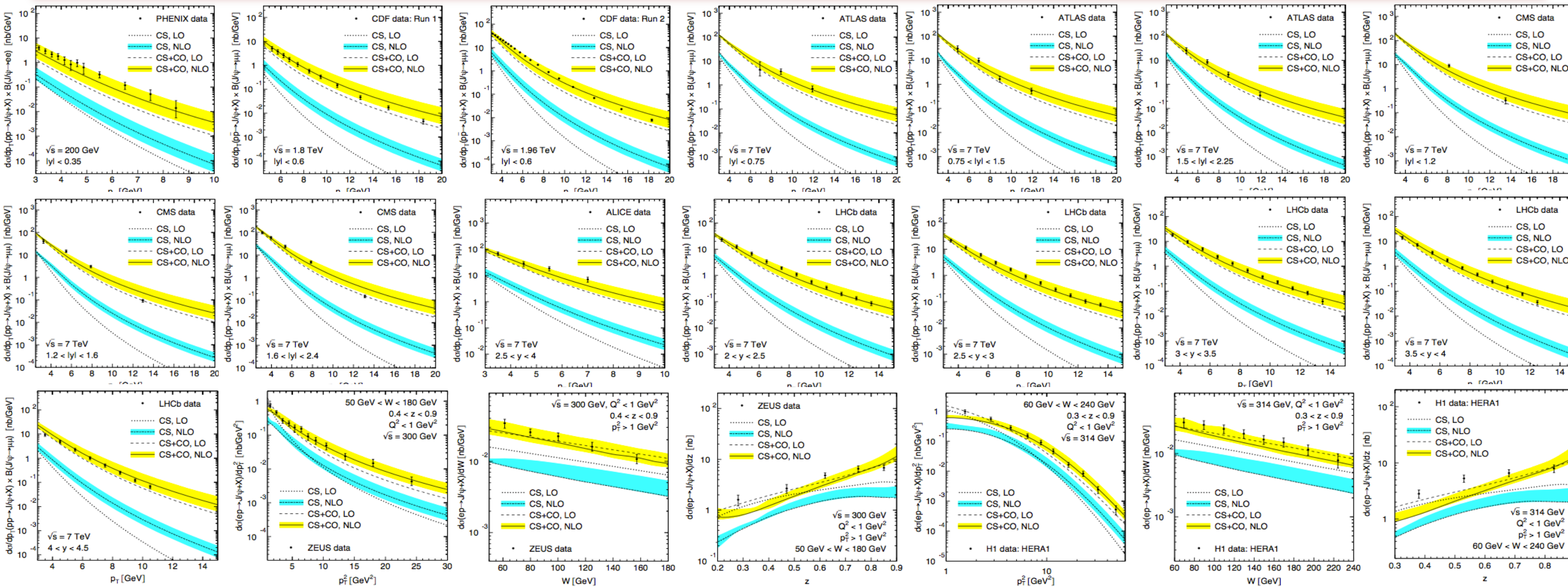
  At leading order of v, p-wave heavy quarkonium production has two contributions:

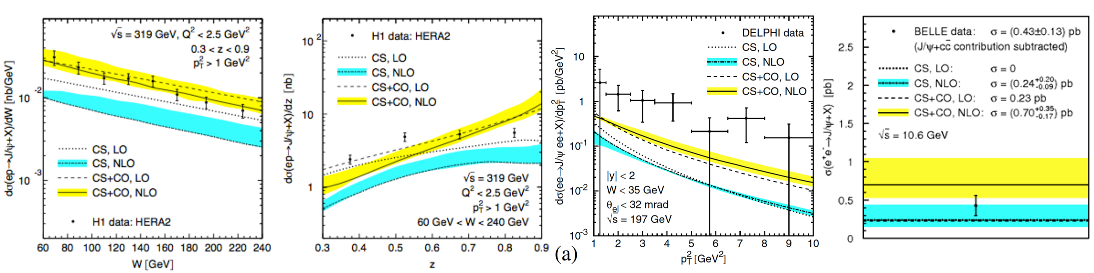
  (1) Hard part produces CS p-wave $Q\bar{Q}$ directly.
  (2) Hard part produces CO s-wave $Q\bar{Q}$, which later transits to p-state by emitting a gluon.

✧ Factorization form is not proved to all orders in α_s.

✧ As in CSM, NRQCD calculation suffers from higher order corrections

NRQCD: Universality Puzzle of LDMEs





$$\langle O[^1S_0^{(8)}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ GeV^3$$

$$\langle O[^3S_1^{(8)}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ GeV^3$$

$$\langle O[^3P_0^{(8)}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ GeV^5$$

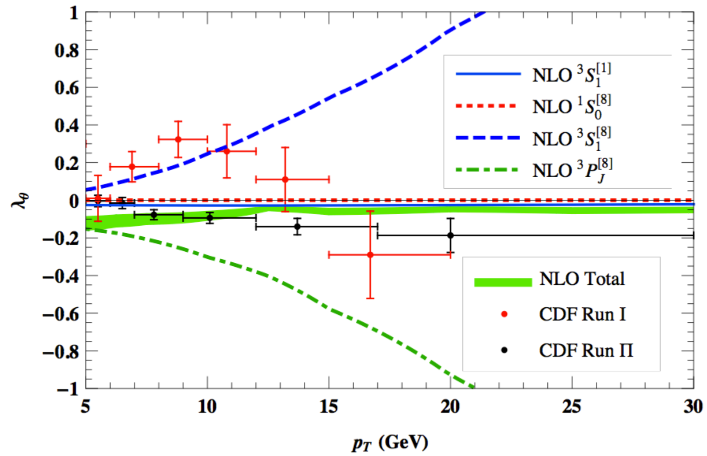
Blue band: NLO CSM

Yellow band: NLO NRQCD

194 data from 10 experiments $\chi^2 / d.o.f. = 857/194 = 4.42$

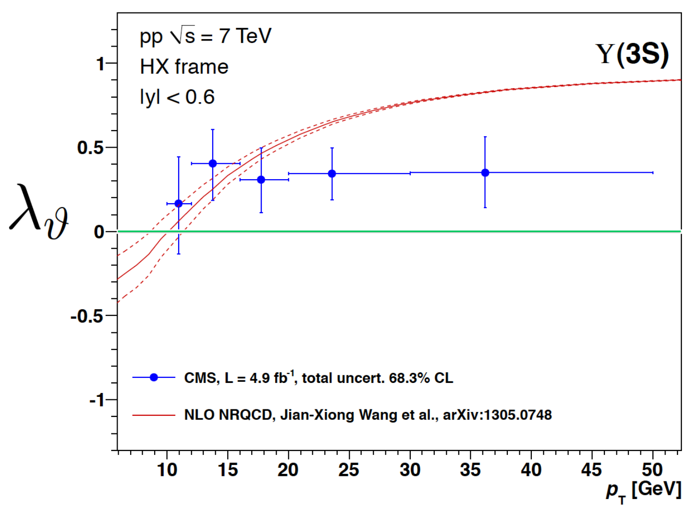
Butenschoen and Kniehl, PRD (2011)

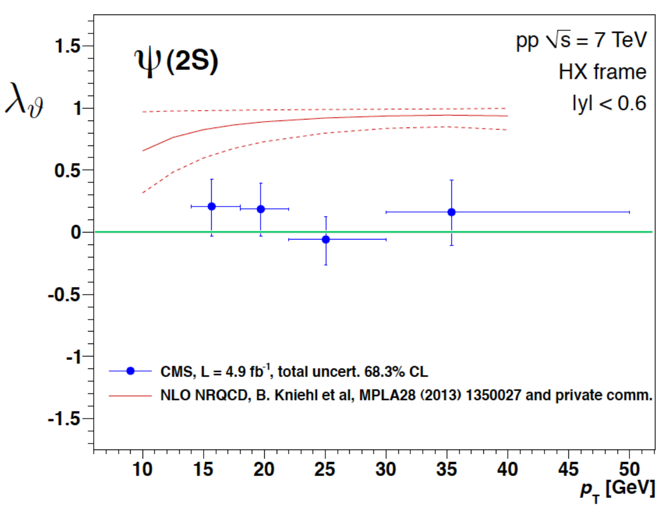
NRQCD: Compare With Data Polarization



Explains J/ψ polarization

K.T. Chao et al, PRL (2012)



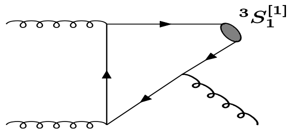


Fails to explain the polarization of ψ(2S) and ϒ(3S)

Lourenco, LHCP 2013

Explain the large HO corrections

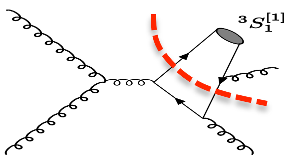
- These inconsistencies may relate to large high-order correction
- Origin of large high-order correction
  - LO in α_s but NNLP in $1/p_T$

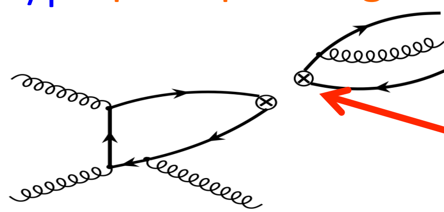


$$\alpha_S^3(p_T)\frac{m_Q^4}{p_T^8}$$

  - NLO in α_s but NLP in $1/p_T$ quark pair fragmentation

Kang, Qiu and Sterman, 1109.1520



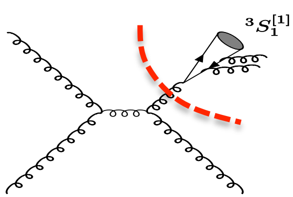


Relativistic Projector to all "spin states"

$$\frac{d\hat{\sigma}^{\rm NLO}}{dp_T^2} \Rightarrow \frac{\alpha_S^3(p_T)}{p_T^6}\otimes\alpha_s(\mu)\log(\mu^2/\mu_0^2)\,m_Q^2 \qquad \mu_0 \gtrsim 2m_Q$$

  - NNLO in α_s but LP in $1/p_T$ gluon fragmentation

Braaten, Yuan, 9303205



$$\frac{\alpha_S^2(p_T)}{p_T^4}\otimes\alpha_S{}^{3+m}(\mu)\log^{\rm m}(\mu^2/\mu_0^2)\,m_Q^2$$

At large P_T, M_Q/P_T gives much stronger suppression than α_S

Need to expand by M_Q/P_T first, and resum Log$[P_T^2/M_Q^2]$

Outline

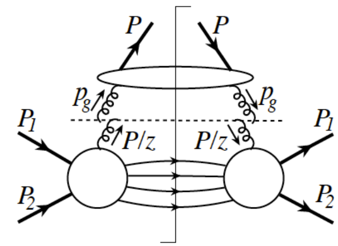
- Introduction
- **PQCD factorization for heavy quarkonia production**
- Apply NRQCD to calculate fragmentation functions
- Summary and outlook

pQCD Factorization Approach: Formalism

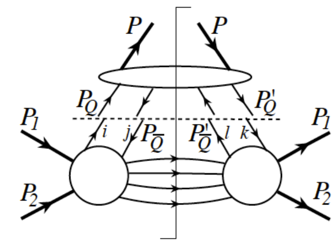
Nayak, Qiu and Sterman, PRD (2005) … , Kang, Qiu and Sterman, PRL (2011)…
Kang, Ma, Qiu and Sterman, arXiv: 1401.0923.
SCET approach: Fleming et.al., PRD (2012)

- Expand first in $1/p_T$, then α_s
- Factorization formalism up to NLP **valid to all orders in α_s**

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$
$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$
$$+ \mathcal{O}(m_Q^4/p_T^4)$$



$\dfrac{1}{p_T^4}$



$\dfrac{1}{p_T^6}$

κ = vector (V), axial-vector (A) and tensor (T) for spin, either CS or CO

- Projection operators

Leading power: top
$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

bottom
$$\mathcal{P}_{\mu\nu}(p) = -g_{\mu\nu} + \bar{n}_\mu n_\nu + n_\mu \bar{n}_\nu \equiv d_{\mu\nu}$$

Next leading power:

top
$$\widetilde{\mathcal{P}}_v^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n$$
$$\widetilde{\mathcal{P}}_a^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma^5$$
$$\widetilde{\mathcal{P}}_t^L(p) = \frac{1}{4p\cdot n}\gamma\cdot n\gamma_\perp^\alpha$$

bottom
$$\mathcal{P}_v^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p} = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$
$$\mathcal{P}_a^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma_5\gamma\cdot\hat{p} = \gamma_5\gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})$$
$$\mathcal{P}_t^L(\hat{p}_Q,\hat{p}_{\bar{Q}}) = \gamma\cdot\hat{p}\gamma_\perp^\alpha = \gamma\cdot(\hat{p}_Q + \hat{p}_{\bar{Q}})\gamma_\perp^\alpha$$

Predictive Power

- Short-distance hard part (at scale p_T) can be calculated perturbatively

  Power series in α_s , with only momentum scale p_T, no large logarithms.

- Evolution equations

  Independence on the factorization scale $$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T) = 0$$

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu) = \sum_j \frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

If only keep LP DGLAP evolution

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu) = \sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

- Evolution kernels can be calculated in pQCD

Flemings et.al., PRD (2013)
Kang, Ma, Qiu and Sterman, arXiv: 1401.0923

Only need input fragmentation functions at a scale μ_0

By solving the evolution equations, resums logarithm Log[μ /μ_0]

Input Fragmentation Functions

- To give prediction, it is crucial to know the fragmentation functions.
  Based on evolution equations, only need to know FFs at initial scale.

- Input fragmentation functions include dynamics with scale < μ_0

  Sensitive to the formation of observed heavy quarkonium
  In contract, hard part and evolution kernel insensitive to final quarkonium state

  - Different quarkonium states require different input distributions!
  - In principle, should be extracted from data.

$D_{H/f}(z, m_Q, \mu_0)$ $\quad\quad$ $\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- Too many unknown fragmentation functions: very difficult to be extracted from data

  Need at least 10 input fragmentation functions for each observed heavy quarkonium.

  Leading power: 4 functions, from g, q, c, and b

  Next-to Leading power: 6 functions, from V, A and T, either CS or CO

Input Quarkonium FFs are Partially Perturbative

- Choose scale $\mu_0 > 2m_Q >> \Lambda_{QCD}$

  Fragmentation functions to heavy quarkonium is partially perturbative!

- Compare with Pions and Kaons fragmentation functions

  No large scale, completely nonperturbative.

Outline

- ✧ Introduction
- ✧ PQCD factorization for heavy quarkonia production
- ✧ **Apply NRQCD to calculate fragmentation functions**
- ✧ Summary and outlook

Motivation

- Apply NRQCD on the input fragmentation functions with $\mu_0 \geq 2m_Q$
  - ✧ No power enhancement or large logarithms.
  - ✧ Obtain the functional form of each fragmentation function
  - ✧ For each heavy quarkonium state

    A few unknown LDMEs v.s. 10 unknown functions

    For $J/\psi(^3S_1)$, **THREE** unknown matrix elements up to order v^4

- No all-order proof of NRQCD factorization yet.

  Valid to NLO.
  NNLO is valid for some special cases. Nayak, Qiu and Sterman, 2006

  - ✧ Modification of our calculated fragmentation functions may be required for better fitting.
  - ✧ Any such modification may shed light on the proof of NRQCD factorization.

Apply NRQCD To FFs

- Definition of FFs

$$D_{g->H}(z,\mu)=\frac{(-g_{\mu\nu})z^{D-4}}{16\pi(D-2)p^+}\int_{-\infty}^{+\infty}dy^- e^{-i(p^+/z)y^-}\langle 0|G_c^{+\mu}(0)[\Phi_{\hat{n}}^{(A)}(0^-)]_{cb}^{+}|H(p^+)X\rangle\langle H(p^+)X|[\Phi_{\hat{n}}^{(A)}(y^-)]_{ba}G_a^{+\nu}(y^-)|0\rangle$$

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q)=\int\frac{p^+dy^-}{2\pi}\frac{p^+/z\,dy_1^-}{2\pi}\frac{p^+/z\,dy_2^-}{2\pi}$$
$$\times e^{-i(p^+/z)y^-}e^{i(p^+/z)[(1-\zeta_2)/2]\,y_1^-}e^{-i(p^+/z)[(1-\zeta_1)/2]\,y_2^-}$$
$$\times\mathcal{P}_{ij,lk}^{(s)}(p_c)\,\mathcal{C}_{ab,cd}^{[I]}\,\langle 0|\bar{\psi}_{c',l}(y_1^-)[\Phi_{\hat{n}}^{(F)}(y_1^-)]_{c'c}^{\dagger}[\Phi_{\hat{n}}^{(F)}(0)]_{dd'}\,\psi_{d',k}(0)|H\,X\rangle$$
$$\times\langle H\,X|\bar{\psi}_{a',i}(y^-)[\Phi_{\hat{n}}^{(F)}(y^-)]_{a'a}^{\dagger}[\Phi_{\hat{n}}^{(F)}(y^-+y_2^-)]_{bb'}\psi_{b',j}(y^-+y_2^-)|0\rangle,$$

complicated gauge link structure,

where $\Phi_{\hat{n}}^{(F)}(y^-)=\mathcal{P}\exp\left[-i\,g\int_{y^-}^{\infty}d\lambda\,\hat{n}\cdot A^{(F)}(\lambda\hat{n})\right]$ is the gauge link.

- Apply NRQCD

Perturbative

$$D_{f\to H}(z,\mu_0,m_Q)\to\sum_{[Q\bar{Q}(c)]}\hat{d}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle O_{[Q\bar{Q}(c)]\to H}\rangle$$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2;\mu_0,m_Q)\to\sum_{[Q\bar{Q}(c)]}\hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta_1,\zeta_2;\mu_0,m_Q)\langle O_{[Q\bar{Q}(c)]\to H}\rangle$$

Non-perturbative NRQCD LDMEs

Same for LP and NLP

A little More about Gauge Links

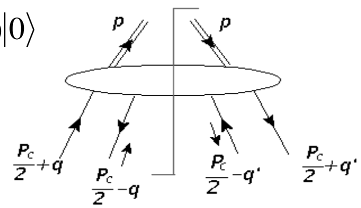
General gauge link structure

$$D_{Q\bar{Q}[\kappa]/Q\bar{Q}[a^{[8]}]}(z,\zeta,\zeta') = \sum_X \int \frac{p^+ dy^-}{2\pi} e^{-i(p^+/z)y^-} \times \int \frac{p^+ dy_1^- \, p^+ dy_2^-}{(2\pi)^2} e^{i(p^+/2z)(1-\zeta)y_1^-} e^{-i(p^+/2z)(1-\zeta')y_2^-}$$

$$\times \frac{4}{N_C^2-1}\langle 0|\psi_{i'}(0) U^{(F)}_{i'i}(0,x^-)\frac{\gamma^+\gamma_5}{4p_c^+}(T_a^{(F)})_{ij} U^{(F)}_{jj'}(x^-,y_2^-)U^{(A)}_{aa'}(x^-,\infty)\bar{\psi}_{j'}(y_2^-)|Q\bar{Q}[\kappa]+X\rangle$$

$$\times\langle Q\bar{Q}[\kappa]+X|\psi_{l'}(y^-+y_1^-)U^{(F)}_{l'l}(y^-+y_1^-,x_1^-)\frac{\gamma^+\gamma_5}{4p_c^+}(T_b^{(F)})_{lk}U^{(F)}_{kk'}(x_1^-,y^-)U^{(A)}_{a'b}(x_1^-,\infty)\bar{\psi}_{k'}(y^-)|0\rangle$$

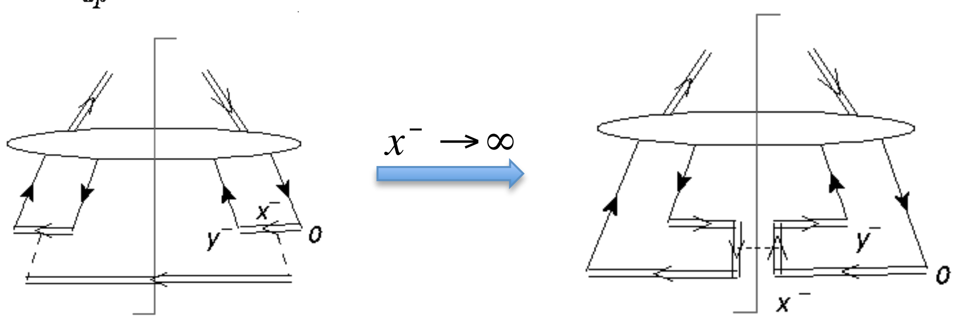
$$U^{(j)}(x_2^-,x_1^-) = \mathcal{P}\exp\left[-ig\int_{x_1^-}^{x_2^-} n\cdot A^{(j)}(n\lambda)\right]$$



$z = p^+/p_C^+$ $\quad \zeta = 2q^+/p_C^+$ $\quad \zeta' = 2q'^+/p_C^+$

Independence of the point where the two gauge links meet,

$$\frac{d}{dx^-}\langle 0|\psi_{i'}(0)\frac{\gamma^+\gamma_5}{4p^+}U^{(f)}_{i'i}(0,x^-)(t^a)_{ij}U^{(f)}_{jj'}(x^-,y^-)U^{(A)}_{aa'}(x^-,\infty)\bar{\psi}_{j'}(y_1^-)|H(p^+)X\rangle = 0$$



4 fundamental + 2 adjoint gauge links

4 fundamental gauge links

- Simpler gauge link structure,
- Each gauge link has the ordinary Feynman rule.

Calculate Coefficients

- Perturbative coefficients are insensitive to final state.
  For LP FF, match final quarkonium states to $Q\bar{Q}[n]$

$$D_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD} = \sum_{[Q\bar{Q}(c)]} \hat{d}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(n)]}\right\rangle\Big|_{Pert.NRQCD}$$

Expand by α_s

$$D^{(0)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q) = \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(n)]}\right\rangle^{(0)} = \hat{d}^{(0)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q)$$

$$D^{(1)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q) = \sum_{[Q\bar{Q}(c)]} \hat{d}^{(1)}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(n)]}\right\rangle^{(0)} + \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(n)]}\right\rangle^{(1)}$$

Solve the above equations

$$\hat{d}^{(0)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q) = D^{(0)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD}$$

$$\hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q) = D^{(1)}_{f\to[Q\bar{Q}(n)]}(z,\mu_0,m_Q)\Big|_{Pert.QCD} - \sum_{[Q\bar{Q}(c)]} \hat{d}^{(0)}_{f\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\left\langle O_{[Q\bar{Q}(c)]\to[Q\bar{Q}(n)]}\right\rangle^{(1)}\Big|_{Pert.NRQCD}$$

Divergences cancel, $\hat{d}^{(1)}_{g\to[Q\bar{Q}(\kappa)]}(z,\mu_0,m_Q)$ is finite.

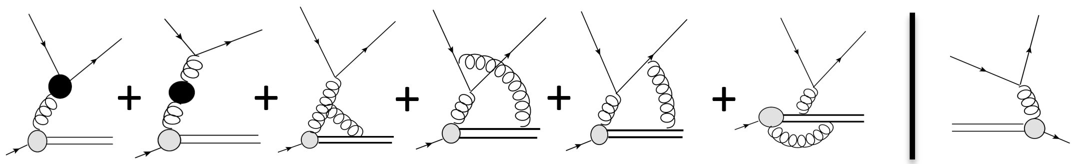
- Perturbative coefficients for NLP is calculated in the same way.

Single Parton FFs

We calculated all channels up to NLO in α_s , with both Feynman gauge and lightcone gauge.

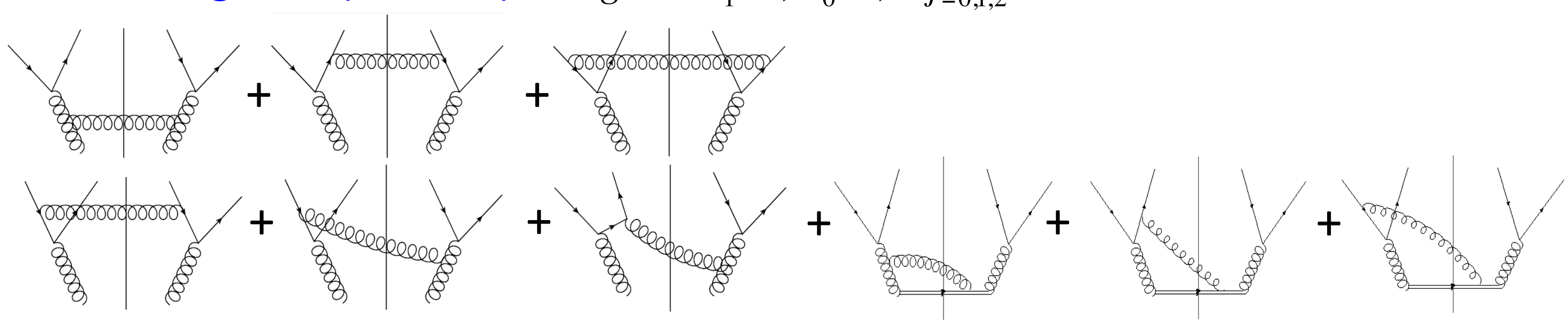
Take gluon fragmentation functions as an example.

- Virtual diagrams (10+10 c.c.) $g \rightarrow {}^3S_1^{[8]}$



+c.c.

- Real diagrams (9+6 c.c.) $g \rightarrow {}^3S_1^{[1,8]}, {}^1S_0^{[1,8]}, {}^3P_{J=0,1,2}^{[1,8]}$



+c.c.

Short-distance coefficients are finite for all channels.

Many results are not new. our calculations confirm most of the previous results.

Heavy Quark Pair FFs: LO

Example: $[Q\bar{Q}(a^{[8]}) \to [Q\bar{Q}({}^1S_0^{[8]})]$

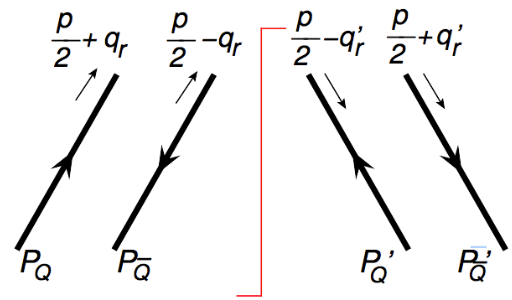
✧ Projection operators:

$$\Gamma_a(p_c) = \frac{[\gamma\cdot\hat{n}\,,\gamma_5]}{8p_c\cdot\hat{n}} \quad , N_a = 1 \qquad \hat{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)^\mu$$

$$\Gamma_i^{\rm NR}(p) = \frac{1}{\sqrt{8m_Q^3}}(\frac{\not p}{2} - \not q_r - m_Q)\gamma_5(\frac{\not p}{2} + \not q_r + m_Q)$$

$$(C_8)_{ij} = \sqrt{2}\left(t_a^{(F)}\right)_{ij} \quad , N_8 = N_c^2 - 1$$

NRQCD state



pQCD state

$$P_Q = \frac{p_c}{2} + q_1, \qquad P_{\bar{Q}} = \frac{p_c}{2} - q_1,$$

$$P'_Q = \frac{p_c}{2} + q_2, \qquad P'_{\bar{Q}} = \frac{p_c}{2} - q_2.$$

$$\mathrm{Tr}\left[\Gamma_a(p_c)\, C_8\, \mathcal{A}^{\rm LO}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(p,z,\zeta_1)\right]$$

$$= \lim_{q_r\to 0}\int \frac{d^Dq_1}{(2\pi)^D}(2\pi)^D\delta^D(q_1-q_r)\delta(\zeta_1 - \frac{2q_1^+}{p^+})\mathrm{Tr}_c\left[\sqrt{2}t^{(F)}_{c_{in}}\sqrt{2}t^{(F)}_{c_{out}}\right]$$

$$\times\,\mathrm{Tr}_\gamma\left[\frac{[\gamma\cdot\hat{n},\gamma_5]}{8p\cdot\hat{n}}\frac{1}{\sqrt{8m_Q^3}}(\frac{\not p}{2} - \not q_r - m_Q)\gamma_5(\frac{\not p}{2} + \not q_r + m_Q)\right] = -\frac{1}{\sqrt{8m_Q}}\,\delta_{c_{in},\,c_{out}}\,\delta(\zeta_1)$$

Relative momenta are different on both sides

$$\zeta_1 = 2q_1^+ / p^+$$

$$\mathcal{D}^{\rm LO}_{[Q\bar{Q}(a^{[8]})\,\to\,[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \frac{\delta(1-z)}{N_sN_bN_i^{\rm NR}N_{b'}^{\rm NR}}[-\frac{1}{\sqrt{8m_Q}}\delta_{c_i,\,c_f}\,\delta(\zeta_1)][-\frac{1}{\sqrt{8m_Q}}\delta_{c_i,\,c_f}\,\delta(\zeta_2)]$$

$$\hat{d}^{\rm LO}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \frac{1}{N_c^2-1}\frac{1}{8m_Q}\delta(1-z)\,\delta(\zeta_1)\,\delta(\zeta_2)$$

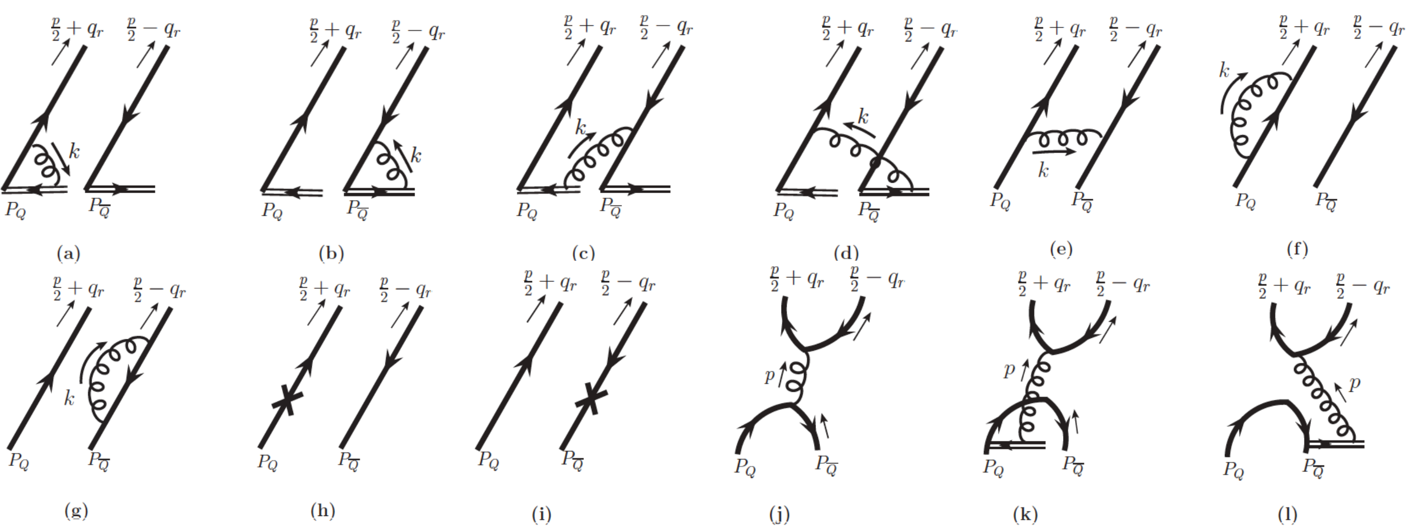
Heavy Quark Pair FFs: NLO

We calculated all channels up to NLO in α_s , with both Feynman gauge and lightcone gauge.

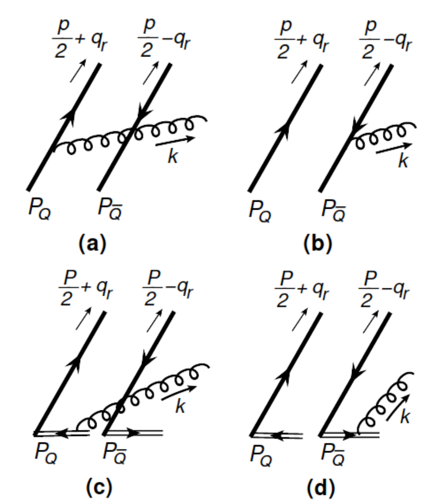
Heavy quark pair fragmentation functions are non-trivial, because of:

- ✧ Large number of channels: 72 channels
- ✧ Complicated gauge link structure.
- ✧ Integration over initial relative momentum, derivative of final relative momentum.
- ✧ Complicated divergence cancellation, new plus/minus distributions.

12 virtual amplitudes



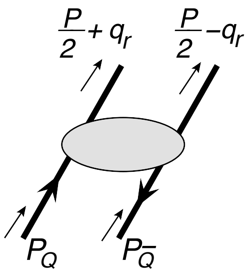
4 real amplitudes



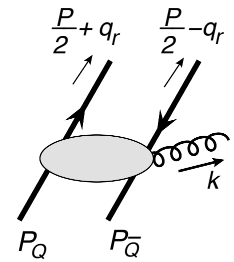
Conservation Law and Symmetry (I)

initial $Q\overline{Q}[\kappa] = V^{[1,8]}, A^{[1,8]}, T^{[1,8]}$ final $Q\overline{Q}[n'] = {}^1S_0^{[1,8]},\ {}^3S_1^{[1,8]},\ {}^1P_1^{[1,8]},\ {}^3P_{J=0,1,2}^{[1,8]}$

✧ Color conservation



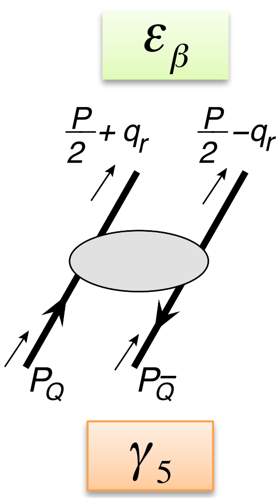
initial and final heavy quark pairs share same color



initial and final heavy quark pairs can not be both color singlet

✧ Lorentz invariance

After loop integration and limit $q_r \to 0$, left momenta: p and n



e.g. $A \to {}^3S_1$ Virtual contribution

Independent directions: p, n

Spin polarization: ε_β

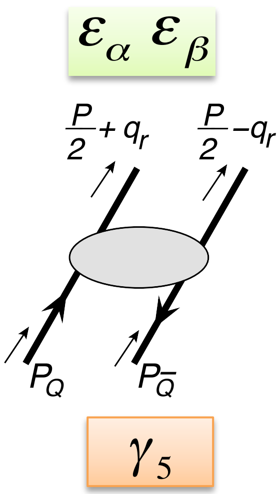
Cannot form Levi-Civita symbol.

Conservation Law and Symmetry (II)

initial $Q\overline{Q}[\kappa] = V^{[1,8]}, A^{[1,8]}, T^{[1,8]}$ final $Q\overline{Q}[n'] = {}^1S_0^{[1,8]},\ {}^3S_1^{[1,8]},\ {}^1P_1^{[1,8]},\ {}^3P_{J=0,1,2}^{[1,8]}$

✧ Lorentz invariance

After loop integration and limit $q_r \to 0$, left momenta: p and n



e.g. $A \to {}^3P_{J=0,1,2}$ Virtual contribution

Indep. Directions: p, n

Orbital polarization: ε_α

Spin polarization: ε_β

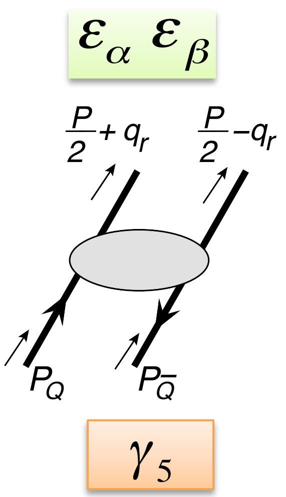
Results ~ $\epsilon^{\alpha\beta\mu\nu} n_\mu p_\nu$ anti-symmetic in α and β

Conservation Law and Symmetry (II)

initial $Q\overline{Q}[\kappa] = V^{[1,8]}, A^{[1,8]}, T^{[1,8]}$ final $Q\overline{Q}[n'] = {}^1S_0^{[1,8]},\ {}^3S_1^{[1,8]},\ {}^1P_1^{[1,8]},\ {}^3P_{J=0,1,2}^{[1,8]}$

✧ Lorentz invariance

After loop integration and limit $q_r \to 0$, left momenta: p and n



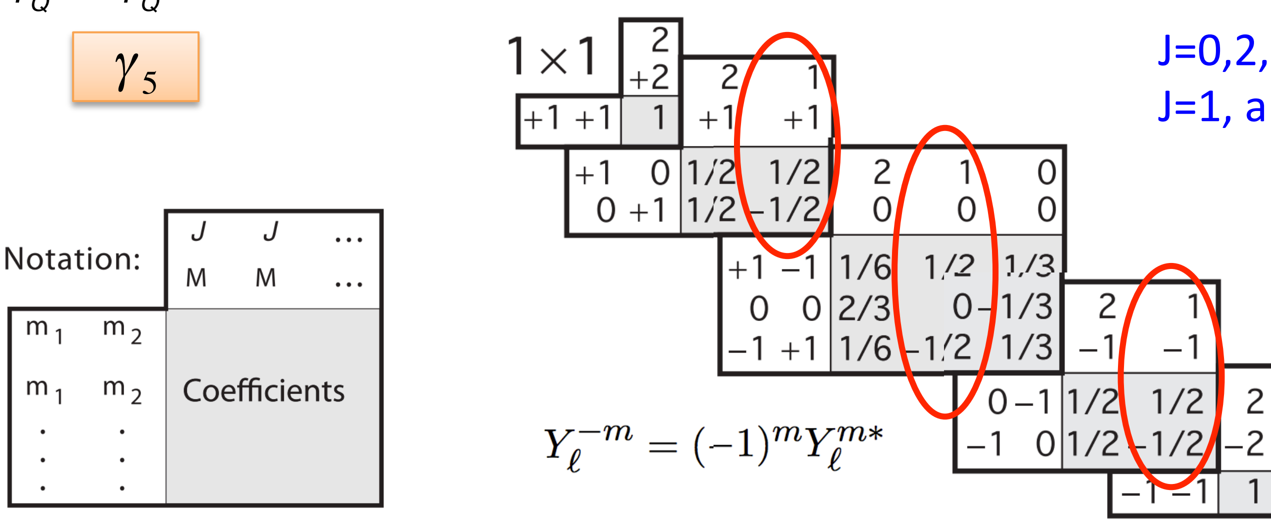
e.g. $A \to {}^3P_{J=0,1,2}$ Virtual contribution

Indep. Directions: p, n

Orbital polarization: ε_α

Spin polarization: ε_β

Results ~ $\epsilon^{\alpha\beta\mu\nu} n_\mu p_\nu$ anti-symmetic in α and β



$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

J=0,2, symmetric

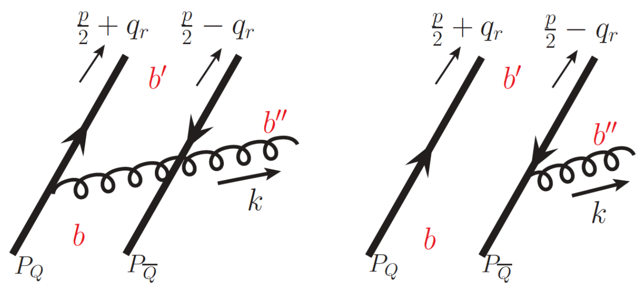
J=1, anti-symmetric

$A \to {}^3P_{J=0,2}$

Virtual contribution must vanish.

Modified Charge Conjugation

✧ Initial heavy quark pair is not charge conjugation invariant



$$P_Q = \frac{p_c}{2} + q_1 \qquad \zeta_1 = \frac{2q_1^+}{p_c^+}$$

$$P_{\bar{Q}} = \frac{p_c}{2} - q_1 \qquad p_c = p + k$$

✧ Modified charge conjugate

Charge conjugation followed by $\zeta_1 \to -\zeta_1$, $q_r \to -q_r$, $q_1 \to -q_1$,

For example, S-wave final heavy quark pair

$$I_{a+b}^S = F(z,0)\left[G(b,b',b'')\delta(1-z-\zeta_1) + (-1)^{S+\delta_{s,a}} G^\dagger(b,b',b'')\delta(1-z+\zeta_1)\right]$$

$S:$ final heavy quark pair spin
$s = v,a,t:$ initial heavy quark pair spin state.

- Case 1: b, b' not both color octet

$$\left[\delta(1-z-\zeta_1) + (-1)^{S+\delta_{s,a}}\delta(1-z+\zeta_1)\right]\left[\delta(1-z-\zeta_2) + (-1)^{S+\delta_{s,a}}\delta(1-z+\zeta_2)\right]$$

- Case 2: b, b', both color octet

$$(N_c^2-2)\left[\delta(1-z+\zeta_1)\delta(1-z+\zeta_2) + \delta(1-z-\zeta_1)\delta(1-z-\zeta_2)\right]$$
$$- (-1)^{S+\delta_{s,a}} 2\left[\delta(1-z+\zeta_1)\delta(1-z-\zeta_2) + \delta(1-z-\zeta_1)\delta(1-z+\zeta_2)\right]$$

e.g. $[Q\bar{Q}(a^{[8]}) \to [Q\bar{Q}({}^1S_0^{[8]})]$, S=0 and δs,a=1 , Positive sign in between.

NLO real contribution

Example: $[Q\bar{Q}(a^{[8]}) \to [Q\bar{Q}({}^1S_0^{[8]})]$

$$\mathcal{D}^{\rm NLO}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \hat{d}^{\,\rm NLO}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q)$$

Real contribution:

$$\mathcal{D}^{{\rm NLO},real}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \frac{\alpha_s}{16\,\pi m_Q N_c(N_c^2-1)}\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)$$
$$\times\Bigg\{-C_A^2\delta(\zeta_1)\delta(\zeta_2)\delta(1-z)\left(\frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}}-\frac{1}{\epsilon_{\rm IR}}\right)+\frac{1}{\epsilon_{\rm UV}}\frac{z}{(1-z)_+}\frac{\Delta_+^{[8]}}{4}$$
$$+2\,(\ln 2)\,C_A^2\,\delta(\zeta_1)\,\delta(\zeta_2)\,\delta(1-z)\frac{1}{\epsilon_{\rm UV}}$$
$$-2\left[(\ln 2)^2+\ln 2\right]C_A^2\,\delta(\zeta_1)\,\delta(\zeta_2)\,\delta(1-z)-(2\ln 2)\frac{z}{(1-z)_+}\frac{\Delta_+^{[8]}}{4}$$
$$+\frac{\Delta_+^{[8]}}{4}\left[-\frac{z}{(1-z)_+}-2\,z\left(\frac{\ln(1-z)}{1-z}\right)_+\right]\Bigg\},$$

$$\Delta_+^{[8]} \equiv 4\Big\{(N_c^2-2)[\delta(1-z+\zeta_1)\delta(1-z+\zeta_2)+\delta(1-z-\zeta_1)\delta(1-z-\zeta_2)]$$
$$+2[\delta(1-z-\zeta_1)\delta(1-z+\zeta_2)+\delta(1-z+\zeta_1)\delta(1-z-\zeta_2)]\Big\}.$$

Same structure as analyzed on last slide.

Double pole: $k\cdot\hat{n}\to 0,\ k_\perp\to\infty$

Soft pole: $$\frac{1}{(1-z)^{1+2\varepsilon}} = -\frac{1}{2\varepsilon_{IR}}\delta(1-z)+\frac{1}{(1-z)_+}-2\varepsilon\left(\frac{\ln(1-z)}{1-z}\right)_+$$

NLO virtual contribution

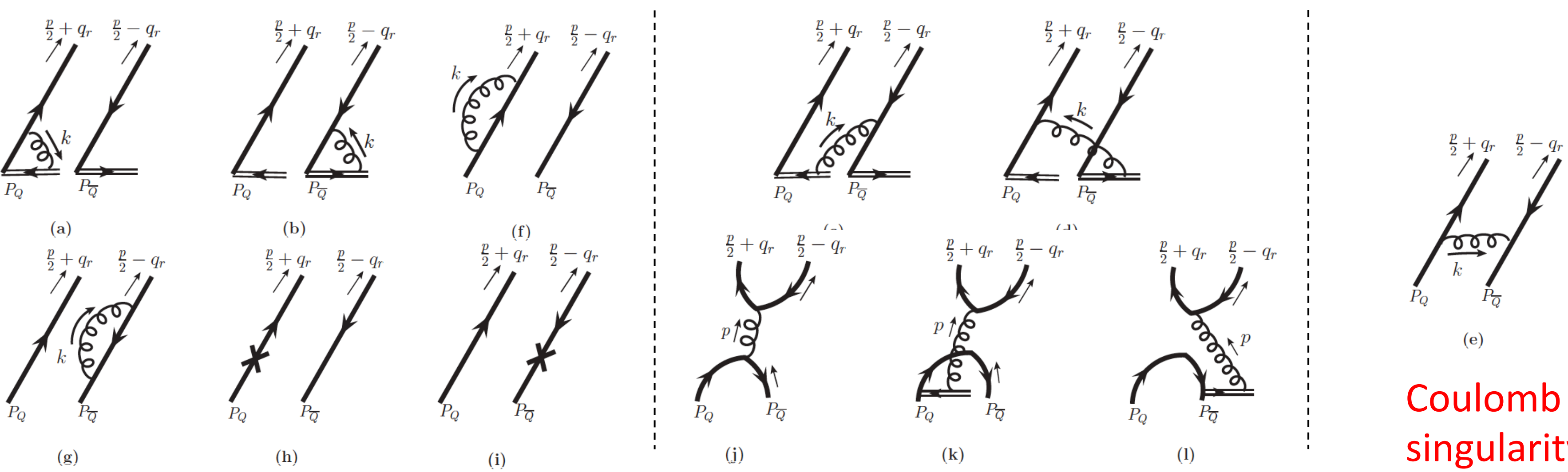
✧ NLO virtual contribution is non-trivial by keeping q_r finite in q_1-integral.

$$A_1(\zeta_1) = \lim_{q_r \to 0} \left(\prod_{j=0}^{l} \frac{\mathrm{d}}{\mathrm{d}q_r^{\alpha_j}} \right) \left\{ \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \, \delta(\zeta_1 - \frac{2q_1^+}{p_c^+}) \, \bar{A}(q_1, q_r) \right\}$$

Much easier if do the derivative and limit first

$$A_2(\zeta_1) = \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \, \delta(\zeta_1 - \frac{2q_1^+}{p_c^+}) \lim_{q_r \to 0} \left(\prod_{j=0}^{l} \frac{\mathrm{d}}{\mathrm{d}q_r^{\alpha_j}} \bar{A}(q_1, q_r) \right)$$

Equal only when the region $q_1 \lesssim q_r \to 0$ is not important.

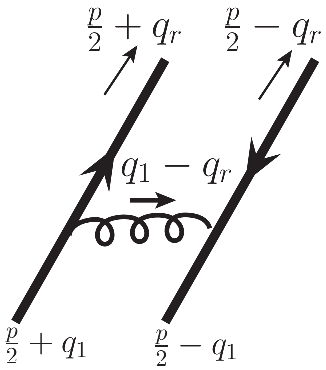


Coulomb singularity

For disconnect diagrams, q_1-integral is trivial.

Region $q_1 \lesssim q_r \to 0$ not important.

Coulomb Singularity (I)



$$A_1(\zeta_1) = \lim_{q_r\to 0}\left(\prod_{j=0}^{l}\frac{\mathrm{d}}{\mathrm{d}q_r^{\alpha_j}}\right)\left\{\int\frac{\mathrm{d}^D q_1}{(2\pi)^D}\,\delta(\zeta_1-\frac{2q_1^+}{p_c^+})\,\bar{A}(q_1,q_r)\right\}$$

$$\bar{A}(q_1,q_r) = \frac{B(q_1,q_r)}{[(q_r-q_1)^2+i\varepsilon]\,[(p/2+q_1)^2-m_Q^2+i\varepsilon]\,[(p/2-q_1)^2-m_Q^2+i\varepsilon]}$$

In the heavy quark pair rest frame $p\sim(2m_Q,\vec{0})$ $\quad q_r\sim(0,m_Q\vec{v})$

In Coulomb region $q_1\sim(m_Q v^2, m_Q\vec{v})$ $\quad d^D q_1\sim m_Q^D v^{D+1}$

$$(q_r-q_1)^2\sim(p/2+q_1)^2-m_Q^2\sim(p/2-q_1)^2-m_Q^2\sim m_Q^2 v^2$$

Leading contribution in this region behaves as 1/v.

qr is a regulator of this divergence,
performing derivative and limit before q1-integral gives a different result.

However, the difference is always absorbed into NRQCD LDMEs

Coulomb Singularity (II)

Consider the simplest case

$$I_1 \equiv \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{\delta(\zeta_1 - \frac{2q_1^+}{p^+})}{\left[(q_r - q_1)^2 + i\varepsilon\right] \left[(p/2 + q_1)^2 - m_Q^2 + i\varepsilon\right] \left[(p/2 - q_1)^2 - m_Q^2 + i\varepsilon\right]}$$

Use Feynman parameter and the identity

$$\int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{\delta(\zeta_1 - \frac{2q_1^+}{p^+})}{\left[(q_1 - q_1')^2 - \Delta\right]^n} = \delta(\zeta_1 - \frac{2q'^+_1}{p^+}) \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{1}{\left[(q_1 - q_1')^2 - \Delta\right]^n}$$

$$(q_1 - q'_1)^2 = 2(q_1^+ - q'^+_1)(q_1^- - q'^-_1) - (q_1 - q'_1)^2_\perp$$

Integration over q_1^{-} equals zero unless $q_1^+ = q'^+_1$

Same trick can be used to calculate integral with lightcone propagators

$$\int \frac{d^D q_1}{(2\pi)^D} \frac{1}{q_1 \cdot n \left[(q_1 - q_1')^2 - \Delta\right]^n} = \frac{1}{q_1' \cdot n} \int \frac{d^D q_1}{(2\pi)^D} \frac{1}{\left[(q_1 - q_1')^2 - \Delta\right]^n}$$

Generalized Plus/Minus Distributions

To regularize the singularities of terms like $\zeta_1^{-1-2\epsilon}\,\mathrm{Sgn}(\zeta_1)$ and $\zeta_1^{-2-2\varepsilon}\mathrm{Sgn}(\zeta_1)$ at $\zeta_1 \to 0$, we introduced the generalized plus/minus distributions

$$\Big(g(\zeta_1)\Big)_{m\pm} \equiv \int_{-1}^{1} [\theta(x) \pm \theta(-x)]\, g(|x|) \times \left(\delta(x-\zeta_1) - \sum_{i=0}^{m-1} \frac{\delta^{(i)}(\zeta_1)}{i\,!}(-x)^i\right) dx$$

So that

$$\frac{\mathrm{Sgn}(\zeta_1)}{(\zeta_1)^{1+2\epsilon}} = -\frac{1}{\epsilon_{\mathrm{IR}}}\delta(\zeta_1) + \left(\frac{1}{\zeta_1}\right)_{1+} - \epsilon\left(\frac{\ln(\zeta_1^2)}{\zeta_1}\right)_{1+}$$

$$\frac{\mathrm{Sgn}(\zeta_1)}{\zeta_1^{\,2+2\varepsilon}} = \frac{1}{\varepsilon_{\mathrm{IR}}}\delta'(\zeta_1) + \left(\frac{1}{\zeta_1^{\,2}}\right)_{2-} - \varepsilon\left(\frac{\ln\zeta_1^{\,2}}{\zeta_1^{\,2}}\right)_{2-}$$

The generalized plus and minus distributions are free of divergence at $\zeta_1 \to 0$.

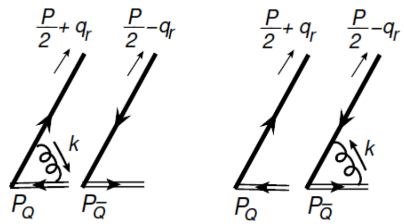
$$\int \left(\frac{1}{\zeta_1^2}\right)_{2-} f(\zeta_1)d\zeta_1 = \int_{-1}^{1}\left(\frac{\theta(\zeta_1)}{\zeta_1^2} - \frac{\theta(-\zeta_1)}{(-\zeta_1)^2}\right)\times\left[f(\zeta_1) - f(0) - f'(0)\,\zeta_1\right]d\zeta_1$$

$$\int \left(\frac{1}{\zeta_1}\right)_{1+} f(\zeta_1)d\zeta_1 = \int_{-1}^{1}\left(\frac{\theta(\zeta_1)}{\zeta_1} + \frac{\theta(-\zeta_1)}{(-\zeta_1)}\right)\times\left[f(\zeta_1) - f(0)\right]d\zeta_1$$

NLO virtual contribution

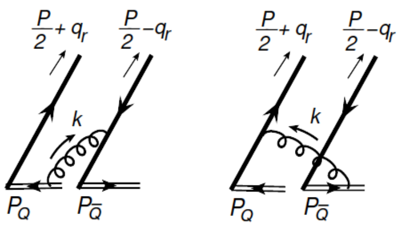
Example: $[Q\bar{Q}(a^{[8]}) \to [Q\bar{Q}(^1S_0^{[8]})]$

Virtual contribution:



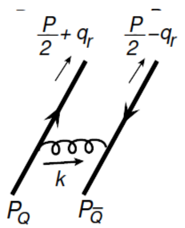
(a) (b)

$$\Lambda(\zeta_1) = \frac{\alpha_s}{16\pi m_Q(N_c^2-1)}\delta(\zeta_1)C_F\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left(\frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}}+\frac{2}{\epsilon_{\rm UV}}+4\right)$$



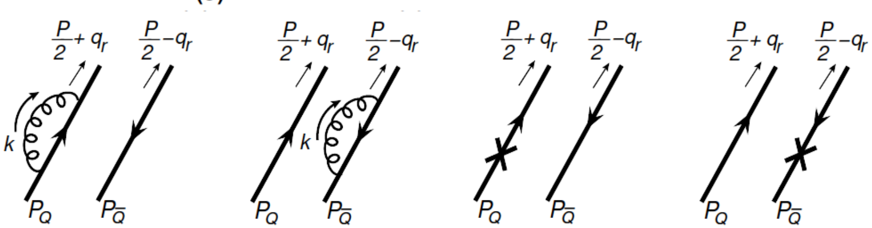
(c) (d)

$$\Sigma(\zeta_1) = \frac{\alpha_s}{32\pi m_Q}\frac{1}{N_c(N_c^2-1)}\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left\{\frac{1}{\epsilon_{\rm UV}\epsilon_{\rm IR}}\delta(\zeta_1)+\frac{1}{\epsilon_{\rm UV}}\left[1-\left(\frac{1}{\zeta_1}\right)_{1+}\right]\right.$$
$$\left.+\left(\frac{\ln(\zeta_1^2)}{\zeta_1}\right)_{1+}-\ln(\zeta_1^2)\right\},$$



(e)

$$\Pi(\zeta_1) = \frac{\alpha_s}{64\pi m_Q}\frac{1}{N_c(N_c^2-1)}\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left\{\frac{1}{\epsilon_{\rm UV}}((\zeta_1)_{0+}-1)-\frac{2}{\epsilon_{\rm IR}}\delta(\zeta_1)+4\,\delta(\zeta_1)\right.$$
$$\left.-(\zeta_1\ln(\zeta_1^2)+\zeta_1)_{0+}+2\left(\frac{1}{\zeta_1}\right)_{1+}-2\left(\frac{1}{\zeta_1^2}\right)_{2+}+\ln(\zeta_1^2)+1\right\},$$



(f) (g) (h) (i)

$$W(\zeta_1) = -\frac{\alpha_s}{32\pi m_Q(N_c^2-1)}\delta(\zeta_1)C_F\left(\frac{4\pi\mu^2}{m_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left(\frac{1}{\epsilon_{\rm UV}}+\frac{2}{\epsilon_{\rm IR}}+4\right)$$

Renormalization of pQCD operator

Adding up real and virtual contribution has leftover UV divergence, which has the same structure as evolution kernel.

Kang, Ma, Qiu and Sterman, arXiv: 1401.0923

$$\mathcal{D}^{\mathrm{NLO},ren}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2;m_Q) = -\frac{\alpha_s}{\pi}(4\pi e^{-\gamma_E})^{\epsilon}\frac{1}{\epsilon_{\mathrm{UV}}}\frac{1}{64\,m_Q N_c(N_c^2-1)}$$
$$\times\left\{\frac{z}{(1-z)_+}\Delta_+^{[8]} + 8(\ln 2)\,C_A^2\,\delta(\zeta_1)\,\delta(\zeta_2)\,\delta(1-z)\right.$$
$$\left. + \delta(1-z)\left[\delta(\zeta_2)F(\zeta_1) + \delta(\zeta_1)F(\zeta_2)\right]\right\},$$
$$F(\zeta_1) \equiv 6\,C_A C_F\,\delta(\zeta_1) - 2\left(\frac{1}{\zeta_1}\right)_{1+} + (\zeta_1)_{0+} + 1$$

After adding this renormalization contribution

$$\hat{d}^{\,\mathrm{NLO}}_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}({}^1S_0^{[8]})]}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \frac{\alpha_s}{64\,\pi m_Q(N_c^2-1)}$$
$$\times\left\{\Gamma_{[Q\bar{Q}(a^{[8]})]\to[Q\bar{Q}(a^{[8]})]}\left(z,\frac{1+\zeta_1}{2},\frac{1+\zeta_2}{2};\frac{1}{2},\frac{1}{2}\right)\ln\left[\frac{\mu_0^2}{m_Q^2}\right]\right.$$
$$\left. + R(z,\zeta_1,\zeta_2) + \delta(1-z)[V(\zeta_1)\delta(\zeta_2) + V(\zeta_2)\delta(\zeta_1)]\right\}$$

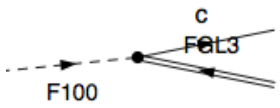
$$V(\zeta_1) = \frac{1}{N_c}\left\{2\left[-\left(\frac{1}{\zeta_1^2}\right)_{2+} + \left(\frac{1}{\zeta_1}\right)_{1+} + \left(\frac{\ln(\zeta_1^2)}{\zeta_1}\right)_{1+}\right] - [(\zeta_1)_{0+}+1]\ln(\zeta_1^2) - (\zeta_1)_{0+} + 1\right\}$$
$$R(z,\zeta_1,\zeta_2) = \frac{1}{N_c}\left\{\Delta_+^{[8]}\left[-2z\left(\frac{\ln(2-2z)}{1-z}\right)_+ - \frac{z}{(1-z)_+}\right] - 8\left[(\ln 2)^2 + \ln 2\right]C_A^2\,\delta(\zeta_1)\,\delta(\zeta_2)\,\delta(1-z)\right\}$$

Short-distance coefficient is finite.

For final P-wave heavy quark pair, may require IR divergence cancellation with LDMEs

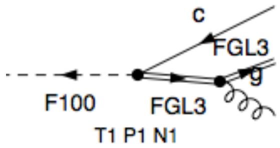
Mathematica Packages

- FeynArts –diagrams and Feynman amplitudes generator



Original package: no gauge link, connected diagrams only.

Improvement:



(1) Include new particle family "gauge link" and its Feynman rule;
(2) Include the nice double line for gauge links;
(3) Allow user-defined "seeds" for generating any type of diagrams.

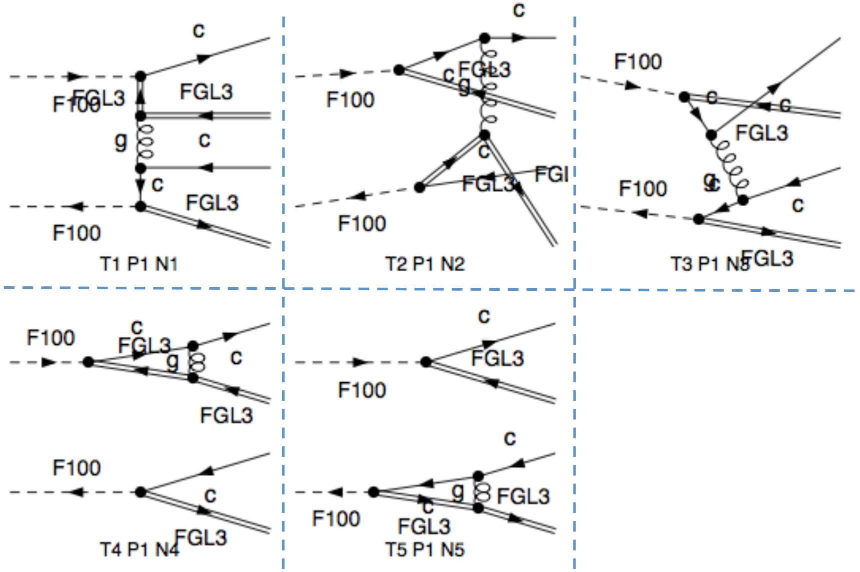
- FeynCalc –Feynman amplitude calculator

Original package: (1) Use different notations as in FeynArts;
(2) Cannot deal with $\int d^4k \frac{1}{k\cdot n + i\varepsilon}\cdots$.

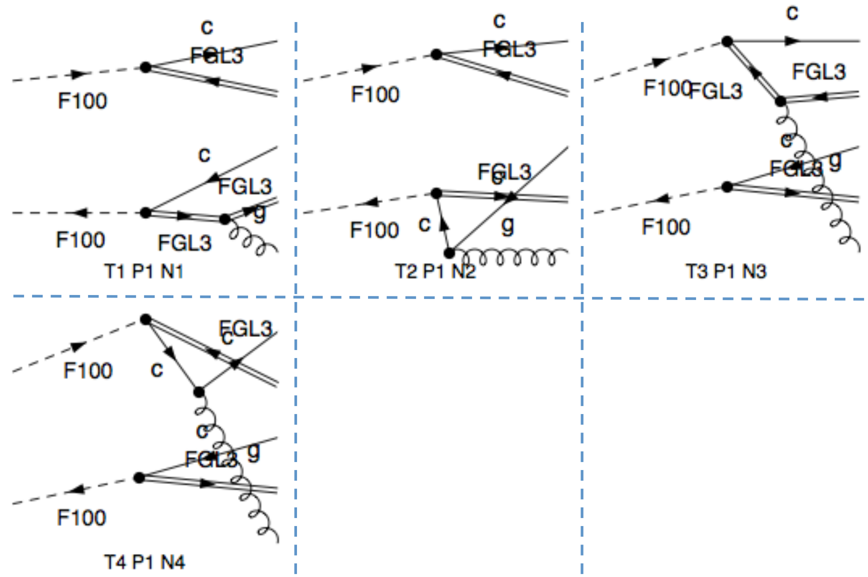
Improvement: (1) Include the translator to link FeynArts and FeynCalc;
(2) Isolate unknown integrations and wait for user's command;
(3) Many new functions to facilitate calculation.

Next-to Leading Power: Diagrams

- Virtual amplitudes



- Real amplitudes



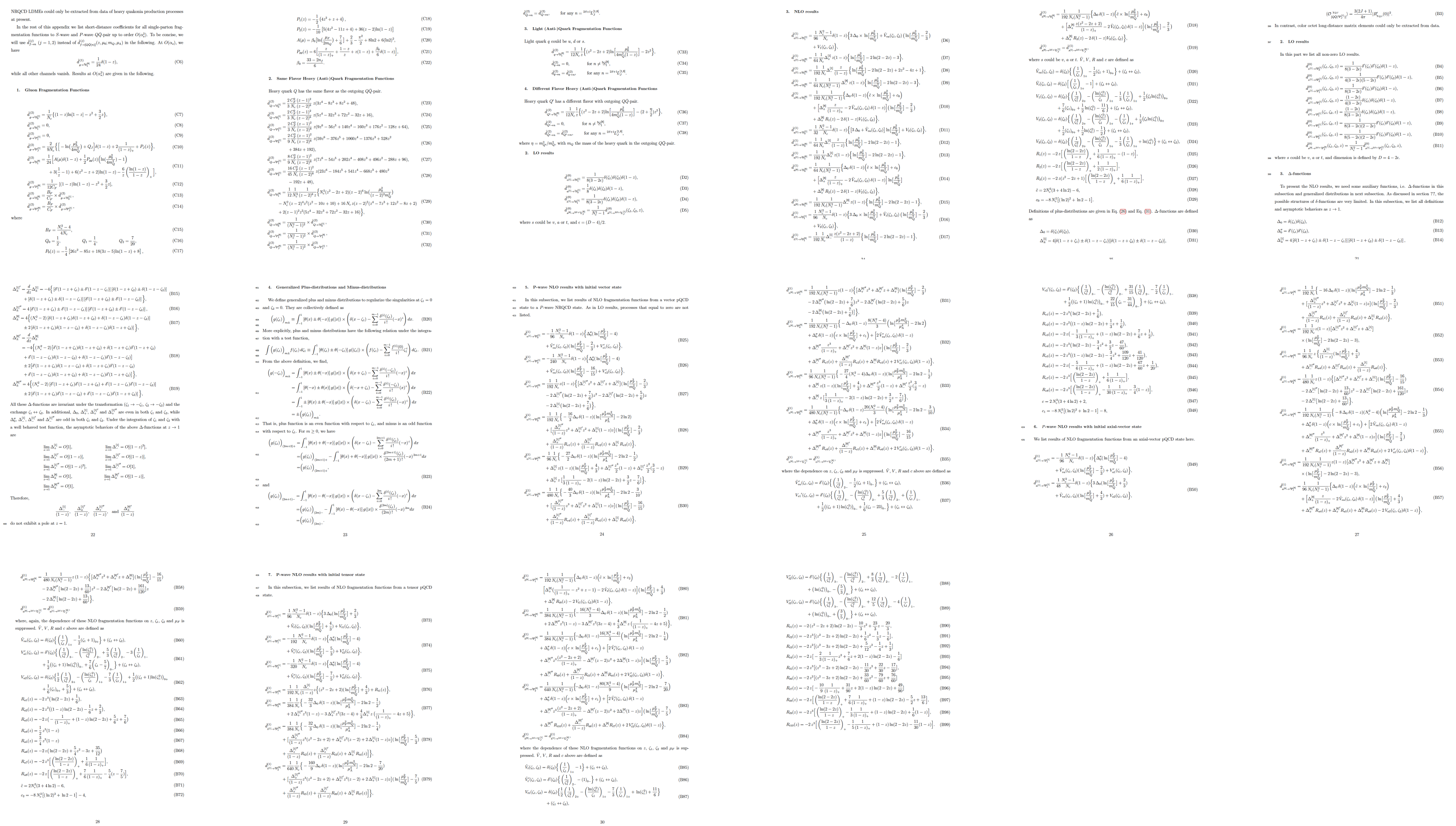
Sample results

$$\hat{d}^{(1)}_{v^{[8]}\to {}^3P_0^{[8]}} = \frac{z}{48}\frac{C_F}{(N_c^2-1)^2}\Big\{\delta(1-z)\Big[\frac{1}{3}(N_c^2-4)\Delta_0\Big(\ln[\frac{\mu_\Lambda^2}{m_Q^2}]+2\ln 2\Big)$$
$$+\frac{1}{4}\Delta_0''\Big(c\times\ln[\frac{\mu_F^2}{m_Q^2}]+c_1\Big)+\tilde{V}_{va}'(\zeta_1,\zeta_2)\big(\ln[\frac{\mu_F^2}{m_Q^2}]-\frac{2}{3}\big)+V_{v1}'(\zeta_1,\zeta_2)\Big]$$
$$+\Big[\frac{\Delta_+^{[8]\prime\prime}}{(1-z)_+}+\Delta_+^{[8]\prime}+\Delta_+^{[8]}(1-z)\Big]\big(\frac{1}{2}\ln[\frac{\mu_F^2}{m_Q^2}]-\frac{1}{3}\big)$$
$$-\Delta_+^{[8]\prime\prime}R_{v7}(z)-\frac{\Delta_+^{[8]\prime}}{(1-z)}R_{v2}(z)-\Delta_+^{[8]}R_{v3}(z)\Big\}, \tag{B32}$$

$$\hat{d}^{(1)}_{v^{[8]}\to {}^3P_1^{[8]}} = \frac{z}{48}\frac{C_F}{(N_c^2-1)^2}\Big\{\frac{27}{16}(N_c^2-4)\Delta_0\,\delta(1-z)\big(\ln[\frac{\mu_\Lambda^2}{m_Q^2}]+2\ln 2+\frac{1}{2}\big)$$
$$+\Delta_+^{[8]}(1-z)\big(\ln[\frac{\mu_F^2}{m_Q^2}]+\frac{4}{3}\big)+\frac{\Delta_+^{[8]\prime\prime}}{2}(1-z)+\frac{\Delta_+^{[8]\prime}}{2}\big(\frac{3}{2}-z\big)$$
$$+\Delta_+^{[8]}\Big[\frac{1}{3}\frac{1}{(1-z)_+}-2(1-z)\ln(2-2z)+\frac{3}{2}z-\frac{7}{6}\Big]\Big\}, \tag{B33}$$

$$\hat{d}^{(1)}_{v^{[8]}\to {}^3P_2^{[8]}} = \frac{z}{120}\frac{C_F}{(N_c^2-1)^2}\Big\{\delta(1-z)\Big[\frac{5}{6}(N_c^2-4)\Delta_0\Big(\ln[\frac{\mu_\Lambda^2}{m_Q^2}]+2\ln 2+\frac{3}{10}\Big)$$
$$+\frac{1}{4}\Delta_0''\Big(c\times\ln[\frac{\mu_F^2}{m_Q^2}]+c_1\Big)+\tilde{V}_{va}'(\zeta_1,\zeta_2)\big(\ln[\frac{\mu_F^2}{m_Q^2}]-\frac{16}{15}\big)+V_{v2}'(\zeta_1,\zeta_2)\Big]$$
$$+\Big[\frac{\Delta_+^{[8]\prime\prime}}{(1-z)_+}+\Delta_+^{[8]\prime}+\Delta_+^{[8]}(1-z)\Big]\big(\frac{1}{2}\ln[\frac{\mu_F^2}{m_Q^2}]-\frac{8}{15}\big)$$
$$-\Delta_+^{[8]\prime\prime}R_{v8}(z)-\frac{\Delta_+^{[8]\prime}}{(1-z)}R_{v5}(z)-\Delta_+^{[8]}R_{v6}(z)\Big\}, \tag{B34}$$

All Results



All short-distance coefficients are IR safe at NLO of α_s.

Summary

- At large $p_T >> m_Q$, calculations based on early models are not perturbatively stable. LO in α_s-expansion does not coincide with LP term in $1/p_T$-expansion.
- PQCD factorization is a strictly proved factorization approach. It expands $1/p_T$ before α_s expansion.
- With the evolution equations, the predictive power of pQCD factorization for heavy quarkonium production relies completely on the initial distributions
- Different from fragmentation to pion, fragmentation to heavy quarkonium is partially perturbative because of the large mass scale. NRQCD is applied to calculate the fragmentation functions at scale $\mu \sim 2m_Q$.
- With our results, many unknown fragmentation functions are reduced into a few parameters, which are much easier to be extracted from data.

What's Next?

- HQ production in e+e- collision
  - the effect of resummation
- HQ production in ep, pp collision
  - size of contribution from color octet channel

} Global fitting of CO NRQCD LDMEs

- HQ production and polarization in unpolarized and polarized pp collision
  - further test this new pQCD formalism with HQ polarization, constrain Generalized Parton Distribution (GPD) E^g

Feng Yuan 2008
Koempel et.al. 2012

Then

- Suppression of HQ production in Cold Nuclear Matter and Quark Gluon Plasma.

What's Next?

- HQ production in e+e- collision

  the effect of resummation

- HQ production in ep, pp collision

  size of contribution from color octet channel

} Global fitting of CO NRQCD LDMEs

- HQ production and polarization in unpolarized and polarized pp collision

  further test this new pQCD formalism with HQ polarization, constrain Generalized Parton Distribution (GPD) E^g

  Feng Yuan 2008
  Koempel et.al. 2012

Then

- Suppression of HQ production in Cold Nuclear Matter and Quark Gluon Plasma.

Still a long way to go.

Thank you!

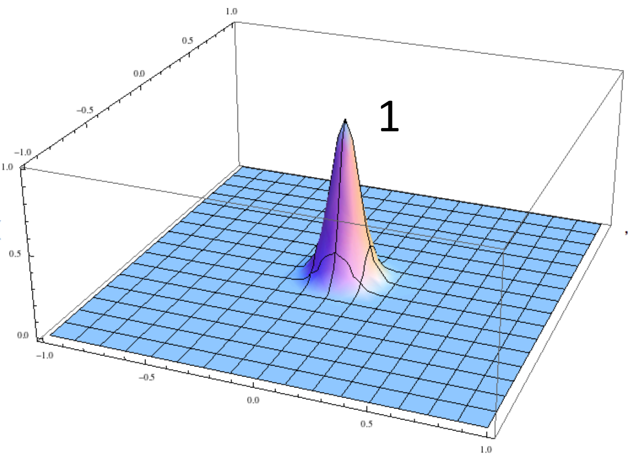
&

Happy Chinese New Year!

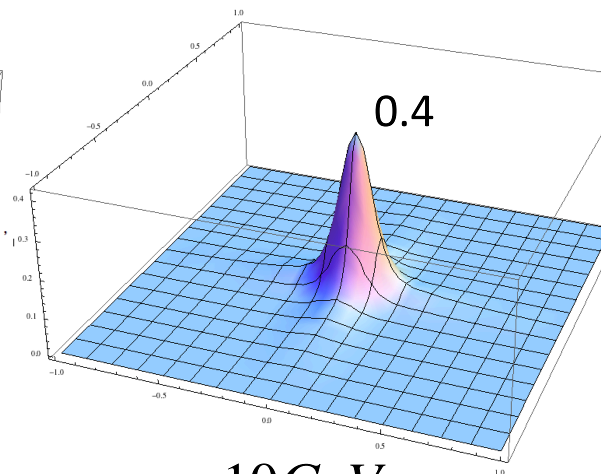
Next-to Leading Power: evolution effect

- Evolution of double parton fragmentation

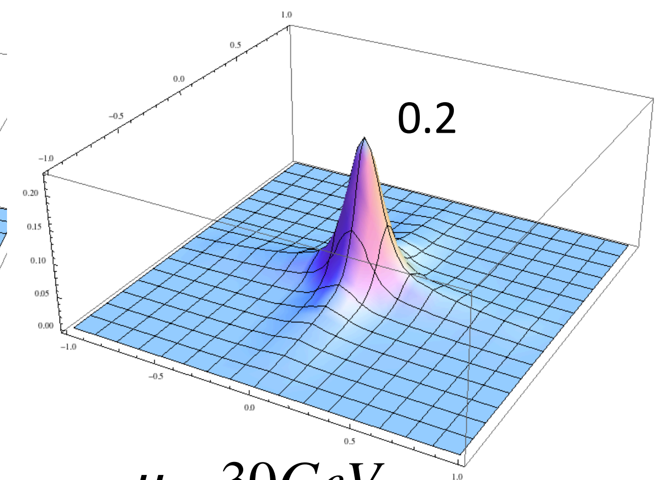
Starting profile: $\delta(\zeta)\delta(\zeta') \rightarrow \mathrm{e}^{-100\zeta^2} e^{-100\zeta'^2}$



$\mu = 3GeV$



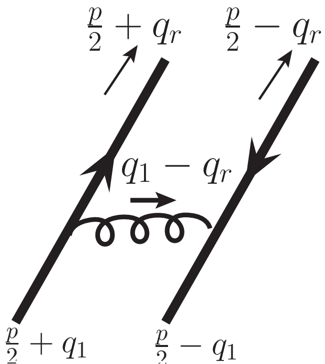
$\mu = 10GeV$



$\mu = 30GeV$

When in higher energy scale,
Peak gets lower and wider while evolving to larger μ.

Coulomb Singularity (I)



$$A_1(\zeta_1) = \lim_{q_r \to 0} \left(\prod_{j=0}^{l} \frac{\mathrm{d}}{\mathrm{d}q_r^{\alpha_j}} \right) \left\{ \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \delta(\zeta_1 - \frac{2q_1^+}{p_c^+}) \bar{A}(q_1, q_r) \right\}$$

$$\bar{A}(q_1, q_r) = \frac{B(q_1, q_r)}{[(q_r - q_1)^2 + i\varepsilon] \, [(p/2 + q_1)^2 - m_Q^2 + i\varepsilon] \, [(p/2 - q_1)^2 - m_Q^2 + i\varepsilon]}$$

In the heavy quark pair rest frame $p \sim (2m_Q, \vec{0})$ $\quad q_r \sim (0, m_Q \vec{v})$

In Coulomb region $q_1 \sim (m_Q v^2, m_Q \vec{v})$ $\quad d^D q_1 \sim m_Q^D v^{D+1}$

$$(q_r - q_1)^2 \sim (p/2 + q_1)^2 - m_Q^2 \sim (p/2 - q_1)^2 - m_Q^2 \sim m_Q^2 v^2$$

Leading contribution in this region behaves as 1/v.

qr is a regulator of this divergence,
performing derivative and limit before q1-integral gives a different result.

However, the difference is always absorbed into NRQCD LDMEs

Coulomb Singularity (II)

Consider the simplest case

$$I_1 \equiv \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{\delta(\zeta_1 - \frac{2q_1^+}{p^+})}{\left[(q_r - q_1)^2 + i\varepsilon\right] \left[(p/2 + q_1)^2 - m_Q^2 + i\varepsilon\right] \left[(p/2 - q_1)^2 - m_Q^2 + i\varepsilon\right]}$$

Use Feynman parameter and the identity $(q_1 - q'_1)^2 = 2(q_1^+ - q'^+_1)(q_1^- - q'^-_1) - (q_1 - q'_1)_\perp^2$

$$\int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{\delta(\zeta_1 - \frac{2q_1^+}{p^+})}{\left[(q_1 - q_1')^2 - \Delta\right]^n} = \delta(\zeta_1 - \frac{2q'^+_1}{p^+}) \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \frac{1}{\left[(q_1 - q_1')^2 - \Delta\right]^n}$$

And we get

$$I_1 = -\frac{i}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon)}{(p^2/4)^{1+\epsilon}} \frac{1}{2} Z(\zeta_1, q_r) \qquad \text{where } \tilde{\beta} = 2q_r^+/p^+ \text{ and } \beta^2 = -4q_r^2/p^2$$

$$Z(\zeta_1, q_r) = \int_{-1}^{1} \frac{dy}{(y^2 - \beta^2 - i\varepsilon)^{1+\epsilon}} \int_0^1 \frac{dx}{x^{1+2\epsilon}} \delta(\zeta_1 - \tilde{\beta} + xy + x\tilde{\beta})$$

Assume δ-function is integrated with a well-behaved function

Expand $\delta(\zeta_1 - \tilde{\beta} + xy + x\tilde{\beta}) = \sum_{i,j \geq 2k, k} C_{i,j,2k} \tilde{\beta}^i x^j y^{2k}$

And the x-integral is trivial, while y-integral is $\int_{-1}^{1} \frac{y^{2k} dy}{(y^2 - \beta^2 - i\varepsilon)^{1+\epsilon}}$

We have not expanded by β yet.

Coulomb Singularity (III)

$$\int_{-1}^{1} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} = \left(\int_{-1}^{-\Lambda} + \int_{\Lambda}^{1}\right) E_k(y^2)dy + \int_{-\Lambda}^{\Lambda} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}$$

Expanded by β before integration at region |y| > Λ. $\Lambda^2 \gg \beta^2$

$$\frac{y^{2k}}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} = \frac{y^{2k}}{y^{2-2\epsilon}} + (1+\epsilon)\frac{y^{2k}}{y^{4+2\epsilon}}\beta^2 + \cdots \equiv E_k(y^2)$$

$$\int_{-1}^{1} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} - \int_{-1}^{1} E_k(y^2)dy = \int_{-\Lambda}^{\Lambda} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} - \int_{-\Lambda}^{\Lambda} E_k(y^2)dy$$

Especially when $\Lambda \to \infty$, the second integration on RHS vanishes.

Beneke and Smirnov (1998)

$$\int_{-1}^{1} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} - \underline{\int_{-1}^{1} E_k(y^2)dy} = \int_{-\infty}^{+\infty} \frac{y^{2k}dy}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}$$

$$= \beta^{2k-1-2\epsilon} \int_{-\infty}^{+\infty} \frac{y^{2k}dy}{(y^2-1-i\varepsilon)^{1+\epsilon}}$$

Contribution From hard region

Contribution From potential Region.

By performing derivative and limit before q1-integral, the contribution from potential region does not show explicitly.