The hyperon puzzle: new hints from Quantum Monte Carlo calculations

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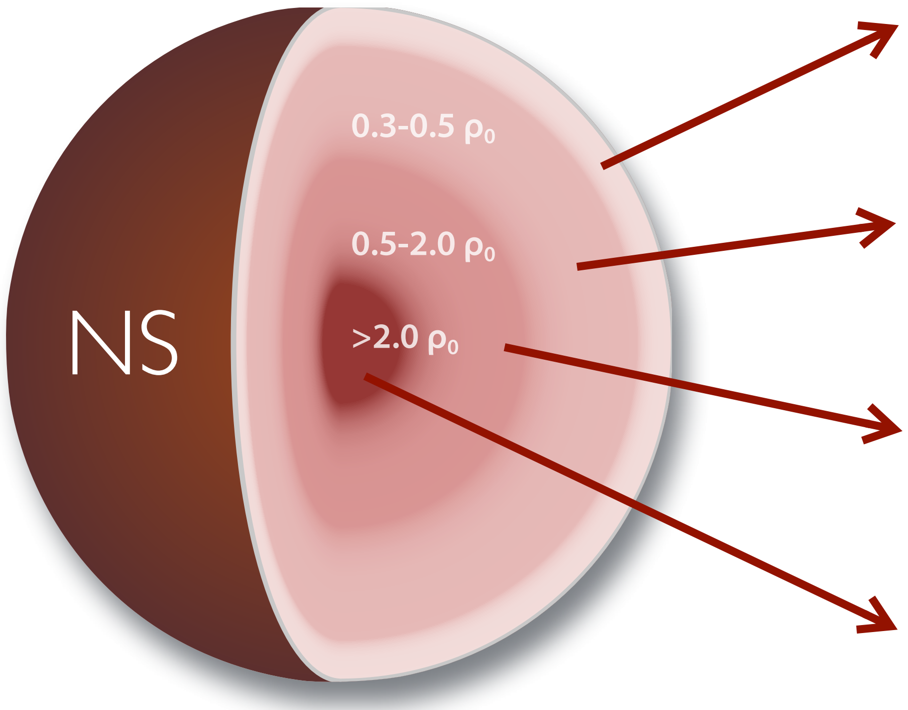
Outline

- ✓ Introduction: strangeness in nuclear systems
  - hyperons in neutron stars: the hyperon puzzle
  - hyperons in finite nuclei
  - hyperon-nucleon interaction
- ✓ The strange QMC project
  - the method
  - the results
- ✓ Conclusions

Introduction: hyperons in neutron stars



$R \sim 12$ km

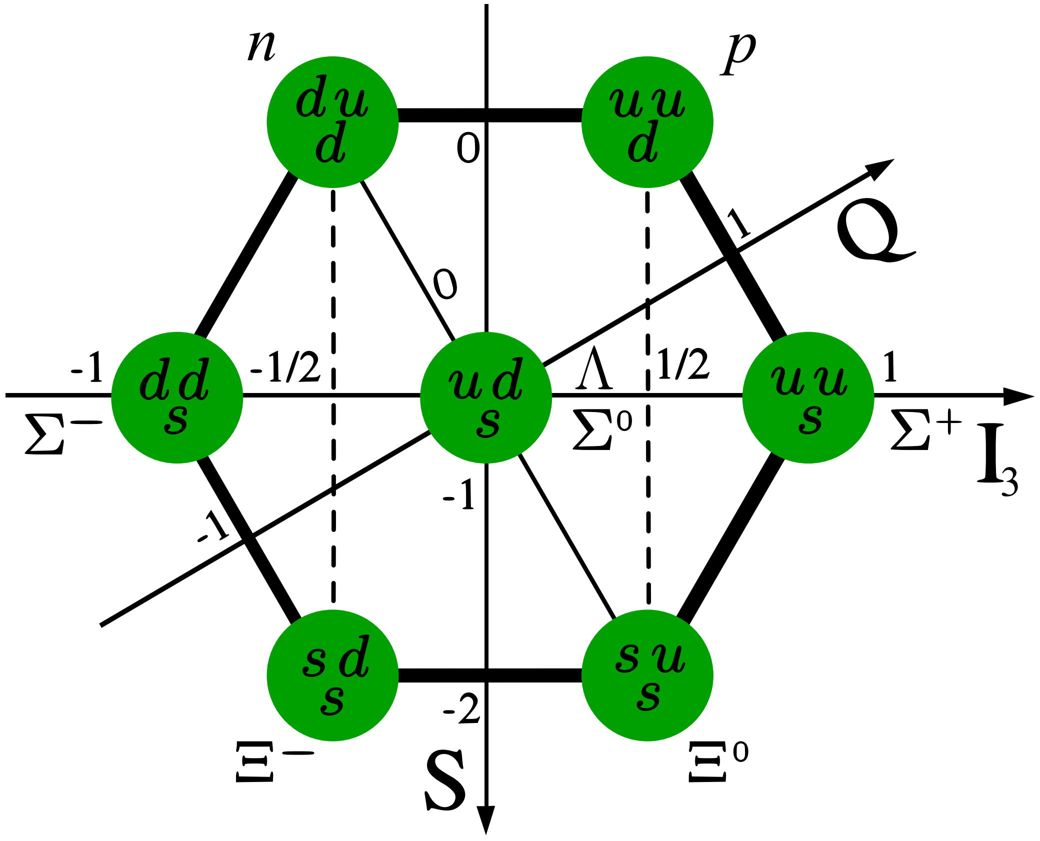
$M \sim 1.4\ M_\odot$

outer crust: $Z\ e$
$(0.3 \div 0.5\ \mathrm{km})$

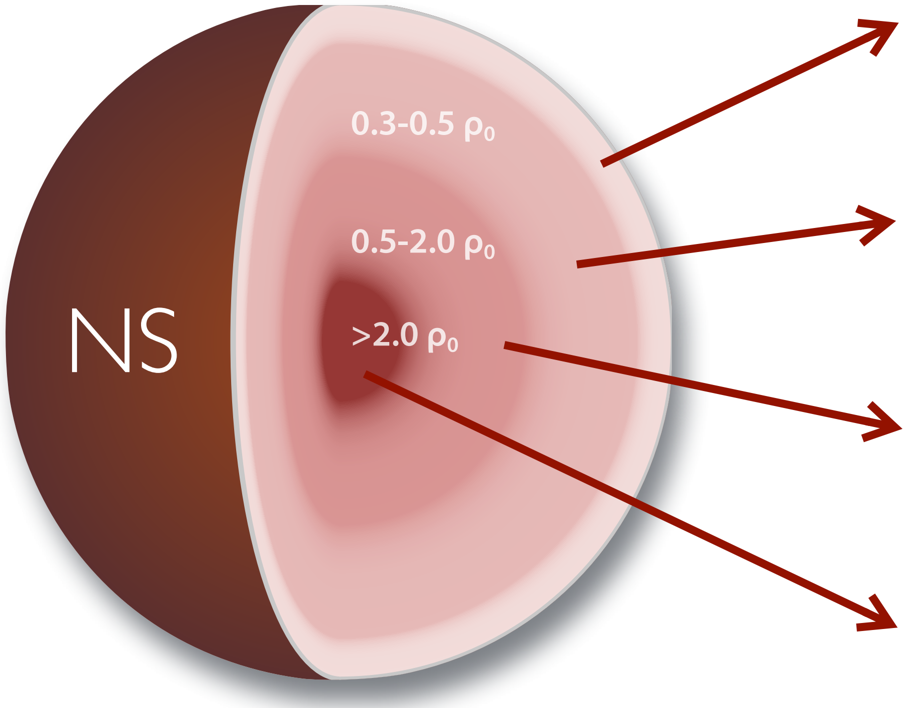
inner crust: $Z\ n\ e$
$(1 \div 2\ \mathrm{km})$

outer core: $n\ p\ e\ \mu$
$(\sim 9\ \mathrm{km})$

inner core: $n\ p\ e\ \mu\ \Lambda\ \Sigma\ \Xi\ \pi_c\ K_c\ q_p$?
$(0 \div 3\ \mathrm{km})$



Introduction: hyperons in neutron stars



outer crust: $Z\ e$
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inner crust: $Z\ n\ e$
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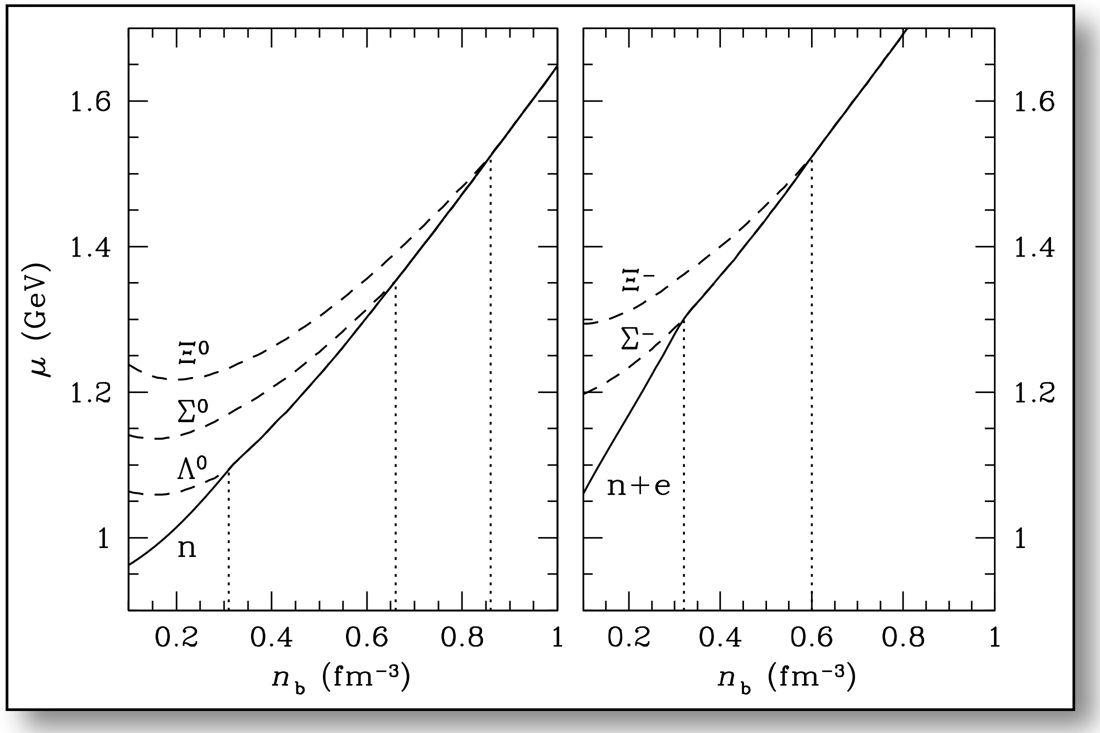
outer core: $n\ p\ e\ \mu$
$(\sim 9\ \text{km})$

inner core: $n\ p\ e\ \mu\ \Lambda\ \Sigma\ \Xi\ \pi_c\ K_c\ q_p$?
$(0 \div 3\ \text{km})$

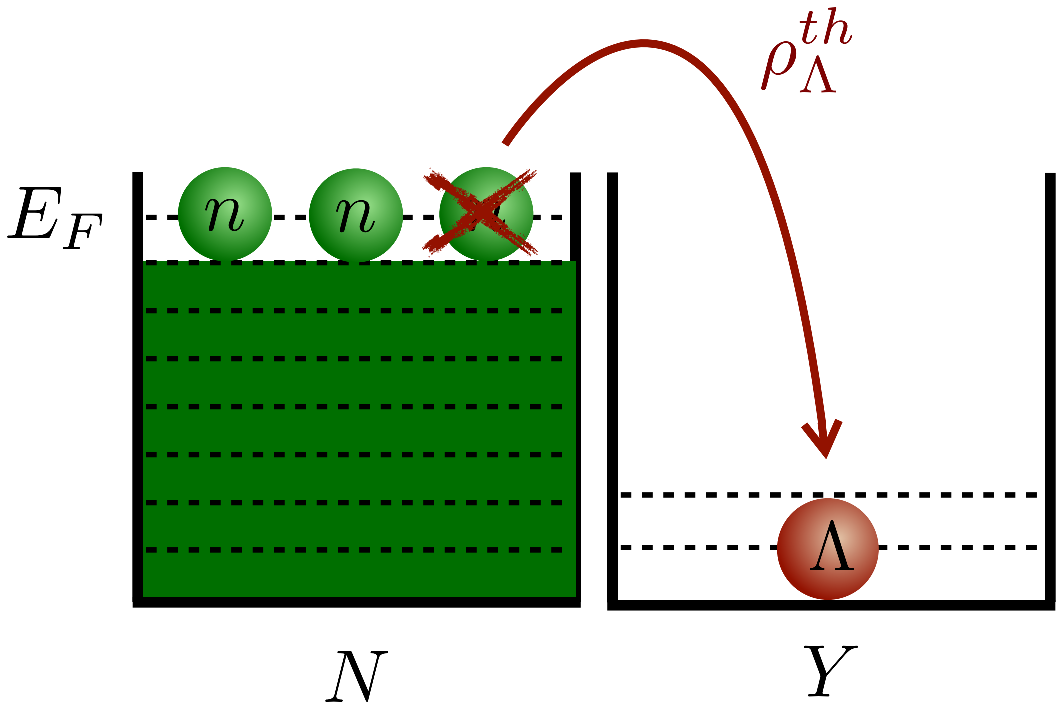
$R \sim 12\ \text{km}$
$M \sim 1.4\ M_\odot$

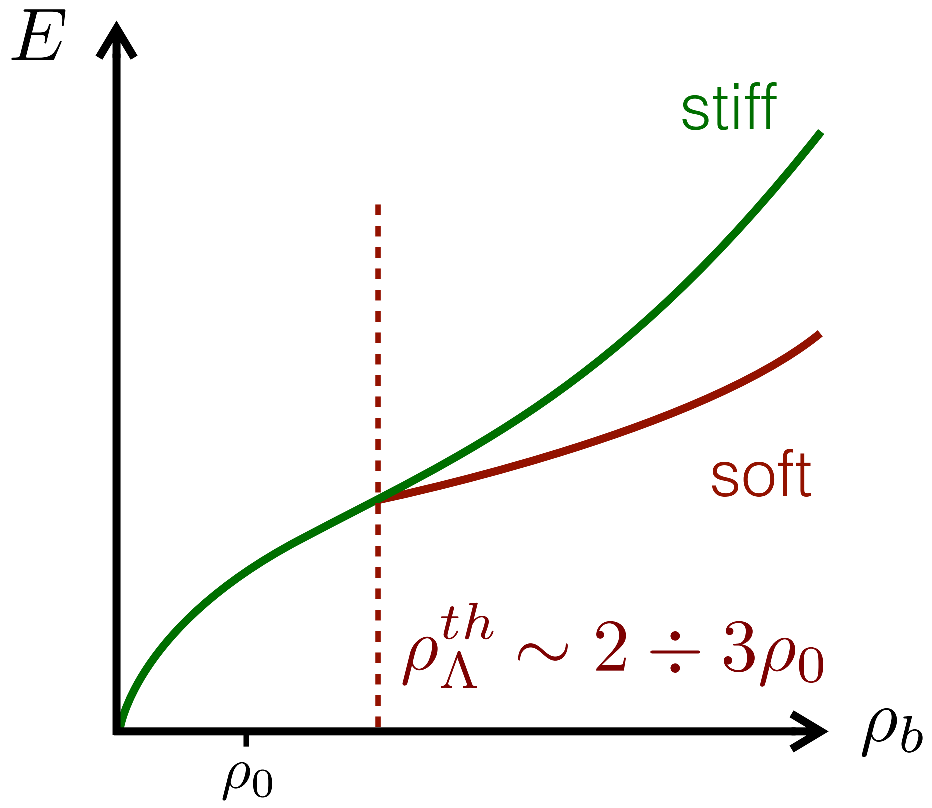
$$Q = -1 \ : \mu_{b^-} = \mu_n + \mu_e$$
$$Q = \ \ 0 \ : \mu_{b^0} = \mu_n$$
$$Q = +1 \ : \mu_{b^+} = \mu_n - \mu_e$$



P. Haensel, A.Y. Potekhin, D.G. Yakovlev
Neutron Stars 1, Springer 2007

Introduction: hyperons in neutron stars

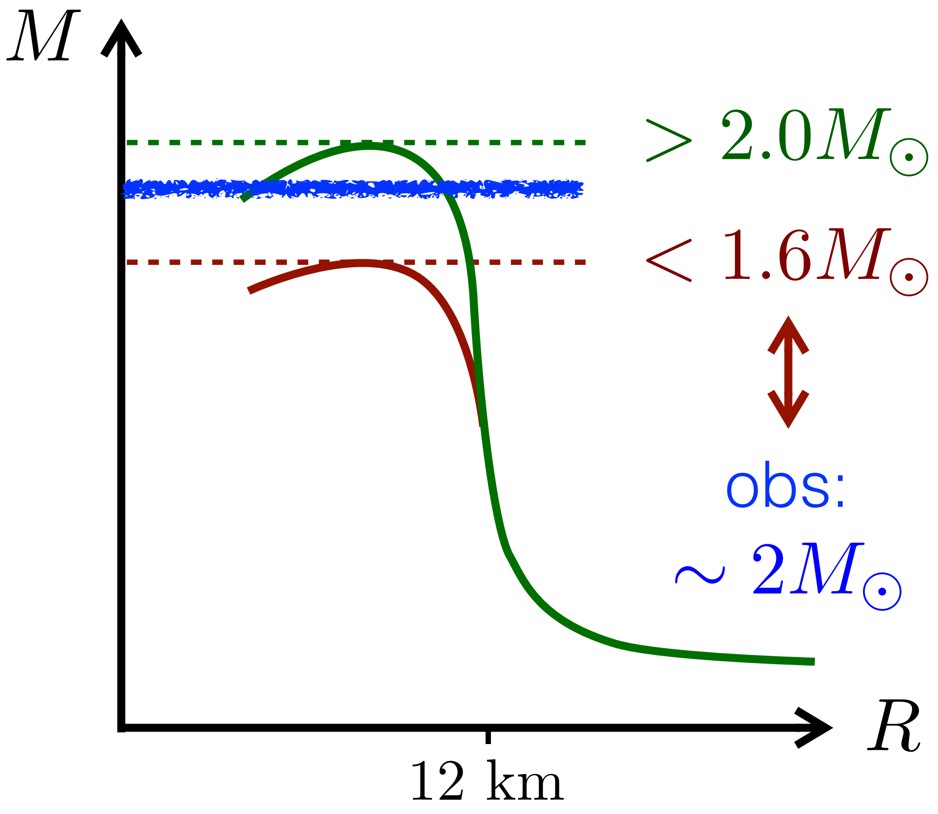




TOV

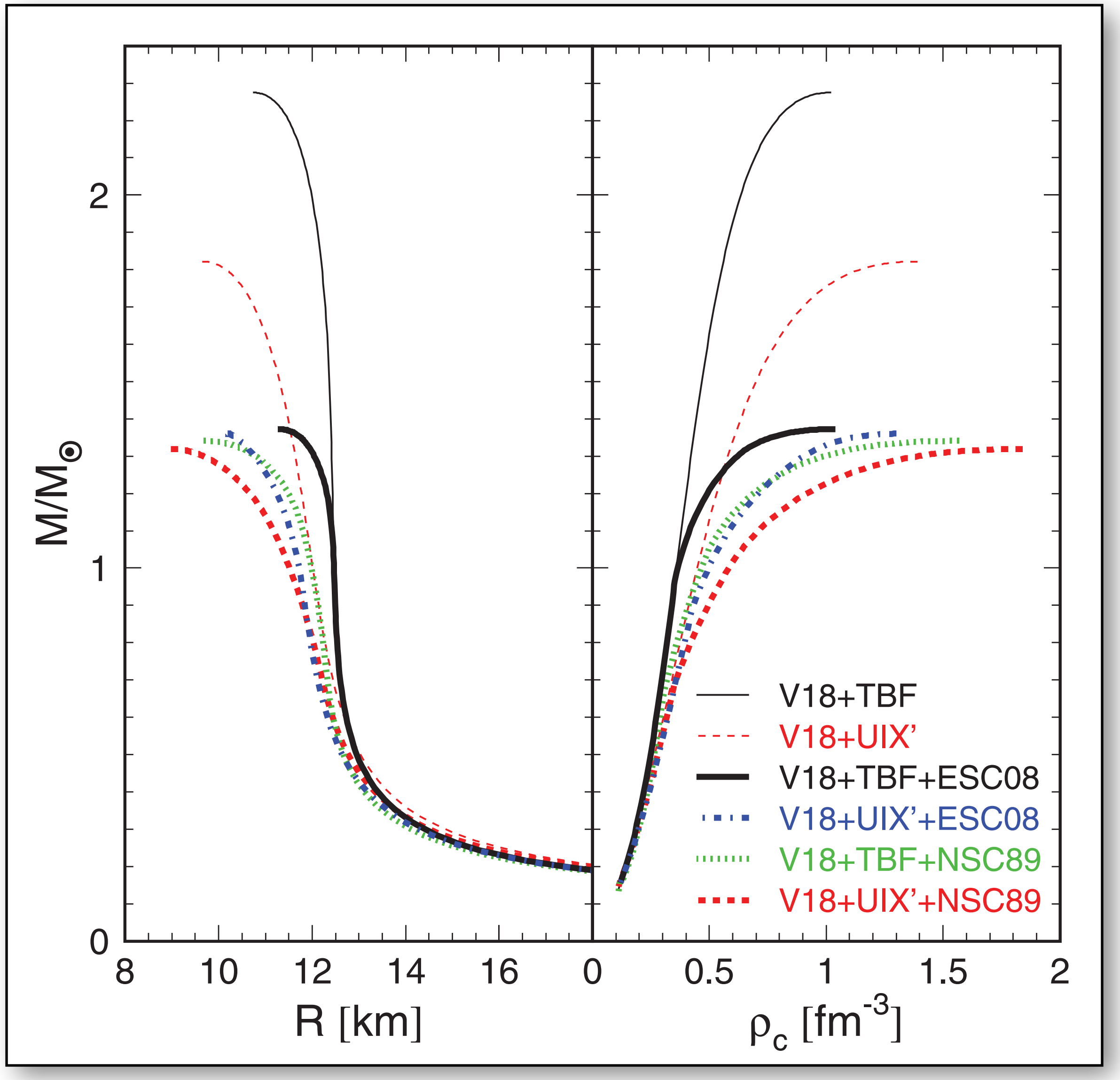
$$\begin{cases} \dfrac{dP(r)}{dr} = \mathcal{F}\big(\mathcal{E}(r), P(r), m(r)\big) \\ \dfrac{dm(r)}{dr} = \mathcal{F}\big(\mathcal{E}(r)\big) \end{cases}$$

$M(R), M_{\max} \longleftrightarrow$ obs

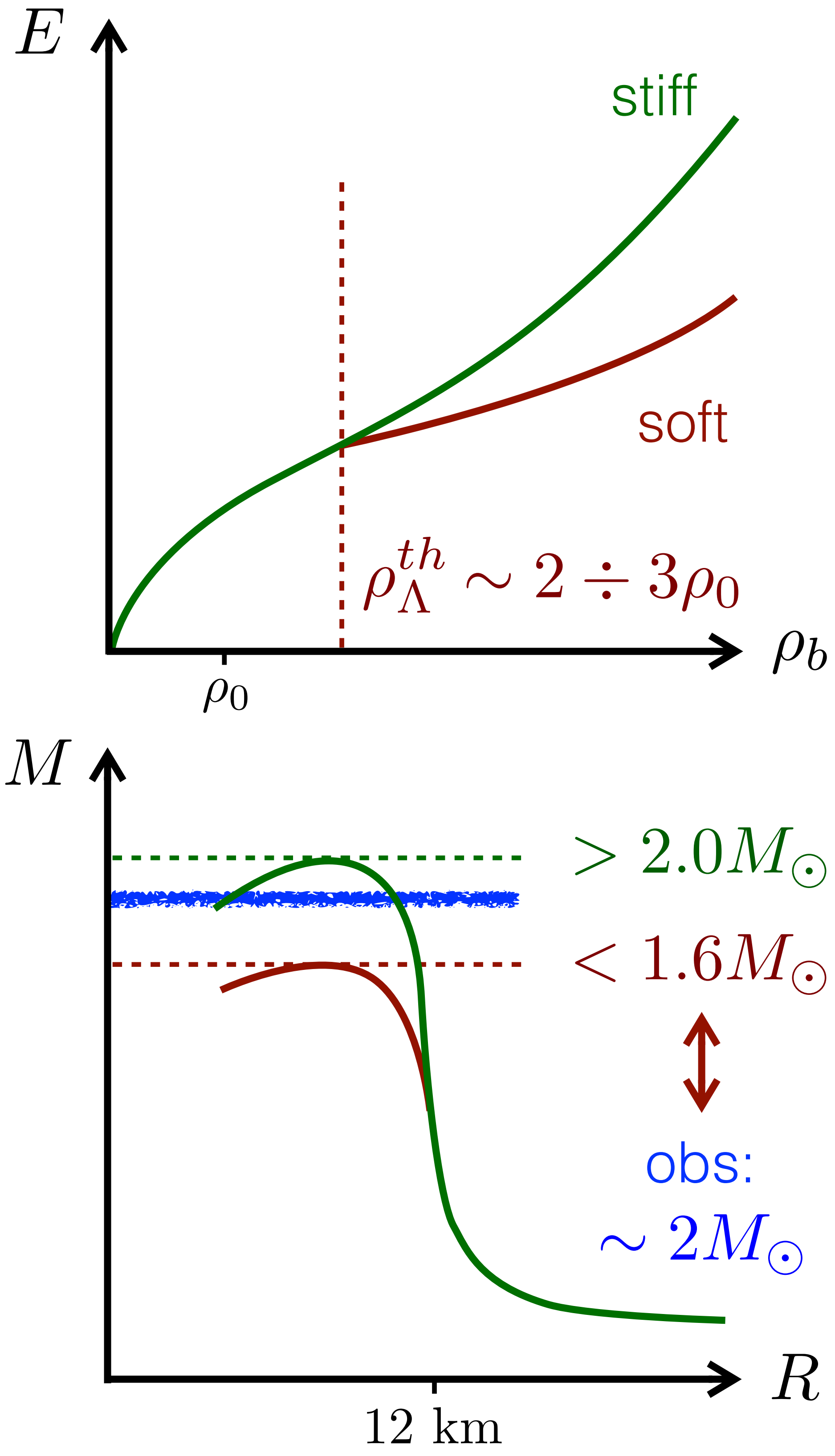


Introduction: hyperons in neutron stars

Brueckner-Hartree-Fock

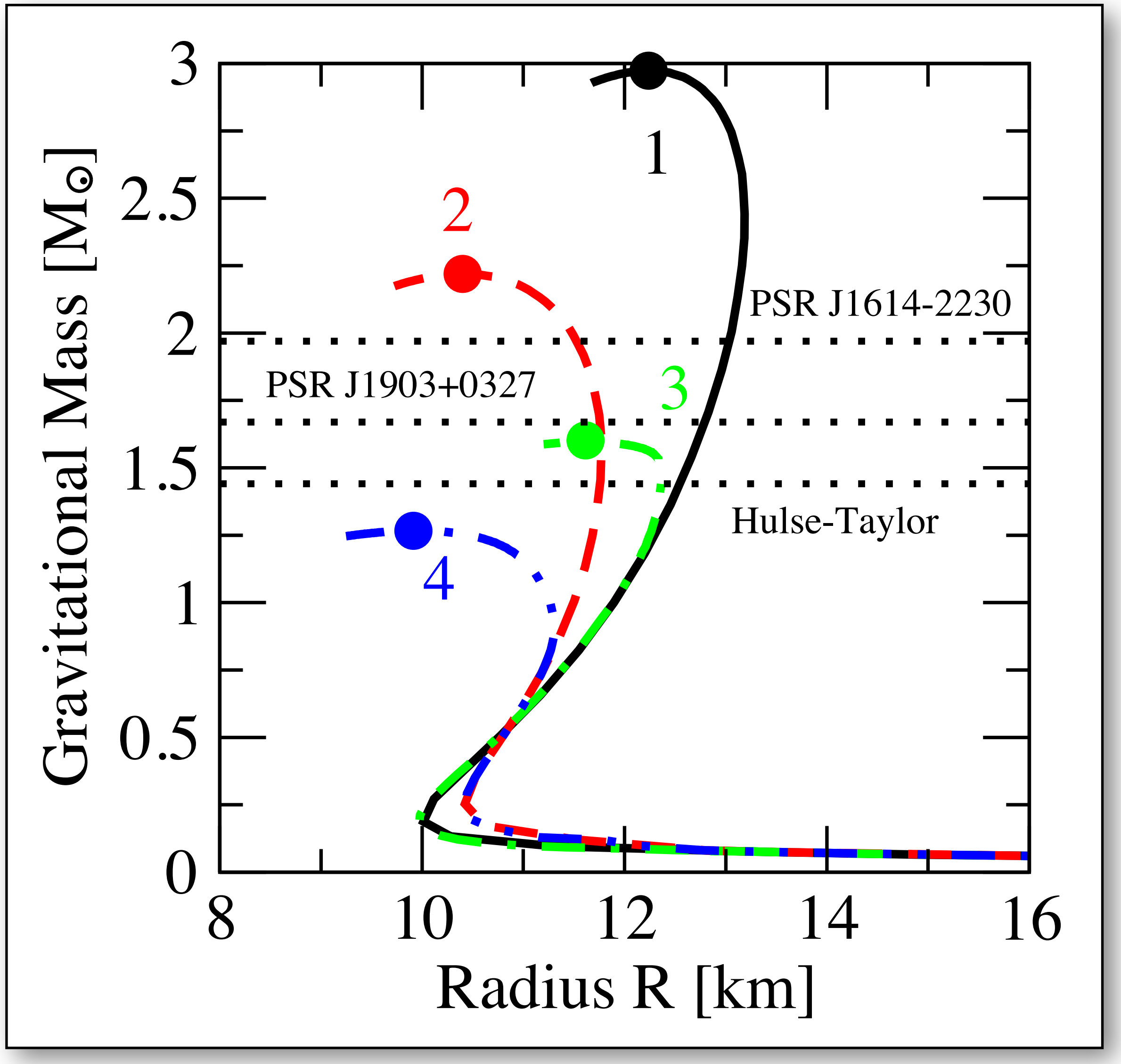


H.-J. Schulze, T. Rijken, Phys. Rev. C 84, 035801 (2011)

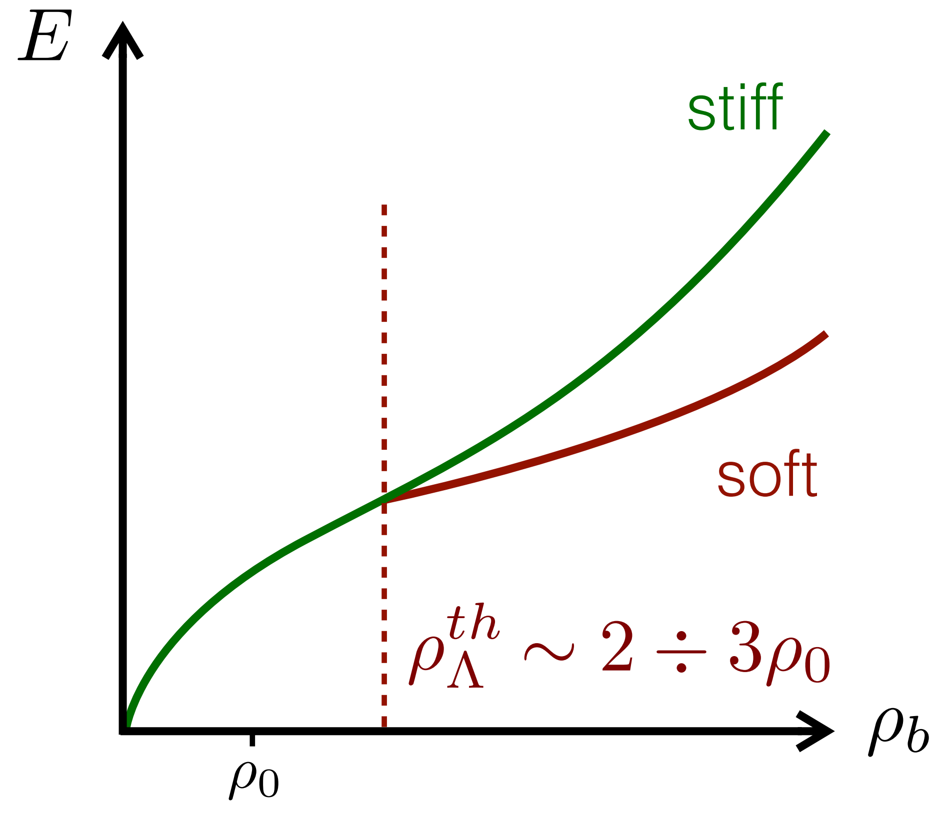


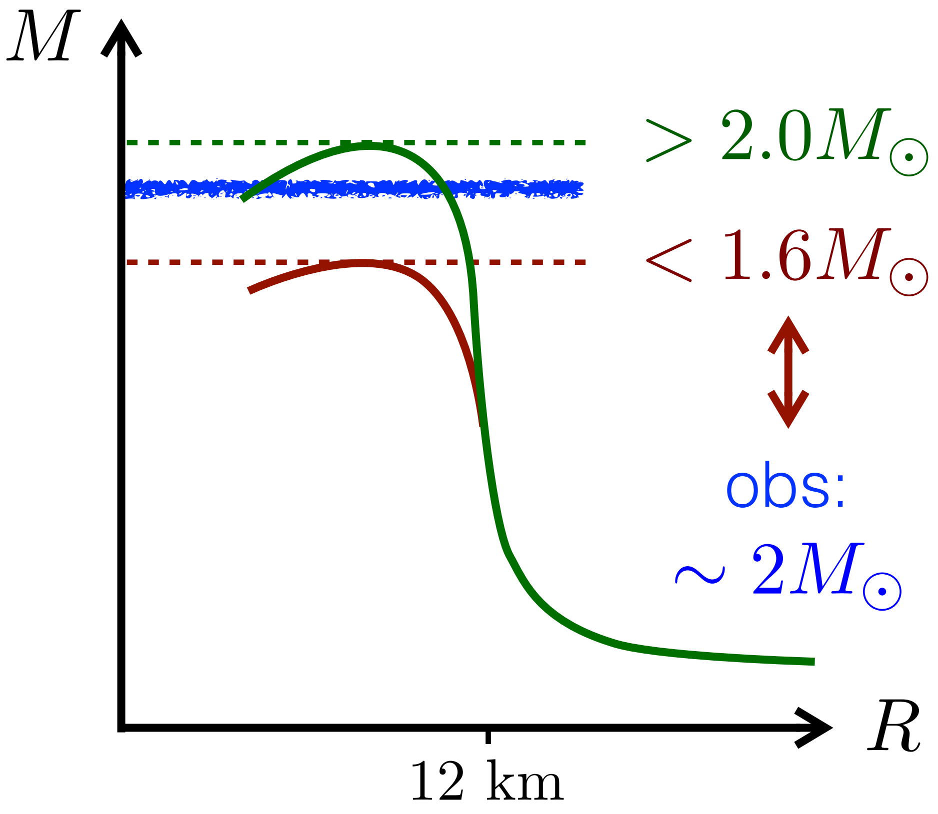
Introduction: hyperons in neutron stars

Brueckner-Hartree-Fock



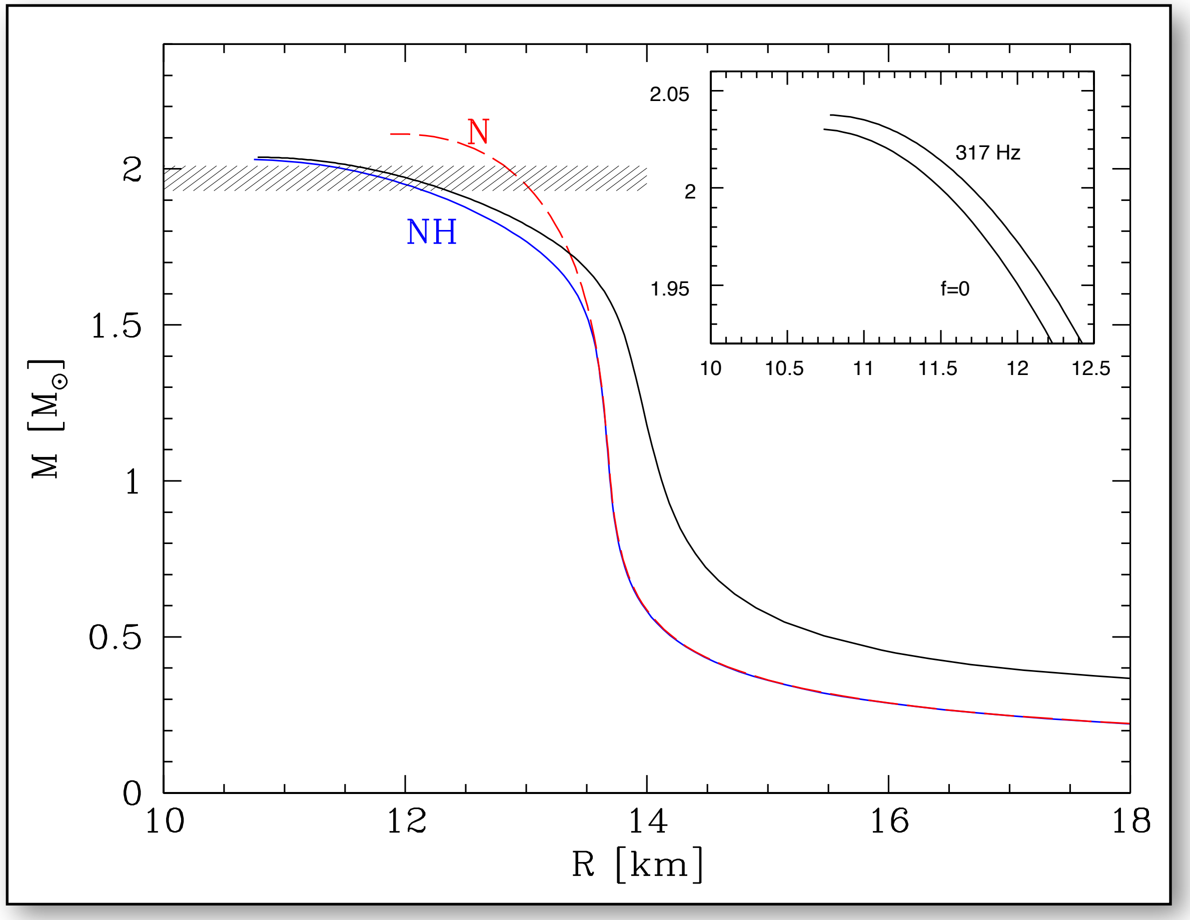
I. Vidaña, D. Logoteta, C. Providência, A. Polls, I. Bombaci, EPL, 94 (2011) 11002



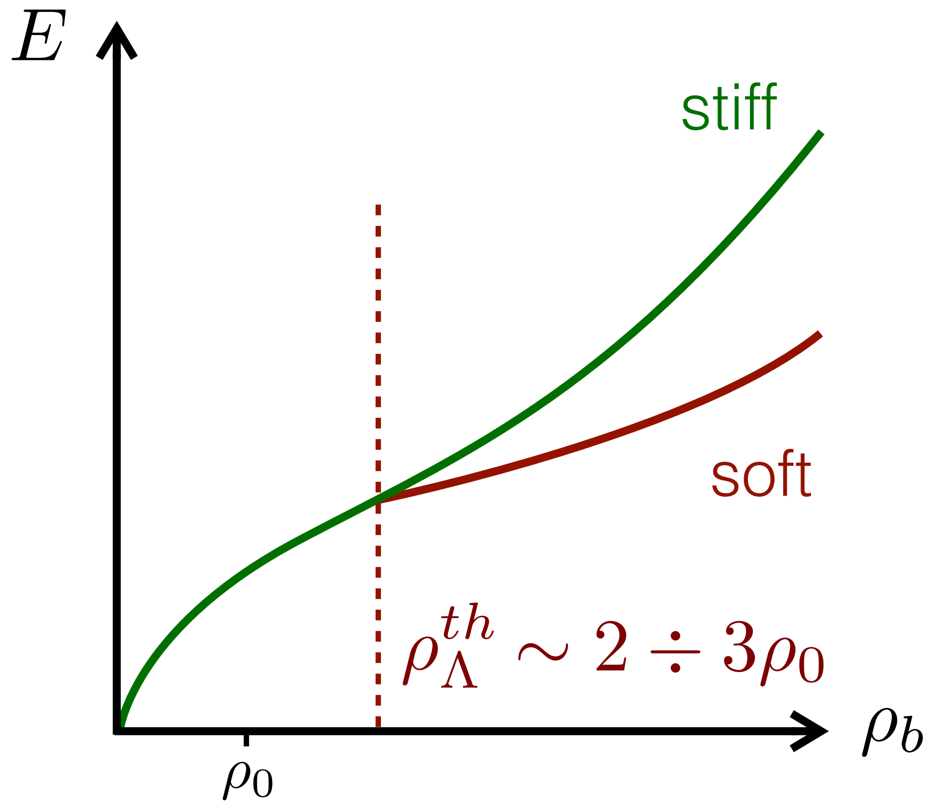


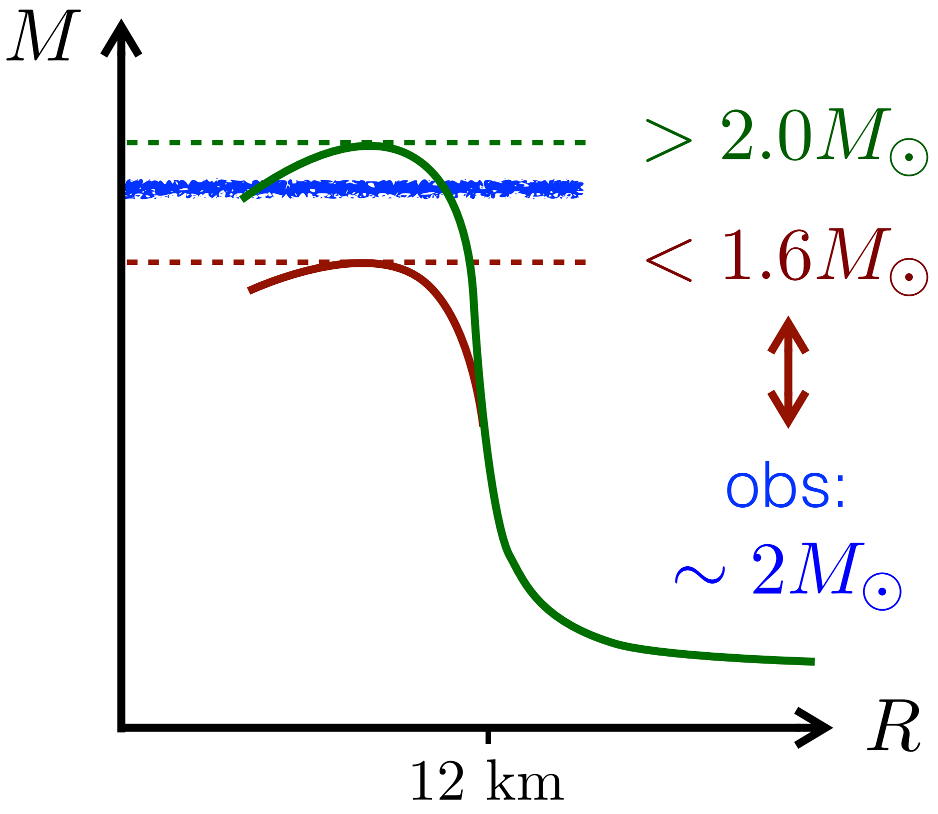
Introduction: hyperons in neutron stars

Relativistic Mean Field



I. Bednarek, P. Haensel, J. L. Zdunik,M. Bejger, R.Manka,
Astron. Astrophys. 543, A157 (2012)



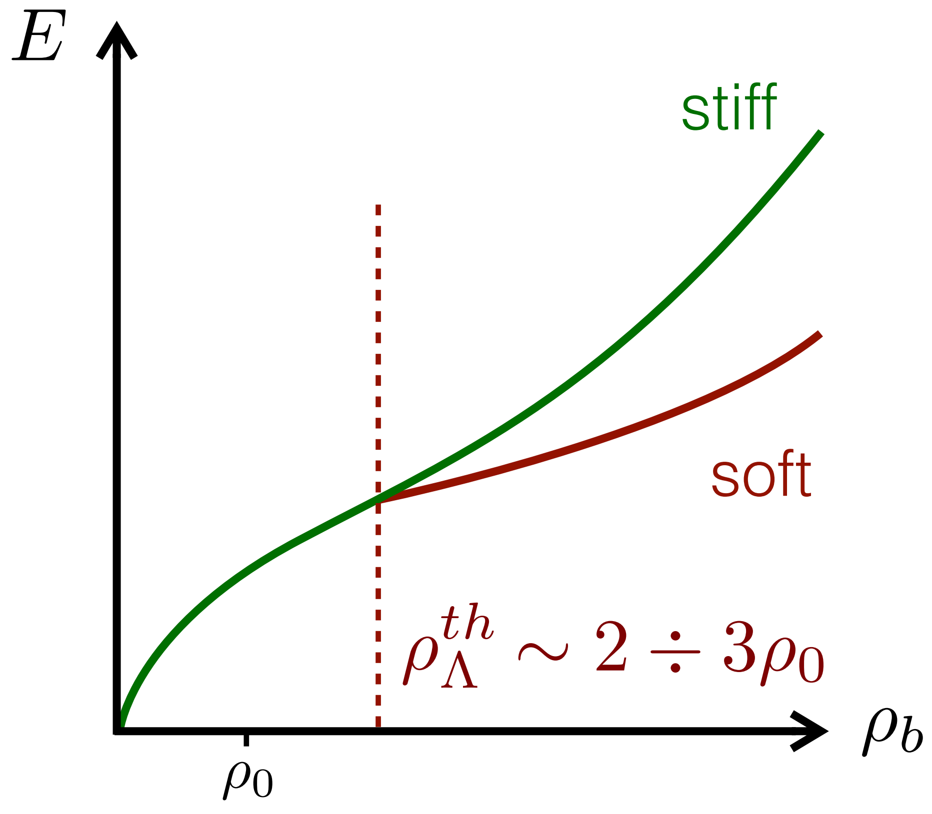


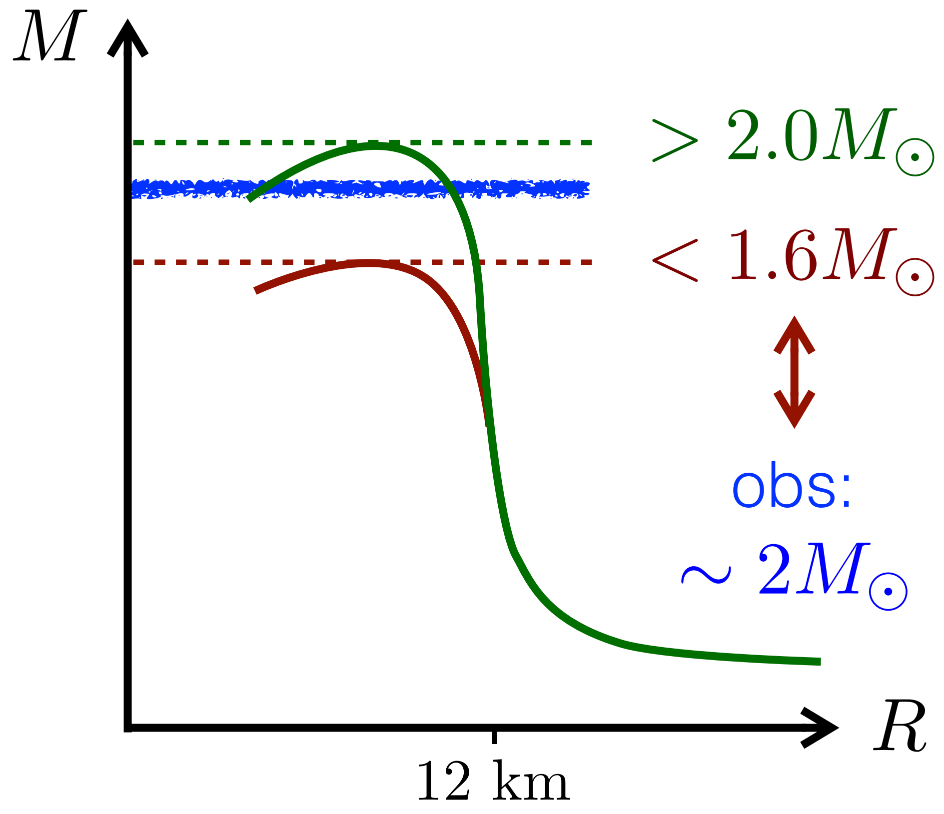
Introduction: hyperons in neutron stars

Hyperon puzzle

- ✓ Theoretical prediction of hyperons in NS core: softening of the EoS
- ✓ Observation of massive NS: stiff EoS
- ✓ Magnitude of the softening: strongly model dependent

- ✓ Non trivial many-body problem: very dense system, strong interactions
- ✓ Interactions poorly known



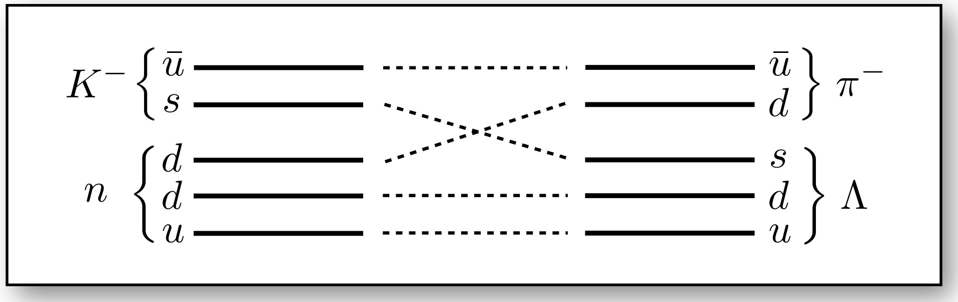


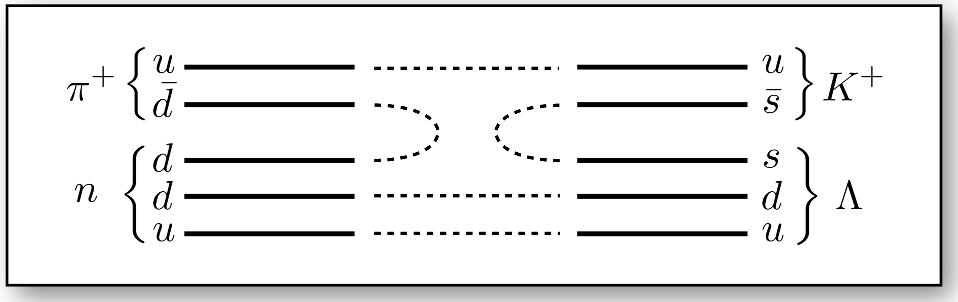
Introduction: hyperons in nuclei

✓ Charge conserving reactions

$$ {}^{A}Z\,(K^-,\pi^-)\,{}^{A}_{\Lambda}Z $$

$$ {}^{A}Z\,(\pi^+,K^+)\,{}^{A}_{\Lambda}Z $$



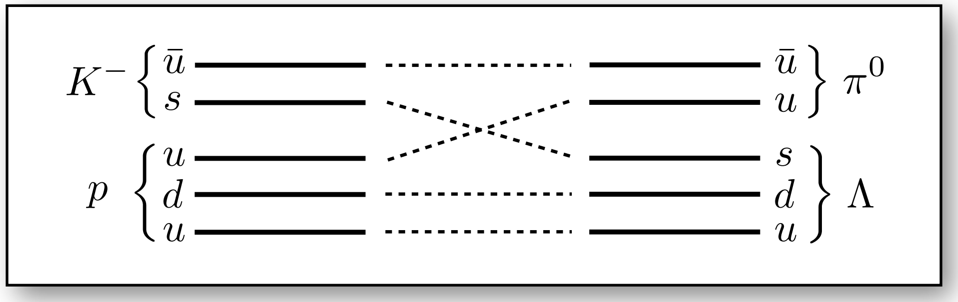


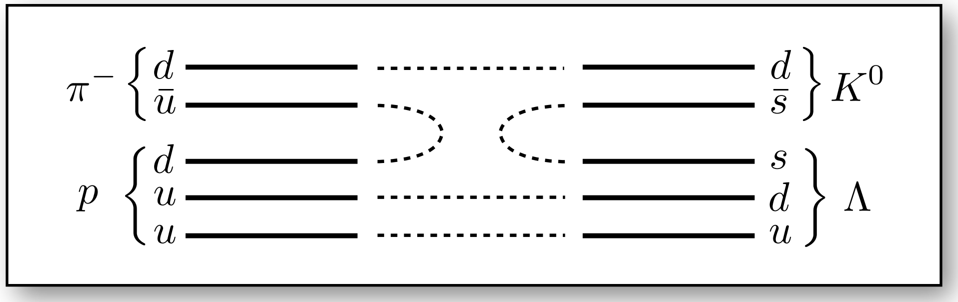
✓ Single charge exchange reactions (SCX)

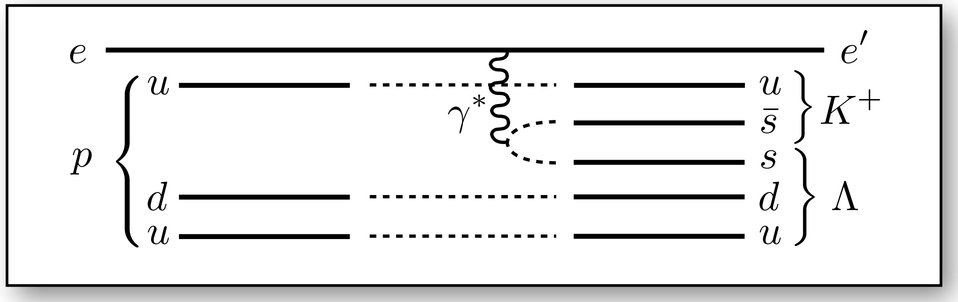
$$ {}^{A}Z\,(K^-,\pi^0)\,{}^{A}_{\Lambda}[Z-1] $$

$$ {}^{A}Z\,(\pi^-,K^0)\,{}^{A}_{\Lambda}[Z-1] $$

$$ {}^{A}Z\,(e,e'K^+)\,{}^{A}_{\Lambda}[Z-1] $$





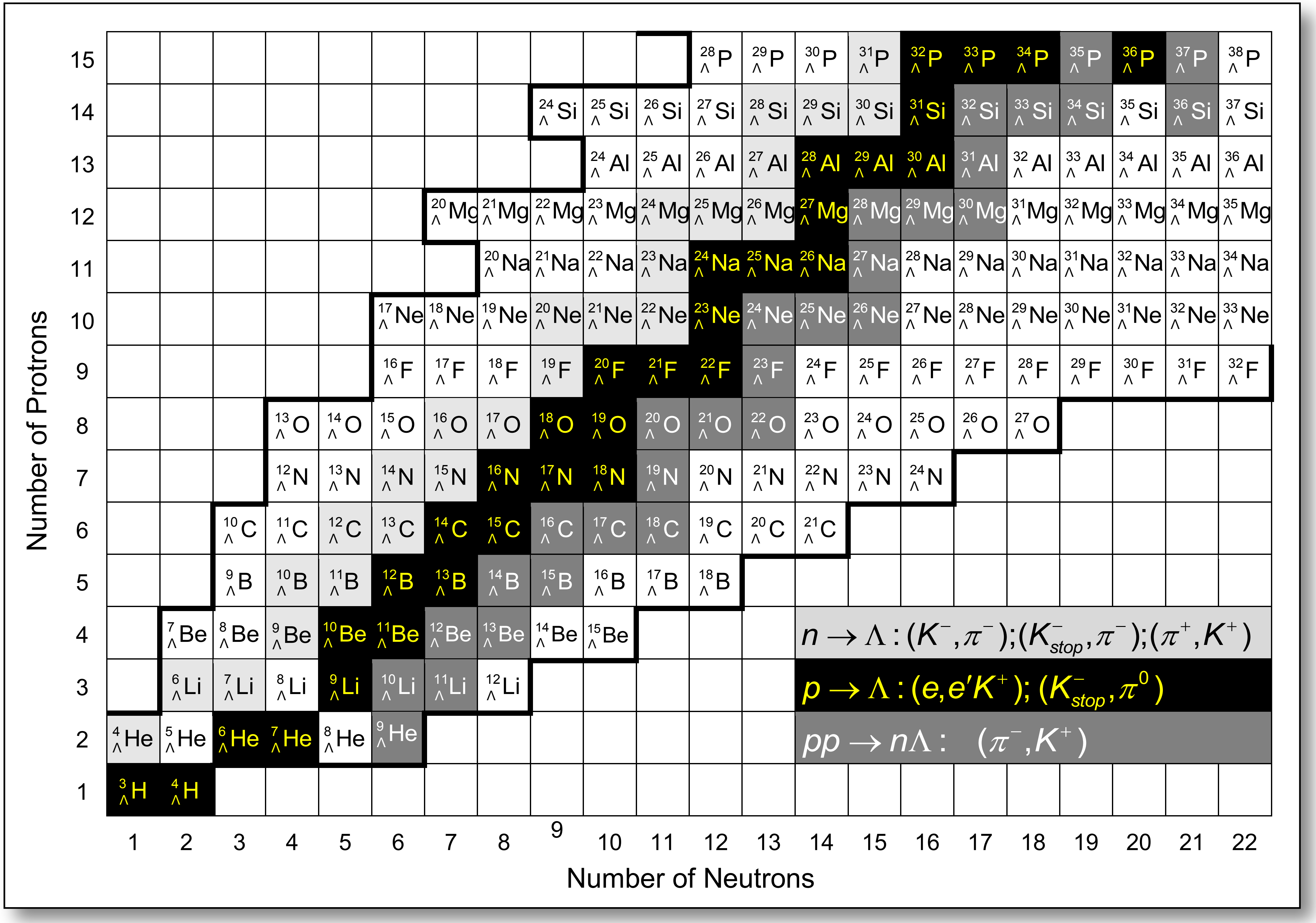


✓ Double charge exchange reactions (DCX)

$$ {}^{A}Z\,(\pi^-,K^+)\,{}^{A+1}_{\quad\Lambda}[Z-2] $$

$$ {}^{A}Z\,(K^-,\pi^+)\,{}^{A+1}_{\quad\Lambda}[Z-2] $$

Introduction: hyperons in nuclei



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
15												$^{28}_{\Lambda}$P	$^{29}_{\Lambda}$P	$^{30}_{\Lambda}$P	$^{31}_{\Lambda}$P	$^{32}_{\Lambda}$P	$^{33}_{\Lambda}$P	$^{34}_{\Lambda}$P	$^{35}_{\Lambda}$P	$^{36}_{\Lambda}$P	$^{37}_{\Lambda}$P	$^{38}_{\Lambda}$P
14									$^{24}_{\Lambda}$Si	$^{25}_{\Lambda}$Si	$^{26}_{\Lambda}$Si	$^{27}_{\Lambda}$Si	$^{28}_{\Lambda}$Si	$^{29}_{\Lambda}$Si	$^{30}_{\Lambda}$Si	$^{31}_{\Lambda}$Si	$^{32}_{\Lambda}$Si	$^{33}_{\Lambda}$Si	$^{34}_{\Lambda}$Si	$^{35}_{\Lambda}$Si	$^{36}_{\Lambda}$Si	$^{37}_{\Lambda}$Si
13										$^{24}_{\Lambda}$Al	$^{25}_{\Lambda}$Al	$^{26}_{\Lambda}$Al	$^{27}_{\Lambda}$Al	$^{28}_{\Lambda}$Al	$^{29}_{\Lambda}$Al	$^{30}_{\Lambda}$Al	$^{31}_{\Lambda}$Al	$^{32}_{\Lambda}$Al	$^{33}_{\Lambda}$Al	$^{34}_{\Lambda}$Al	$^{35}_{\Lambda}$Al	$^{36}_{\Lambda}$Al
12							$^{20}_{\Lambda}$Mg	$^{21}_{\Lambda}$Mg	$^{22}_{\Lambda}$Mg	$^{23}_{\Lambda}$Mg	$^{24}_{\Lambda}$Mg	$^{25}_{\Lambda}$Mg	$^{26}_{\Lambda}$Mg	$^{27}_{\Lambda}$Mg	$^{28}_{\Lambda}$Mg	$^{29}_{\Lambda}$Mg	$^{30}_{\Lambda}$Mg	$^{31}_{\Lambda}$Mg	$^{32}_{\Lambda}$Mg	$^{33}_{\Lambda}$Mg	$^{34}_{\Lambda}$Mg	$^{35}_{\Lambda}$Mg
11								$^{20}_{\Lambda}$Na	$^{21}_{\Lambda}$Na	$^{22}_{\Lambda}$Na	$^{23}_{\Lambda}$Na	$^{24}_{\Lambda}$Na	$^{25}_{\Lambda}$Na	$^{26}_{\Lambda}$Na	$^{27}_{\Lambda}$Na	$^{28}_{\Lambda}$Na	$^{29}_{\Lambda}$Na	$^{30}_{\Lambda}$Na	$^{31}_{\Lambda}$Na	$^{32}_{\Lambda}$Na	$^{33}_{\Lambda}$Na	$^{34}_{\Lambda}$Na
10						$^{17}_{\Lambda}$Ne	$^{18}_{\Lambda}$Ne	$^{19}_{\Lambda}$Ne	$^{20}_{\Lambda}$Ne	$^{21}_{\Lambda}$Ne	$^{22}_{\Lambda}$Ne	$^{23}_{\Lambda}$Ne	$^{24}_{\Lambda}$Ne	$^{25}_{\Lambda}$Ne	$^{26}_{\Lambda}$Ne	$^{27}_{\Lambda}$Ne	$^{28}_{\Lambda}$Ne	$^{29}_{\Lambda}$Ne	$^{30}_{\Lambda}$Ne	$^{31}_{\Lambda}$Ne	$^{32}_{\Lambda}$Ne	$^{33}_{\Lambda}$Ne
9						$^{16}_{\Lambda}$F	$^{17}_{\Lambda}$F	$^{18}_{\Lambda}$F	$^{19}_{\Lambda}$F	$^{20}_{\Lambda}$F	$^{21}_{\Lambda}$F	$^{22}_{\Lambda}$F	$^{23}_{\Lambda}$F	$^{24}_{\Lambda}$F	$^{25}_{\Lambda}$F	$^{26}_{\Lambda}$F	$^{27}_{\Lambda}$F	$^{28}_{\Lambda}$F	$^{29}_{\Lambda}$F	$^{30}_{\Lambda}$F	$^{31}_{\Lambda}$F	$^{32}_{\Lambda}$F
8				$^{13}_{\Lambda}$O	$^{14}_{\Lambda}$O	$^{15}_{\Lambda}$O	$^{16}_{\Lambda}$O	$^{17}_{\Lambda}$O	$^{18}_{\Lambda}$O	$^{19}_{\Lambda}$O	$^{20}_{\Lambda}$O	$^{21}_{\Lambda}$O	$^{22}_{\Lambda}$O	$^{23}_{\Lambda}$O	$^{24}_{\Lambda}$O	$^{25}_{\Lambda}$O	$^{26}_{\Lambda}$O	$^{27}_{\Lambda}$O				
7				$^{12}_{\Lambda}$N	$^{13}_{\Lambda}$N	$^{14}_{\Lambda}$N	$^{15}_{\Lambda}$N	$^{16}_{\Lambda}$N	$^{17}_{\Lambda}$N	$^{18}_{\Lambda}$N	$^{19}_{\Lambda}$N	$^{20}_{\Lambda}$N	$^{21}_{\Lambda}$N	$^{22}_{\Lambda}$N	$^{23}_{\Lambda}$N	$^{24}_{\Lambda}$N						
6			$^{10}_{\Lambda}$C	$^{11}_{\Lambda}$C	$^{12}_{\Lambda}$C	$^{13}_{\Lambda}$C	$^{14}_{\Lambda}$C	$^{15}_{\Lambda}$C	$^{16}_{\Lambda}$C	$^{17}_{\Lambda}$C	$^{18}_{\Lambda}$C	$^{19}_{\Lambda}$C	$^{20}_{\Lambda}$C	$^{21}_{\Lambda}$C								
5			$^{9}_{\Lambda}$B	$^{10}_{\Lambda}$B	$^{11}_{\Lambda}$B	$^{12}_{\Lambda}$B	$^{13}_{\Lambda}$B	$^{14}_{\Lambda}$B	$^{15}_{\Lambda}$B	$^{16}_{\Lambda}$B	$^{17}_{\Lambda}$B	$^{18}_{\Lambda}$B										
4		$^{7}_{\Lambda}$Be	$^{8}_{\Lambda}$Be	$^{9}_{\Lambda}$Be	$^{10}_{\Lambda}$Be	$^{11}_{\Lambda}$Be	$^{12}_{\Lambda}$Be	$^{13}_{\Lambda}$Be	$^{14}_{\Lambda}$Be	$^{15}_{\Lambda}$Be												
3		$^{6}_{\Lambda}$Li	$^{7}_{\Lambda}$Li	$^{8}_{\Lambda}$Li	$^{9}_{\Lambda}$Li	$^{10}_{\Lambda}$Li	$^{11}_{\Lambda}$Li	$^{12}_{\Lambda}$Li														
2	$^{4}_{\Lambda}$He	$^{5}_{\Lambda}$He	$^{6}_{\Lambda}$He	$^{7}_{\Lambda}$He	$^{8}_{\Lambda}$He	$^{9}_{\Lambda}$He																
1	$^{3}_{\Lambda}$H	$^{4}_{\Lambda}$H																				

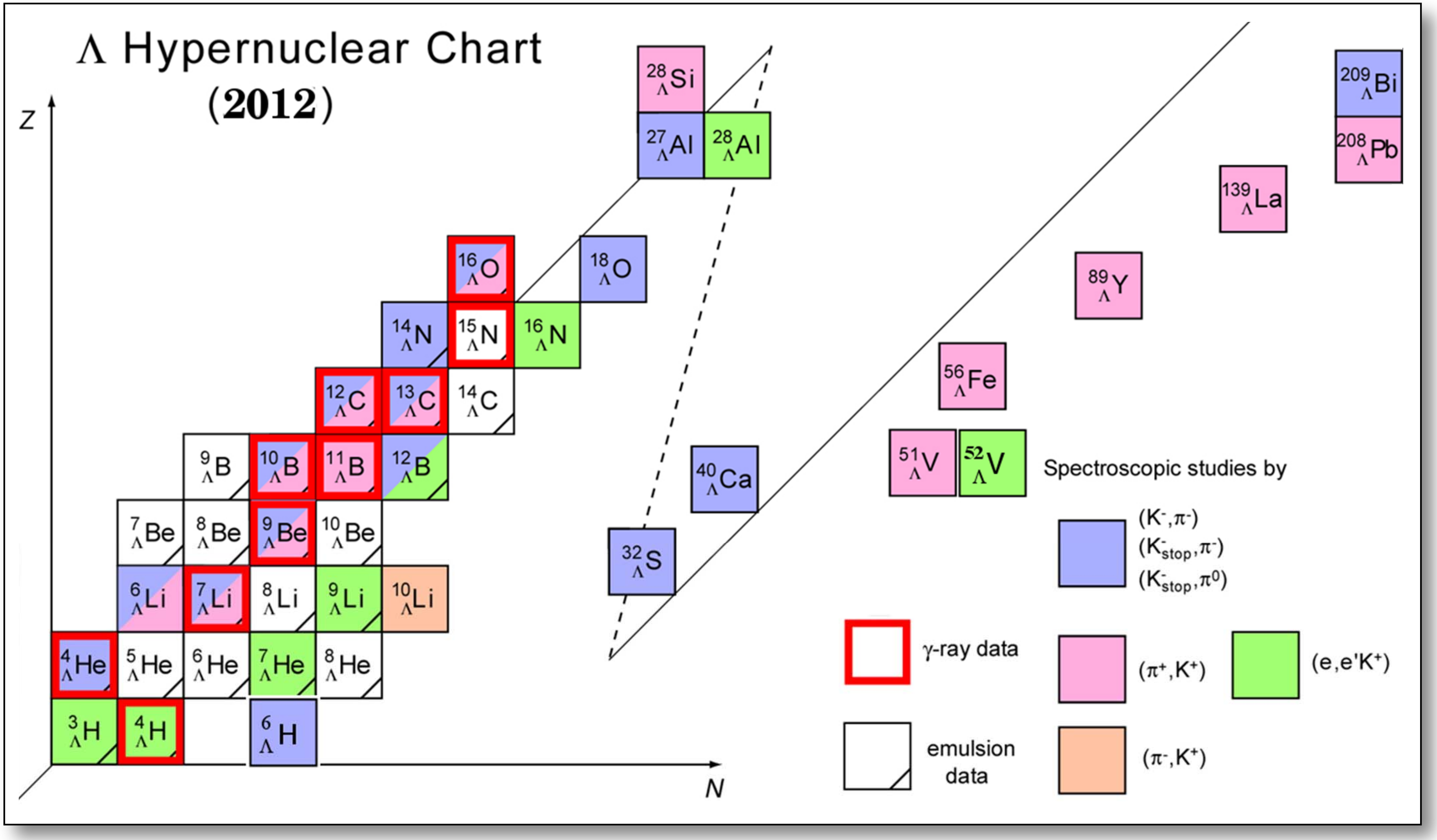
$n \rightarrow \Lambda : (K^-,\pi^-);(K^-_{stop},\pi^-);(\pi^+,K^+)$

$p \rightarrow \Lambda : (e,e'K^+);\ (K^-_{stop},\pi^0)$

$pp \rightarrow n\Lambda : \quad (\pi^-,K^+)$

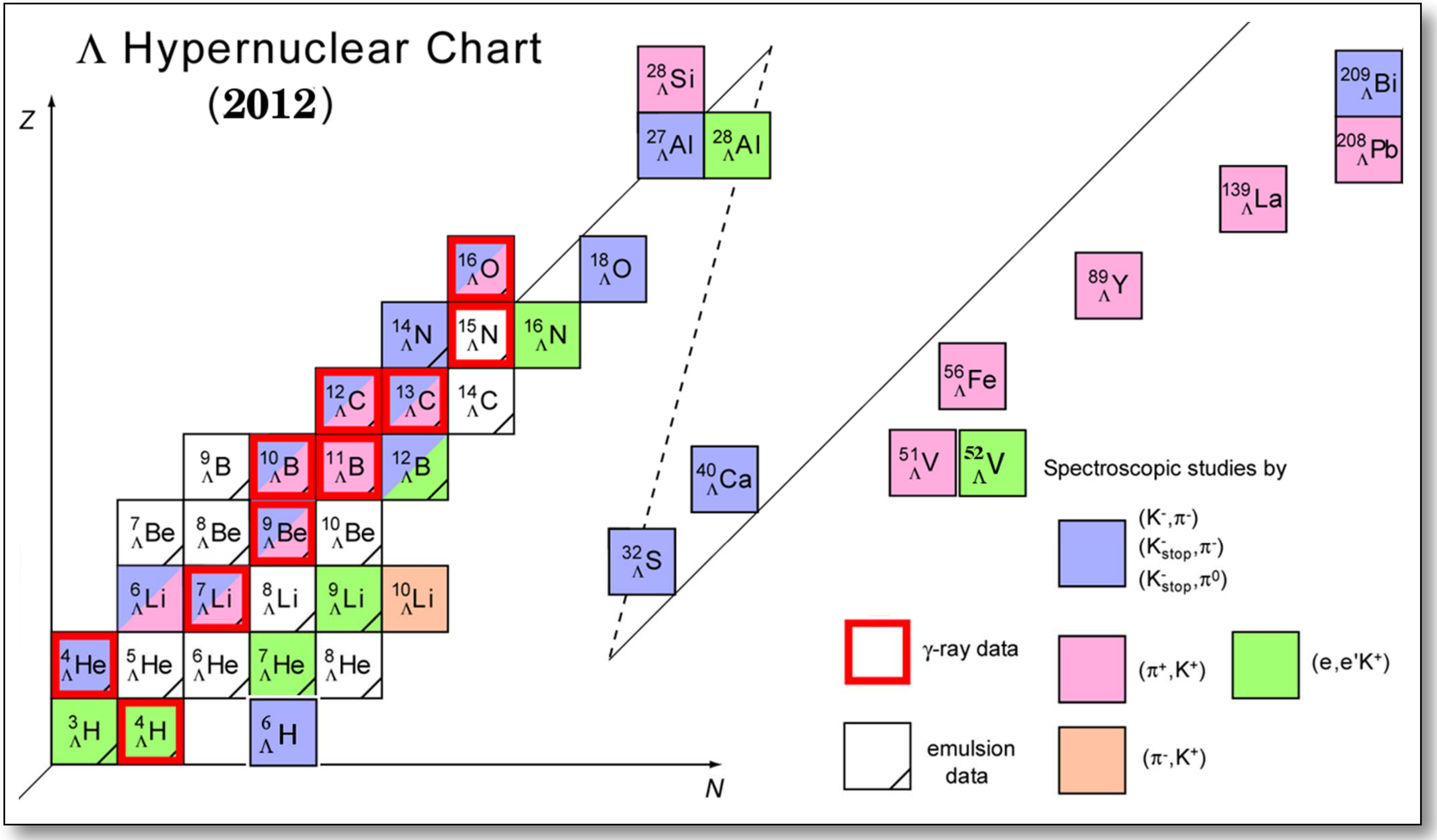
J. Pochodzalla, Acta Phys. Polon. B 42, 833– 842 (2011)

Introduction: hyperons in nuclei



Updated from O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006)

Introduction: hyperons in nuclei



scattering data:

$$NN : \sim 4300 \quad (pp, np)$$

$$YN : \quad 52 \quad (\Lambda p, \Sigma p) \longrightarrow$$ 1971 - 2005

Introduction: hyperons in nuclei

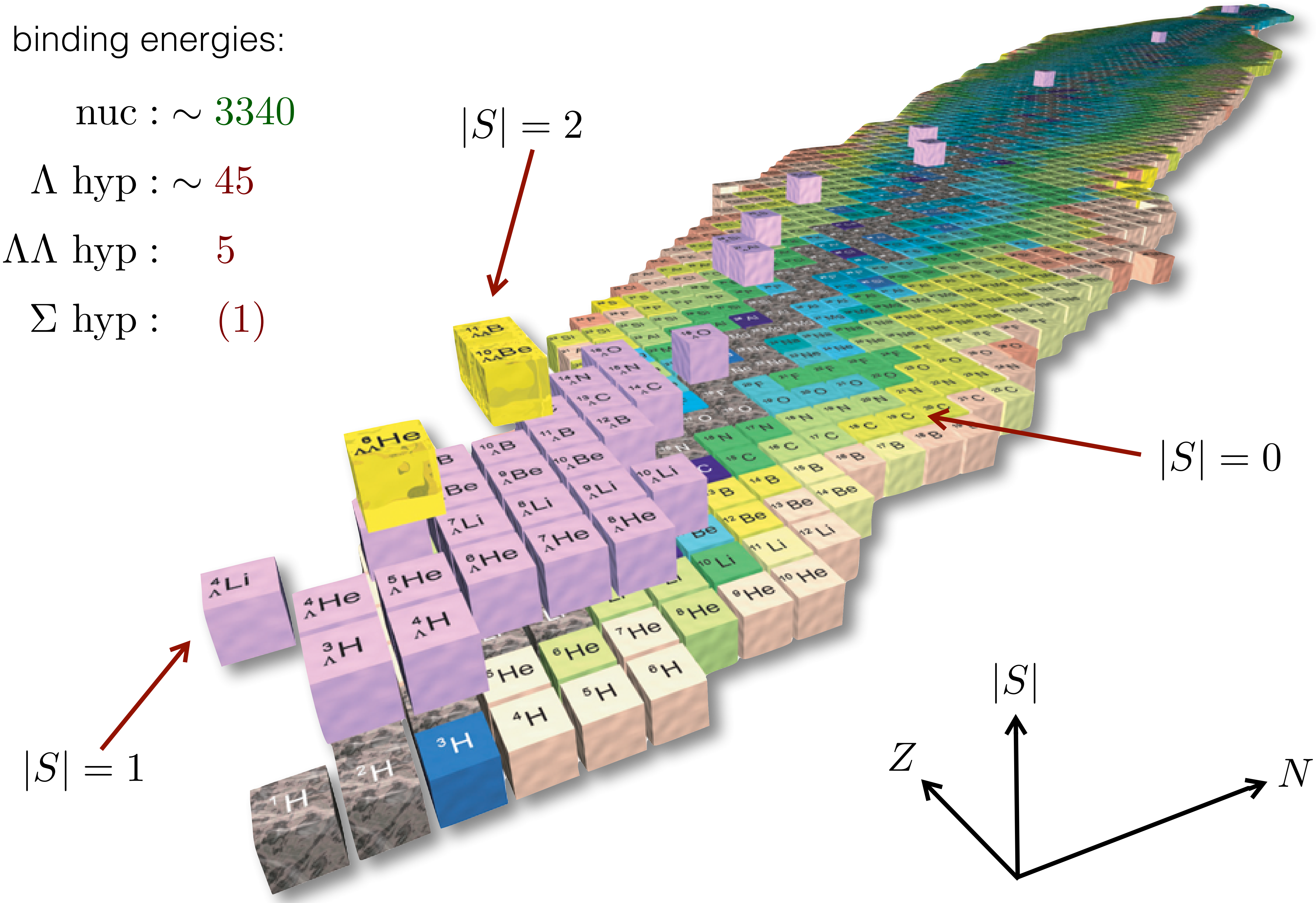
binding energies:

$\text{nuc} : \sim 3340$

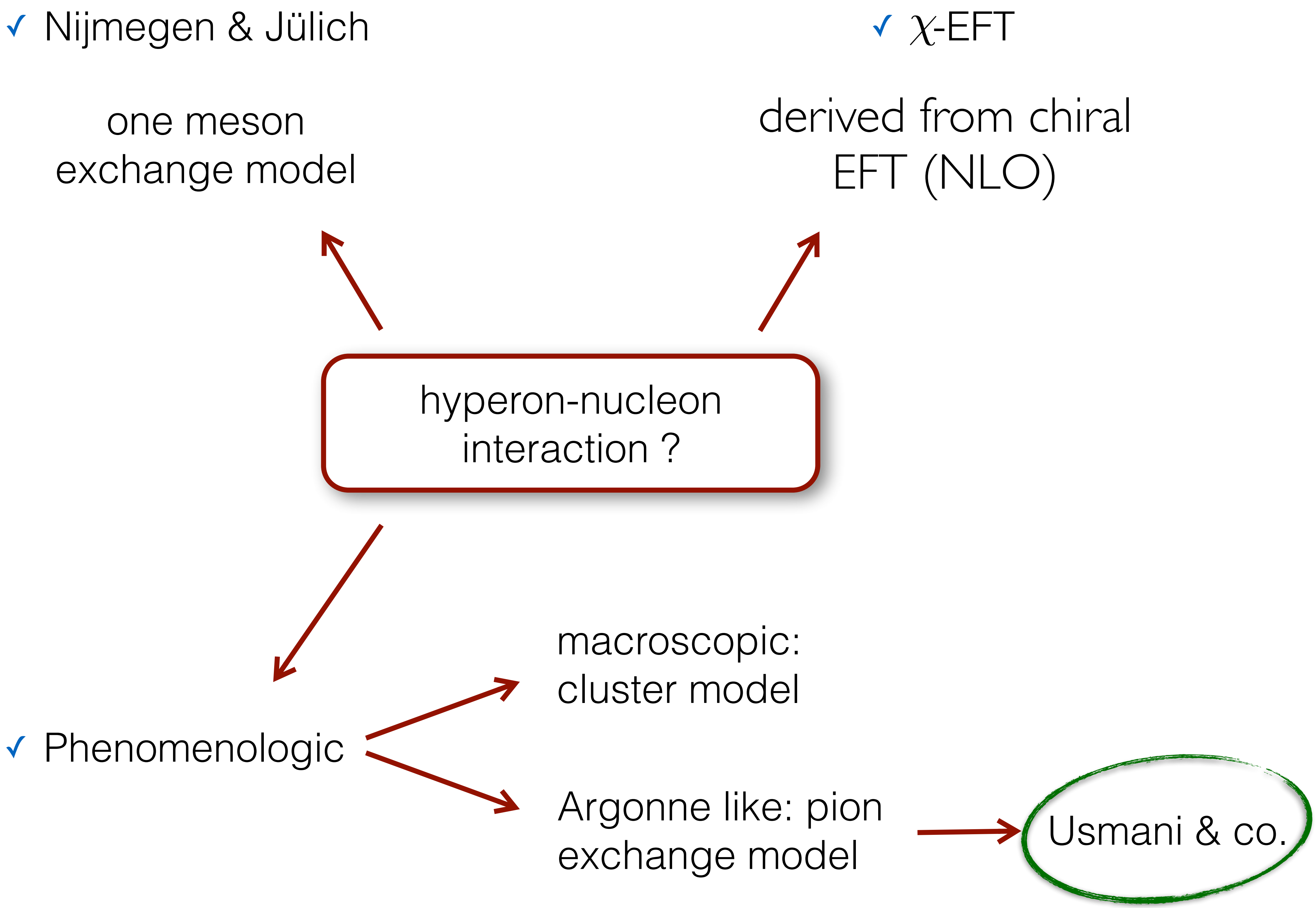
$\Lambda\ \text{hyp} : \sim 45$

$\Lambda\Lambda\ \text{hyp} : \quad 5$

$\Sigma\ \text{hyp} : \quad (1)$



Introduction: hyperon-nucleon interaction



✓ Nijmegen & Jülich

one meson
exchange model

✓ χ-EFT

derived from chiral
EFT (NLO)

hyperon-nucleon
interaction ?

✓ Phenomenologic

macroscopic:
cluster model

Argonne like: pion
exchange model

Usmani & co.

Introduction: hyperon-nucleon interaction

- ✓ 2-body interaction: AV18 & Usmani

$$NN \quad \begin{cases} v_{ij} = \displaystyle\sum_{p=1,18} v_p(r_{ij})\,\mathcal{O}_{ij}^{p} \\ \mathcal{O}_{ij}^{p=1,8} = \Big\{1, \sigma_{ij}, S_{ij}, \boldsymbol{L}_{ij}\cdot\boldsymbol{S}_{ij}\Big\} \otimes \Big\{1, \tau_{ij}\Big\} \end{cases}$$

NN
scattering

$$\Lambda N \quad \begin{cases} v_{\lambda i} = \displaystyle\sum_{p=1,4} v_p(r_{\lambda i})\,\mathcal{O}_{\lambda i}^{p} \\ \mathcal{O}_{\lambda i}^{p=1,4} = \Big\{1, \sigma_{\lambda i}\Big\} \otimes \Big\{1, \tau_i^z\Big\} \end{cases}$$

ΛN scattering

$A = 4$ CSB

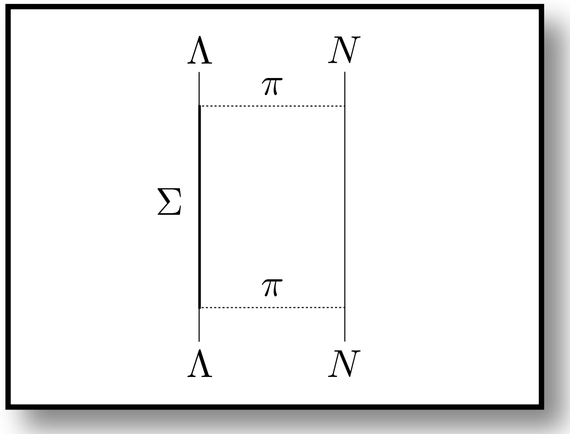
Note:

~~$\Lambda\pi\Lambda$ vertex~~

forbidden

$\Lambda\pi\Sigma$ vertex

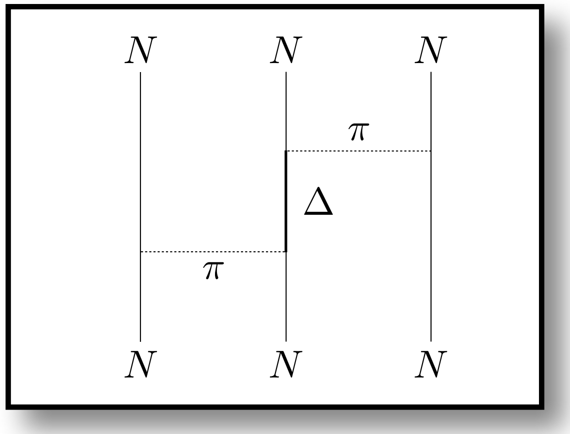
2π exchange

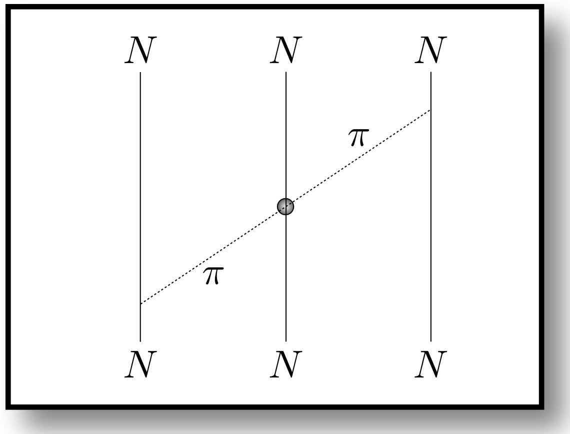


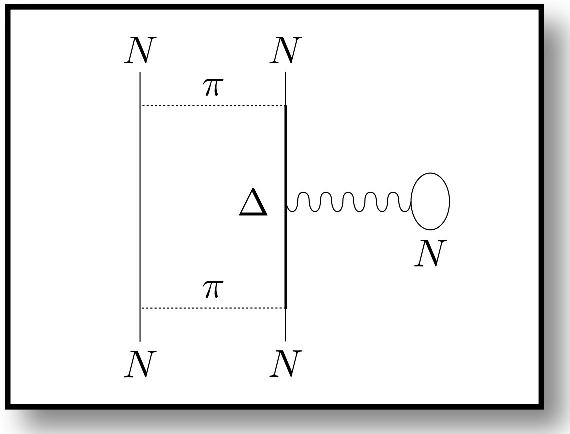
Introduction: hyperon-nucleon interaction

- ✓ 3-body interaction: Urbana IX & Usmani

NNN

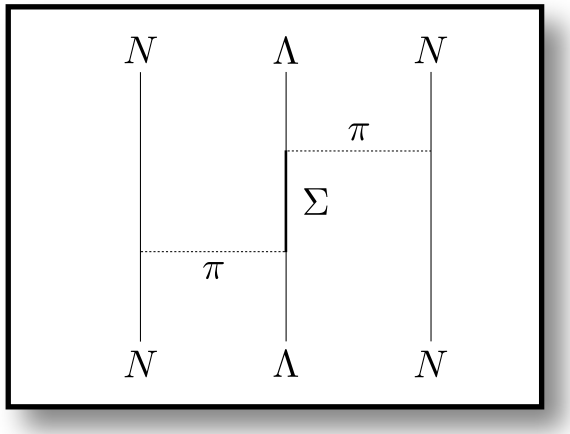


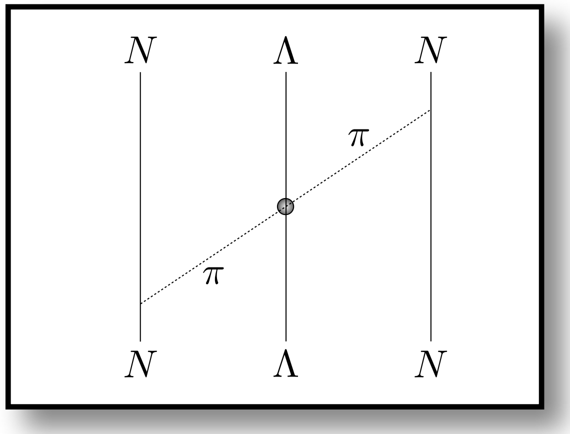


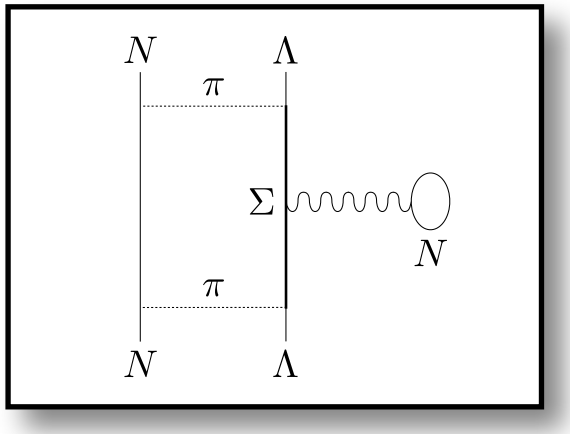


$$v_{ijk} = A_{2\pi}^{P}\,\mathcal{O}_{ijk}^{2\pi,P} + A_{2\pi}^{S}\,\mathcal{O}_{ijk}^{2\pi,S} + A_{R}\,\mathcal{O}_{ijk}^{R}$$

ΛNN





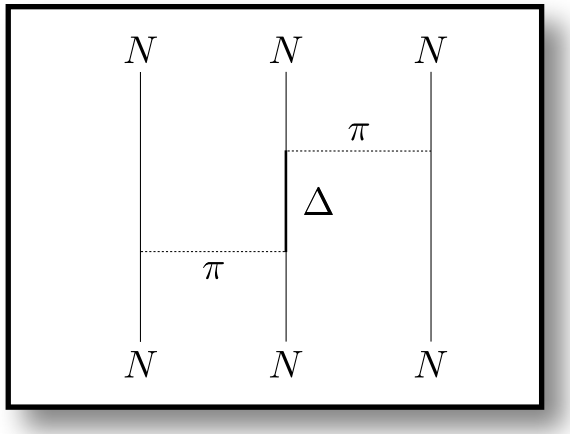


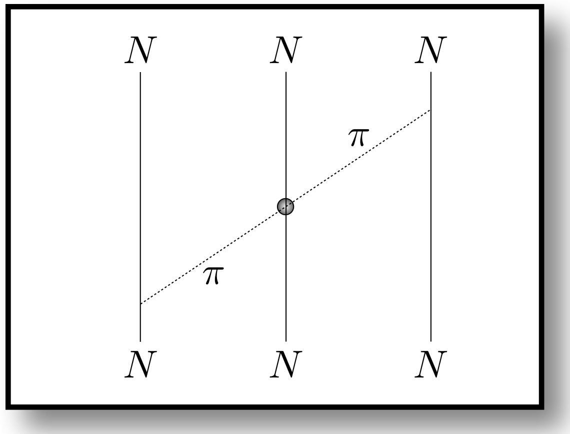
$$v_{\lambda ij} = A_{2\pi}^{P}\,\mathcal{O}_{\lambda ij}^{2\pi,P} + A_{2\pi}^{S}\,\mathcal{O}_{\lambda ij}^{2\pi,S} + A_{R}\,\mathcal{O}_{\lambda ij}^{R}$$

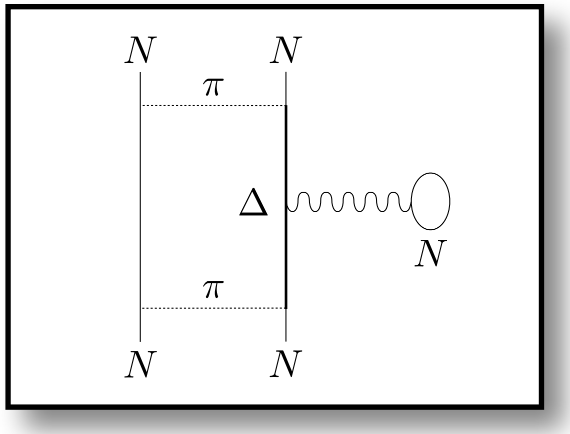
Introduction: hyperon-nucleon interaction

- ✓ 3-body interaction: Urbana IX & Usmani

NNN

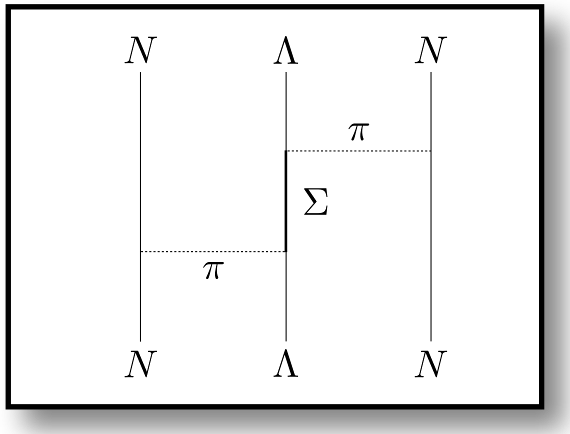


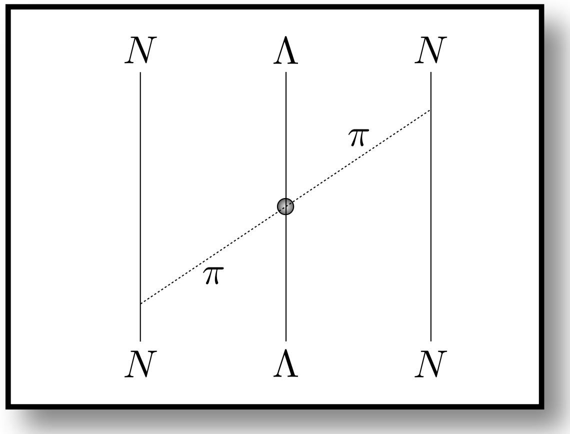


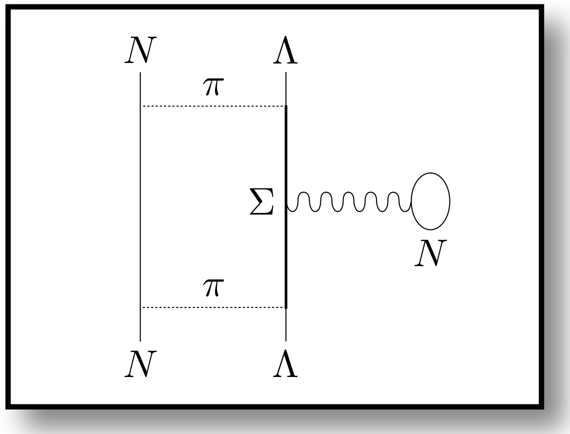


$$v_{ijk} = A_{2\pi}^{P}\,\mathcal{O}_{ijk}^{2\pi,P} + A_{2\pi}^{S}\,\mathcal{O}_{ijk}^{2\pi,S} + A_R\,\mathcal{O}_{ijk}^{R}$$

ΛNN





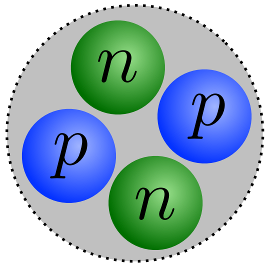


fit on B_Λ

$$v_{\lambda ij} = C_P\,\mathcal{O}_{\lambda ij}^{2\pi,P} + C_S\,\mathcal{O}_{\lambda ij}^{2\pi,S} + W_D\,\mathcal{O}_{\lambda ij}^{R}$$

The strange QMC project: the idea

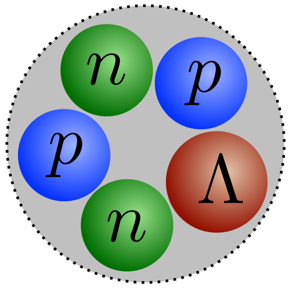
Diffusion Monte Carlo → realistic hyperon-nucleon interaction



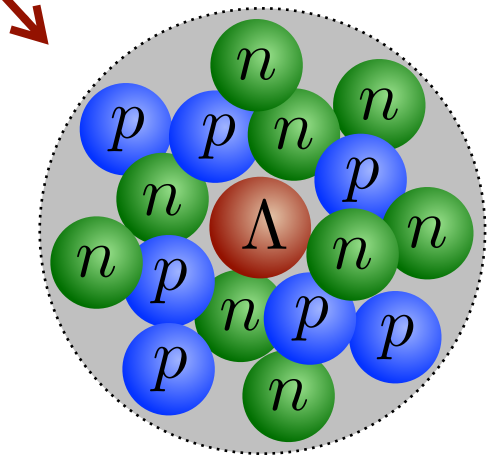
^{A-1}Z

nucleus

hypernucleus



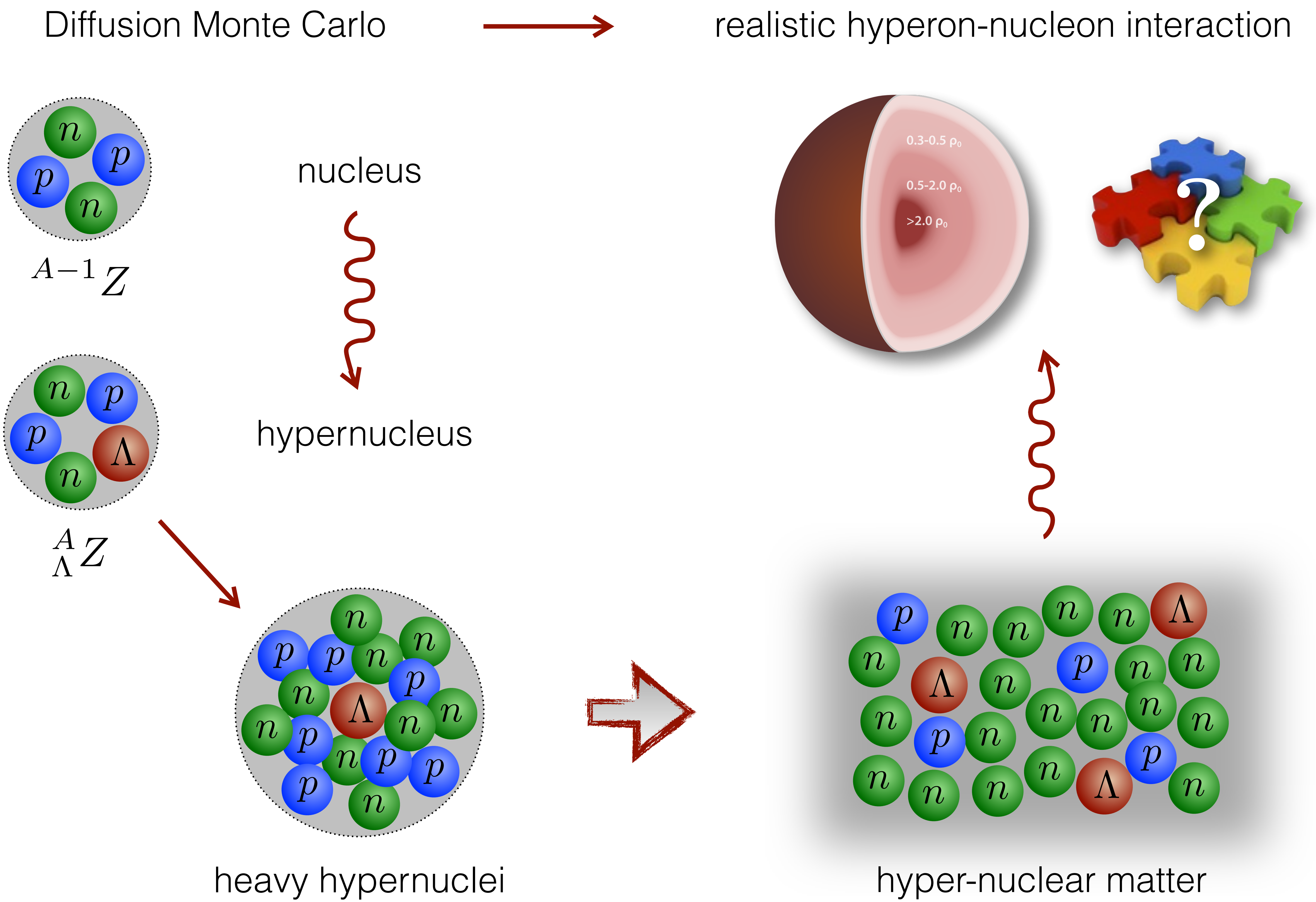
$^{A}_{\Lambda}Z$



heavy hypernuclei

comparison with experimental data in a wide mass range

The strange QMC project: the idea



Diffusion Monte Carlo → realistic hyperon-nucleon interaction

nucleus

^{A-1}Z

hypernucleus

$^{A}_{\Lambda}Z$

heavy hypernuclei

hyper-nuclear matter

The strange QMC project: the method

- ✓ Diffusion Monte Carlo

$$|\psi(\tau+d\tau)\rangle = \mathrm{e}^{-(H-E_T)d\tau}\,|\psi(\tau)\rangle \xrightarrow{\tau\to\infty}$$ ground state

$|S\rangle$?

$$\langle R|\psi(\tau+d\tau)\rangle = \int dR'\,\langle R|\,\mathrm{e}^{-(H-E_T)d\tau}\,|R'\rangle\langle R'|\psi(\tau)\rangle$$

walkers

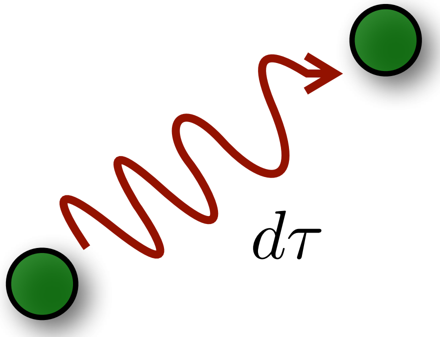
$D = \hbar^2/2m$

diffusion

branching

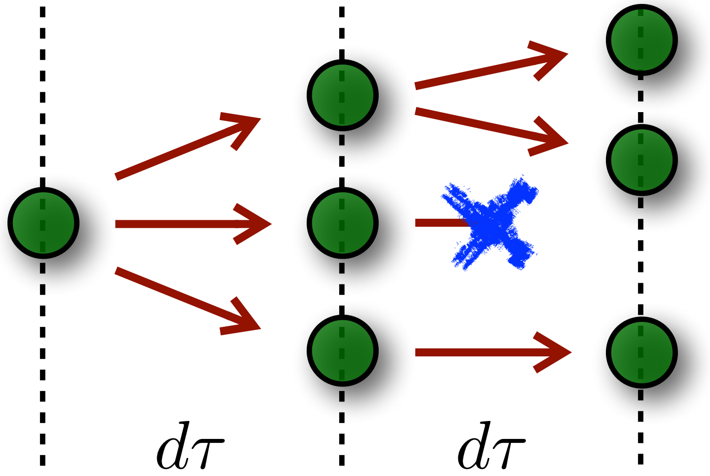
$$(4\pi D d\tau)^{-\frac{3A}{2}}\,\mathrm{e}^{-\frac{(R-R')^2}{4Dd\tau}}$$

$$\mathrm{e}^{-\left(\frac{V(R)+V(R')}{2}-E_T\right)d\tau}$$



kinetic term

potential term



The strange QMC project: the method

- ✓ Auxiliary Field

many body $\quad |S\rangle : \quad 2^A \dfrac{A!}{(A-Z)!Z!}$ components $\quad$ GFMC: $A \leq 12$

single particle $\quad |S\rangle = \bigotimes_i |S\rangle_i : \quad 4A$ components $\quad$ AFDMC: $A \sim 90$

$$\mathcal{P} \sim \mathrm{e}^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} \quad \longrightarrow \quad \mathrm{e}^{-\frac{1}{2}\lambda d\tau \mathcal{O}^2} \bigotimes_i |S\rangle_i \neq \bigotimes_i |\widetilde{S}\rangle_i$$

Idea: Hubbard-Stratonovich transformation

$$\mathrm{e}^{-\frac{1}{2}\lambda \mathcal{O}^2} = \frac{1}{\sqrt{2\pi}} \int dx\, \mathrm{e}^{-\frac{x^2}{2} + \sqrt{-\lambda}\, x \mathcal{O}}$$

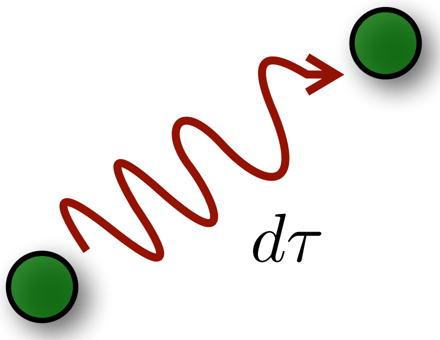
auxiliary field

rotation over spin-isospin configurations

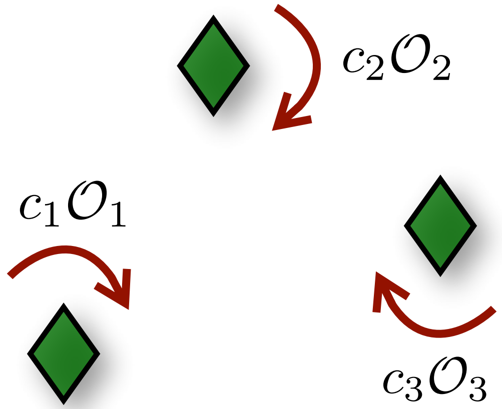
The strange QMC project: the method

- ✓ Auxiliary Field Diffusion Monte Carlo

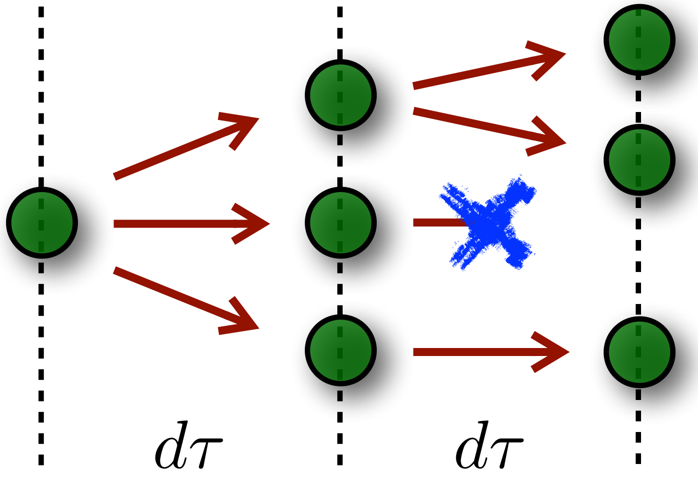
$$(4\pi D d\tau)^{-\frac{3A}{2}}\, \mathrm{e}^{-\frac{(R-R')^2}{4Dd\tau}}$$



$$\prod_n \mathrm{e}^{\sqrt{-\lambda_n d\tau}\, x_n \mathcal{O}_n} |S\rangle$$



$$\mathrm{e}^{-\left(\frac{\tilde{V}(R)+\tilde{V}(R')}{2}-E_T\right)d\tau}$$



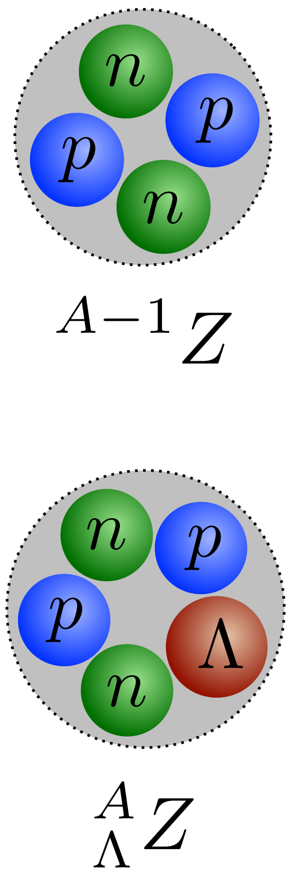
- ✓ nucleon systems: Argonne V4 & V6 like potentials
  no 3-body
- ✓ neutron systems: Argonne V8 like potentials
  plus 3-body

- ✓ hyperons ?

ΛN ✓

$\Lambda N N$ ✓

The strange QMC project: hypernuclei



nucleus $\longrightarrow$ $$E_{\text{nuc}} = \frac{\langle\psi^0_{\text{nuc}}|H_{\text{nuc}}|\psi^0_{\text{nuc}}\rangle}{\langle\psi^0_{\text{nuc}}|\psi^0_{\text{nuc}}\rangle}$$

H_{NN}

AFDMC

hypernucleus $\longrightarrow$ $$E_{\text{hyp}} = \frac{\langle\psi^0_{\text{hyp}}|H_{\text{hyp}}|\psi^0_{\text{hyp}}\rangle}{\langle\psi^0_{\text{hyp}}|\psi^0_{\text{hyp}}\rangle}$$

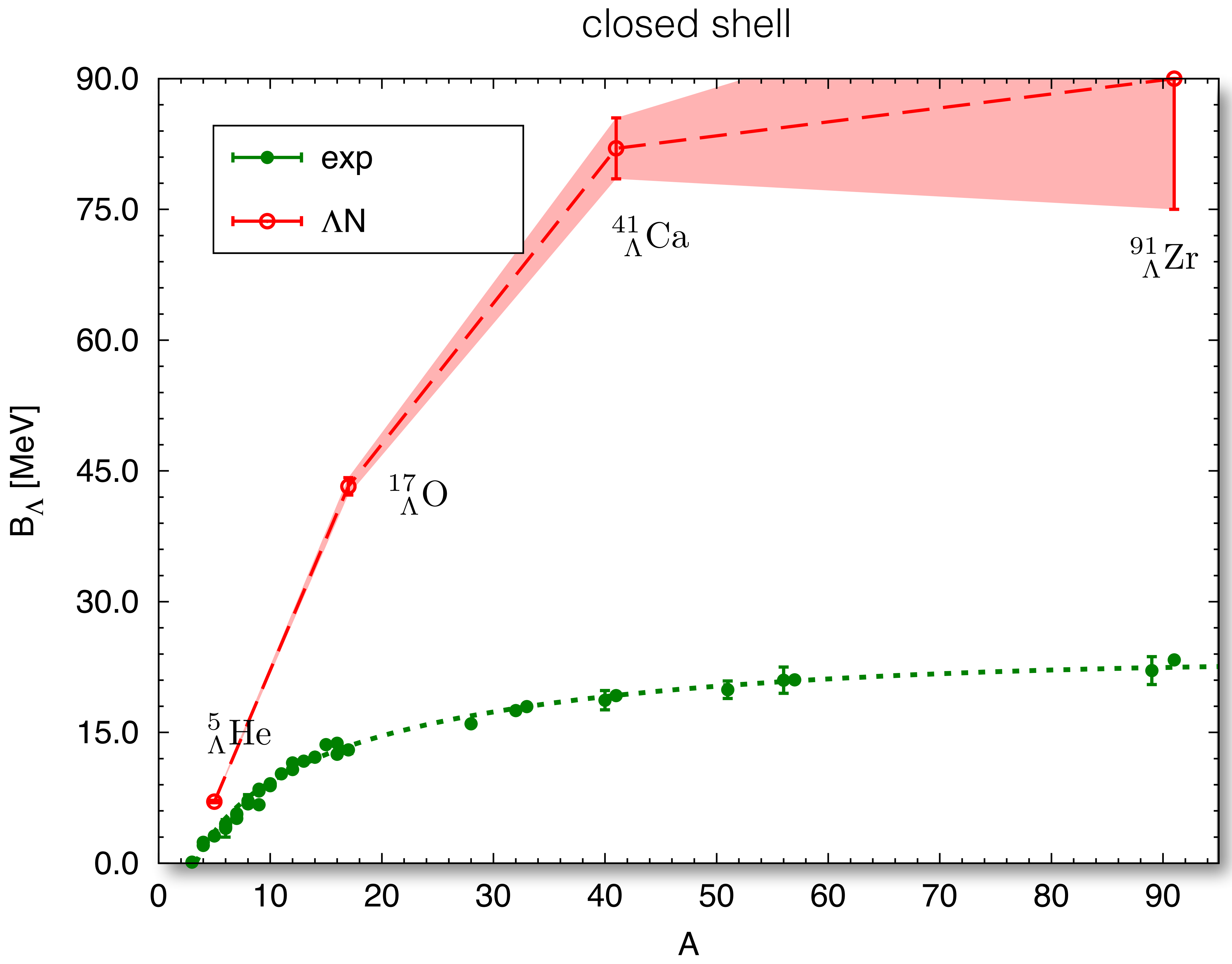
$H_{NN} + H_{\Lambda N}$

$$B_\Lambda = E_{\text{nuc}} - E_{\text{hyp}}$$

Hyp. : nuclear effects cancel at most

information about the hyperon-nucleon interaction

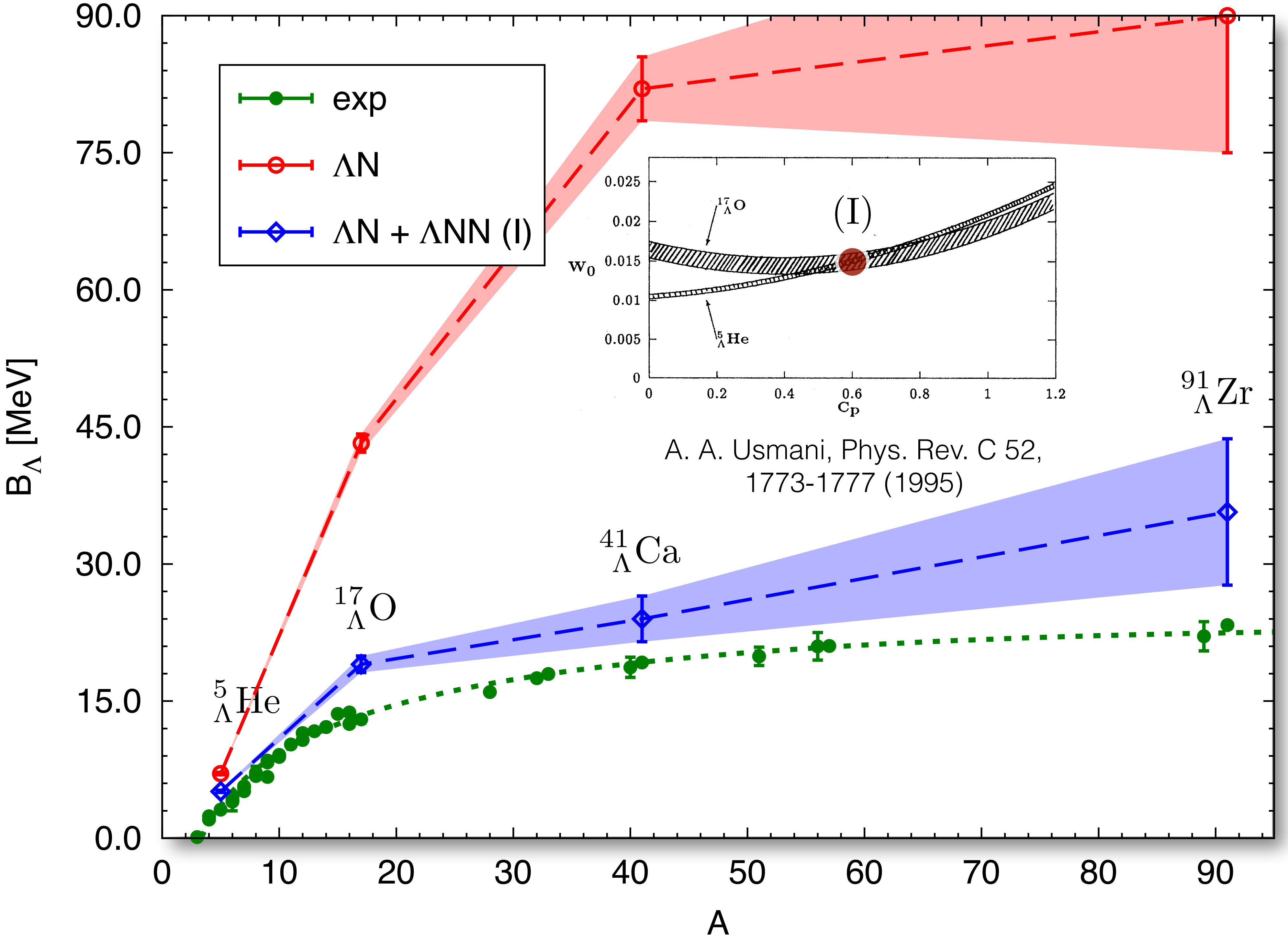
The strange QMC project: hypernuclei



D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

The strange QMC project: hypernuclei

closed shell



A. A. Usmani, Phys. Rev. C 52,
1773-1777 (1995)

D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

The strange QMC project: hypernuclei

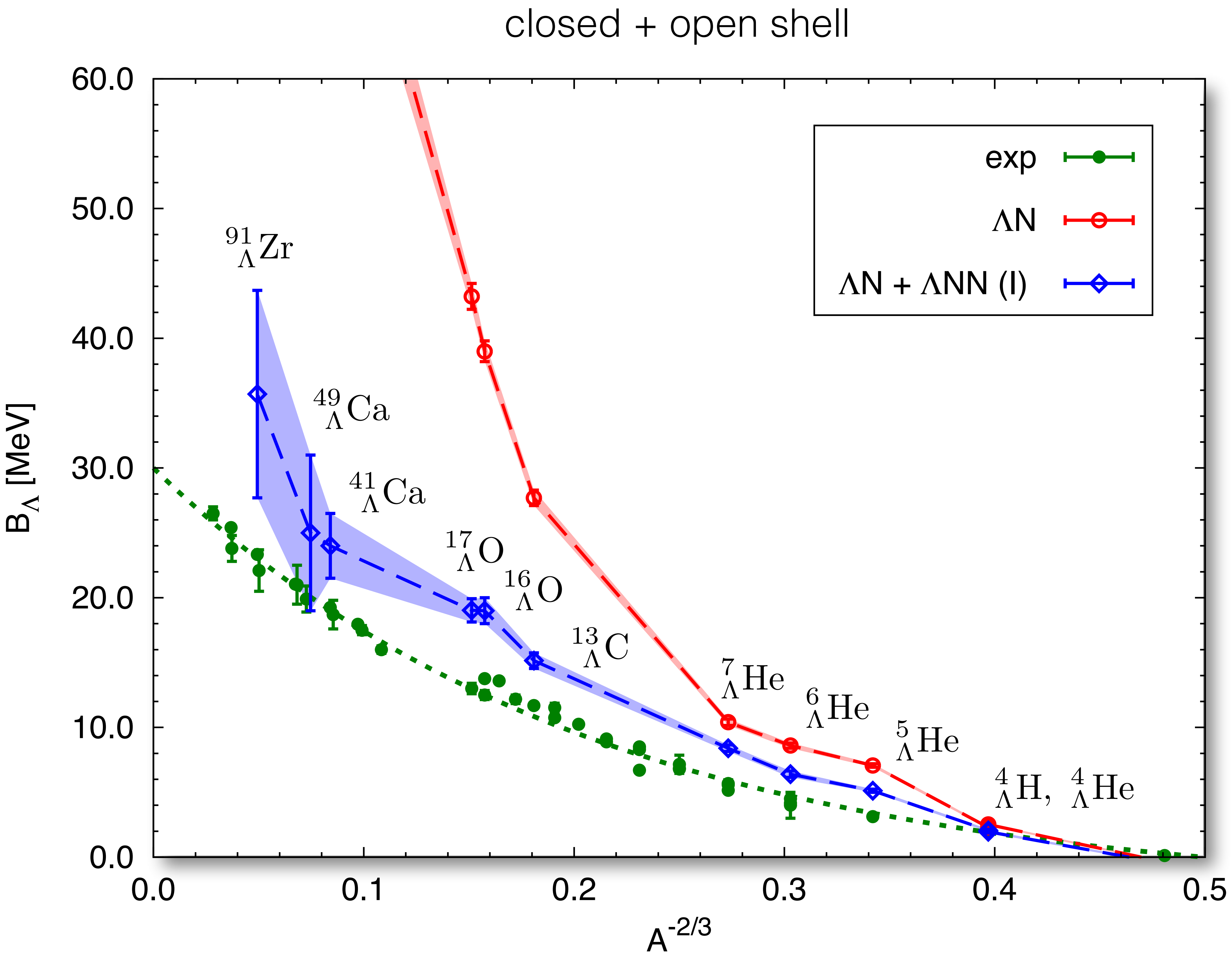
Hyp. : nuclear effects cancel at most ✓

NN potential	${}^{5}_{\Lambda}\mathrm{He}$		${}^{17}_{\Lambda}\mathrm{O}$	
	$V_{\Lambda N}$	$V_{\Lambda N}+V_{\Lambda NN}$	$V_{\Lambda N}$	$V_{\Lambda N}+V_{\Lambda NN}$
Argonne V4’	7.1(1)	5.1(1)	43(1)	19(1)
Argonne V6’	6.3(1)	5.2(1)	34(1)	21(1)
Minnesota	7.4(1)	5.2(1)	50(1)	17(2)
Expt.	3.12(2)		13.0(4)	

D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

B_{Λ} not sensitive to the details of nucleon-nucleon interaction

The strange QMC project: hypernuclei

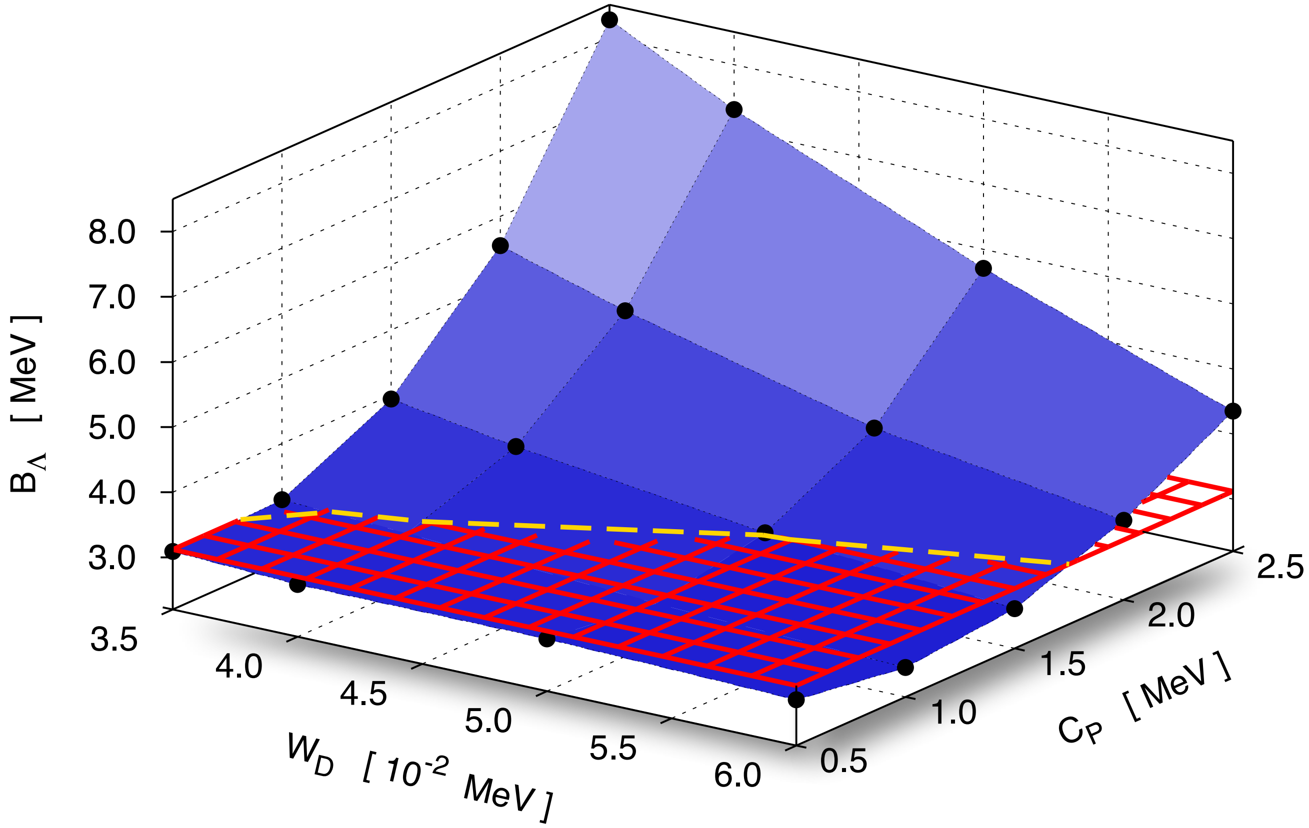


D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

The strange QMC project: hypernuclei

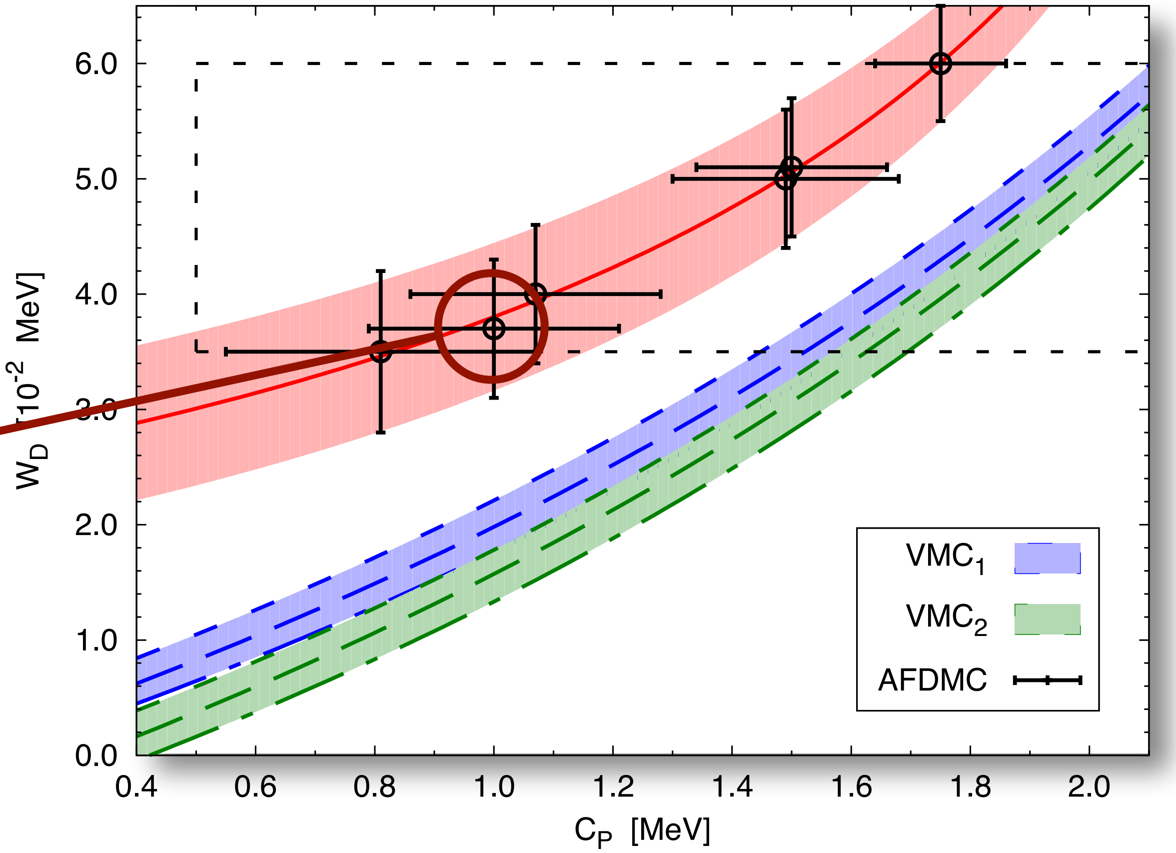
$^{5}_{\Lambda}\mathrm{He}$ study

- ✓ C_S fixed
- ✓ C_P, W_D analysis



D. L., F. Pederiva, S. Gandolfi,
Phys. Rev. C 89, 014314 (2014)

VMC results: A. A. Usmani, F. C. Khanna,
J. Phys. G: Nucl. Part. Phys. 35, 025105 (2008)

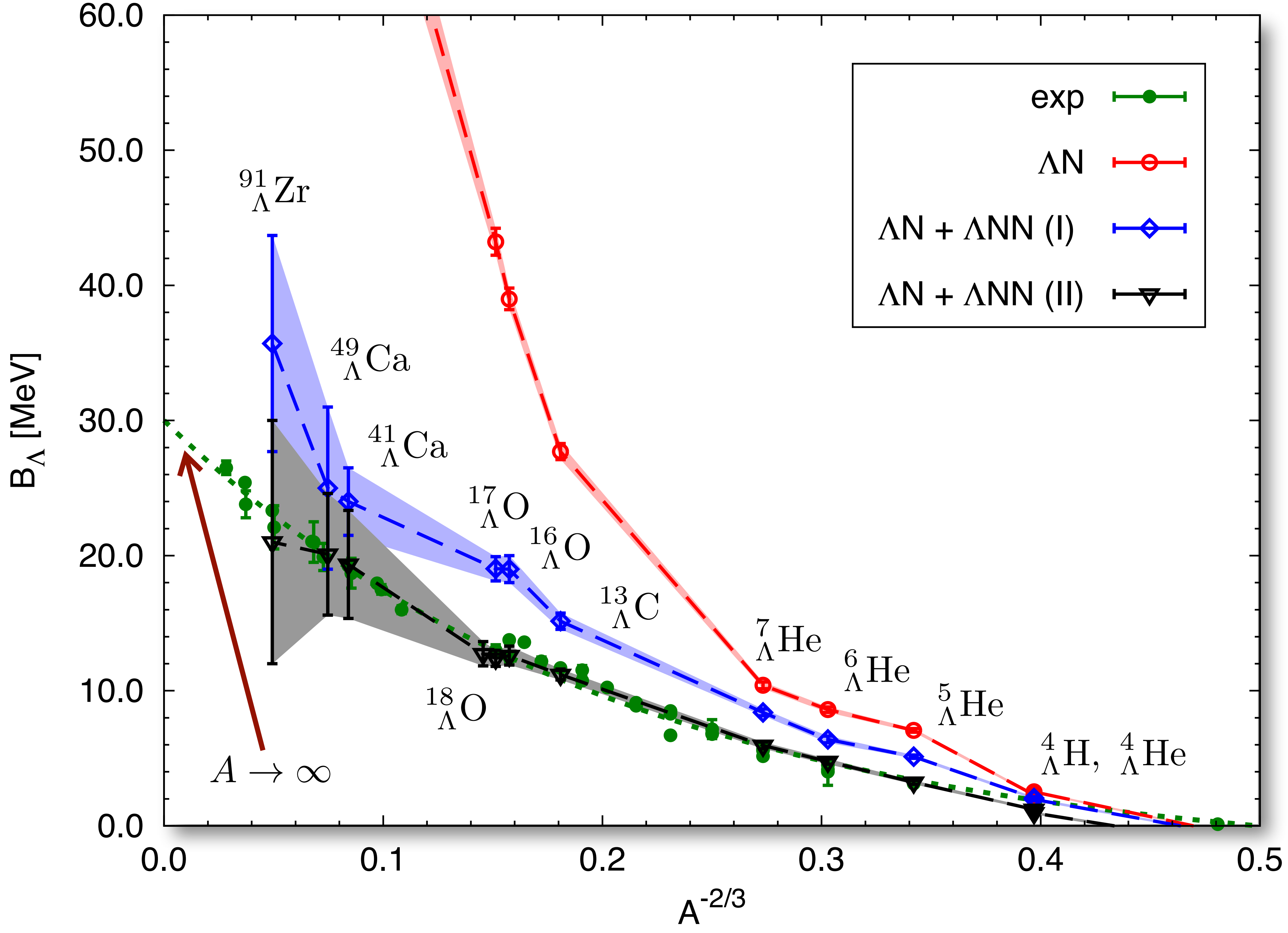


good for both

$^{5}_{\Lambda}\mathrm{He}$ and $^{17}_{\Lambda}\mathrm{O}$

The strange QMC project: hypernuclei

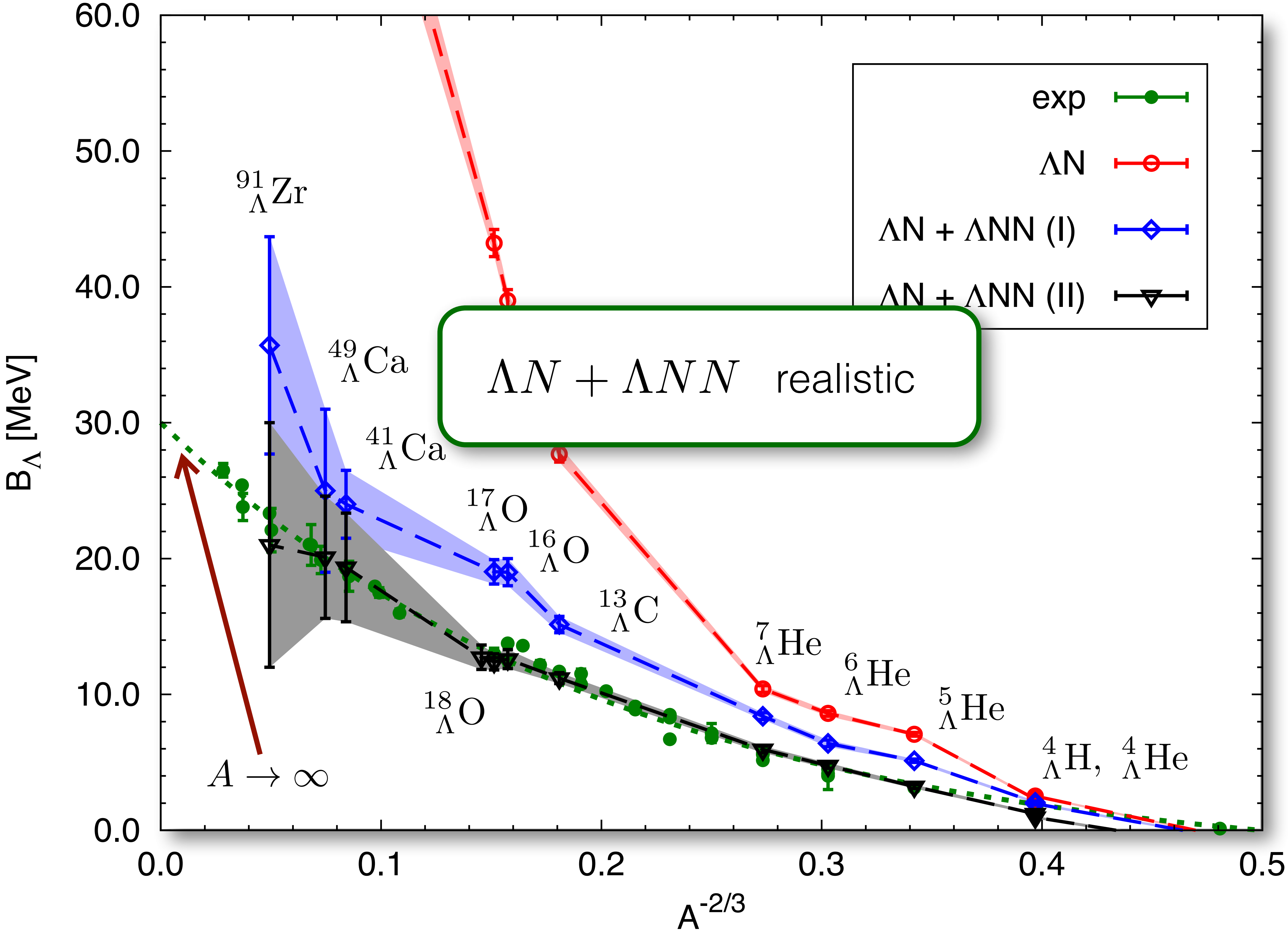
closed + open shell: new parametrization



D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

The strange QMC project: hypernuclei

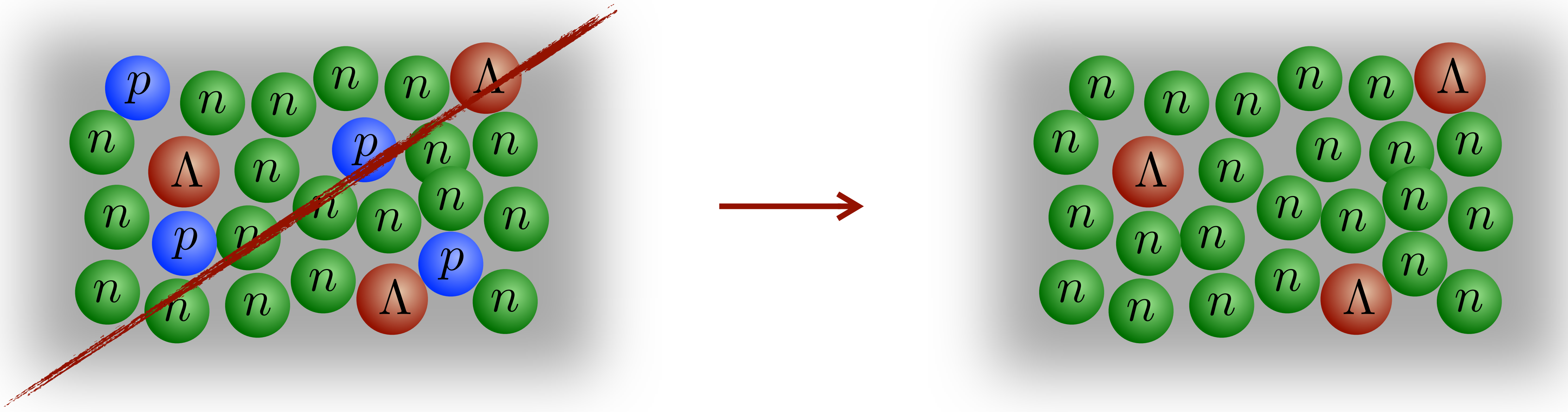
closed + open shell: new parametrization



$\Lambda N + \Lambda NN$ realistic

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

The strange QMC project: hypermatter



hyper-nuclear matter → hyper-neutron matter

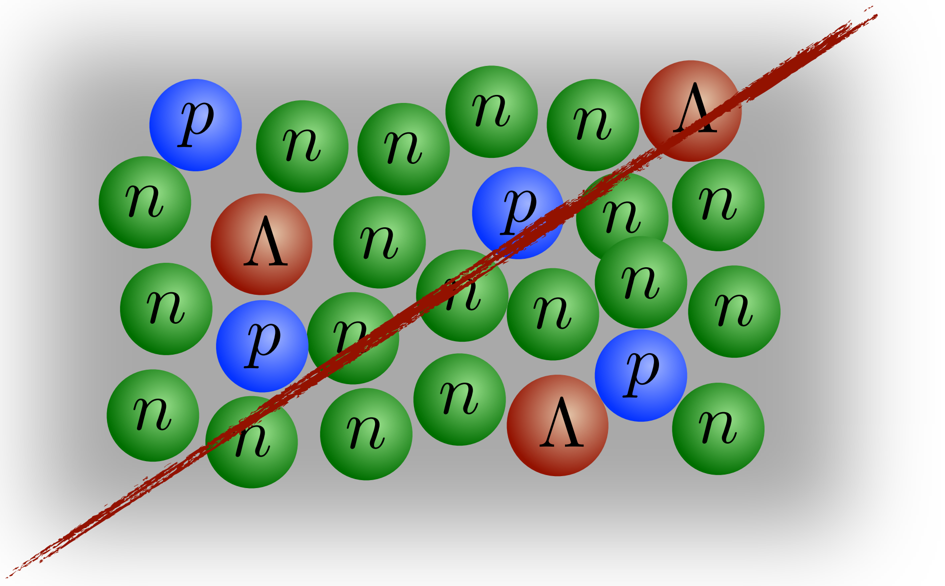
PNM → hyperon fraction → energy per particle: $E_{\rm HNM} \equiv E_{\rm HNM}(\rho_b, x_\Lambda)$

PNM → hyperon fraction → energy density: $\mathcal{E}_{\rm HNM} \equiv \mathcal{E}_{\rm HNM}(\rho_b, x_\Lambda)$

equilibrium condition : chemical potentials

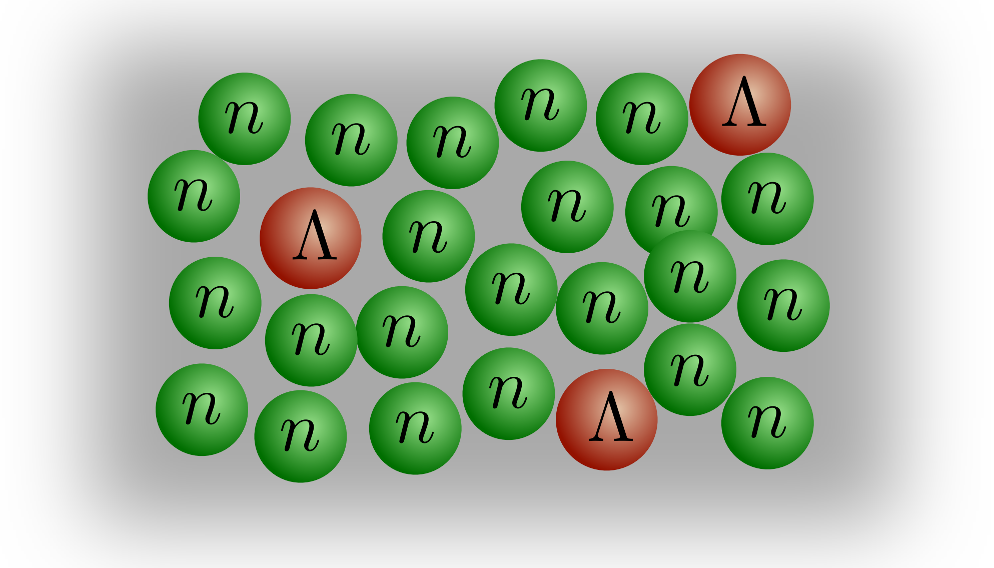
$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda) \Rightarrow x_\Lambda \equiv x_\Lambda(\rho_b)$$

The strange QMC project: hypermatter



hyper-nuclear matter

→



hyper-neutron matter

EoS
$$\left\{\begin{array}{lcl} E_{\mathrm{HNM}} & \equiv & E_{\mathrm{HNM}}(\rho_b) \\ \mathcal{E}_{\mathrm{HNM}} & \equiv & \mathcal{E}_{\mathrm{HNM}}(\rho_b) \\ P_{\mathrm{HNM}} & \equiv & P_{\mathrm{HNM}}(\rho_b) \end{array}\right.$$

TOV
$$\left\{\begin{array}{l} M(R) \\ M_{\max} \end{array}\right.$$

equilibrium condition : chemical potentials

$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$x_\Lambda \equiv x_\Lambda(\rho_b)$$

The strange QMC project: hypermatter

- ✓ pedagogical example: two components Fermi gas

neutrons
+
lambdas

$$\begin{cases} \rho_b &= \rho_n + \rho_\Lambda \\ x_\Lambda &= \dfrac{\rho_\Lambda}{\rho_b} \end{cases} \qquad \begin{cases} \rho_n &= (1-x_\Lambda)\rho_b \\ \rho_\Lambda &= x_\Lambda \rho_b \end{cases}$$

$$\begin{cases} E_{\mathrm{HNM}}(\rho_b, x_\Lambda) &= \Big[E_n^F(\rho_n) + m_n\Big](1-x_\Lambda) + \Big[E_\Lambda^F(\rho_\Lambda) + m_\Lambda\Big]x_\Lambda \\ \mathcal{E}_{\mathrm{HNM}}(\rho_b, x_\Lambda) &= \Big[E_n^F(\rho_n) + m_n\Big]\rho_n + \Big[E_\Lambda^F(\rho_\Lambda) + m_\Lambda\Big]\rho_\Lambda \end{cases}$$

$$\mu_\alpha(\rho_\alpha) = \frac{\partial \mathcal{E}}{\partial \rho_\alpha} = \frac{5}{3}E_\alpha^F(\rho_\alpha) + m_\alpha$$

$m_\Lambda > m_n$

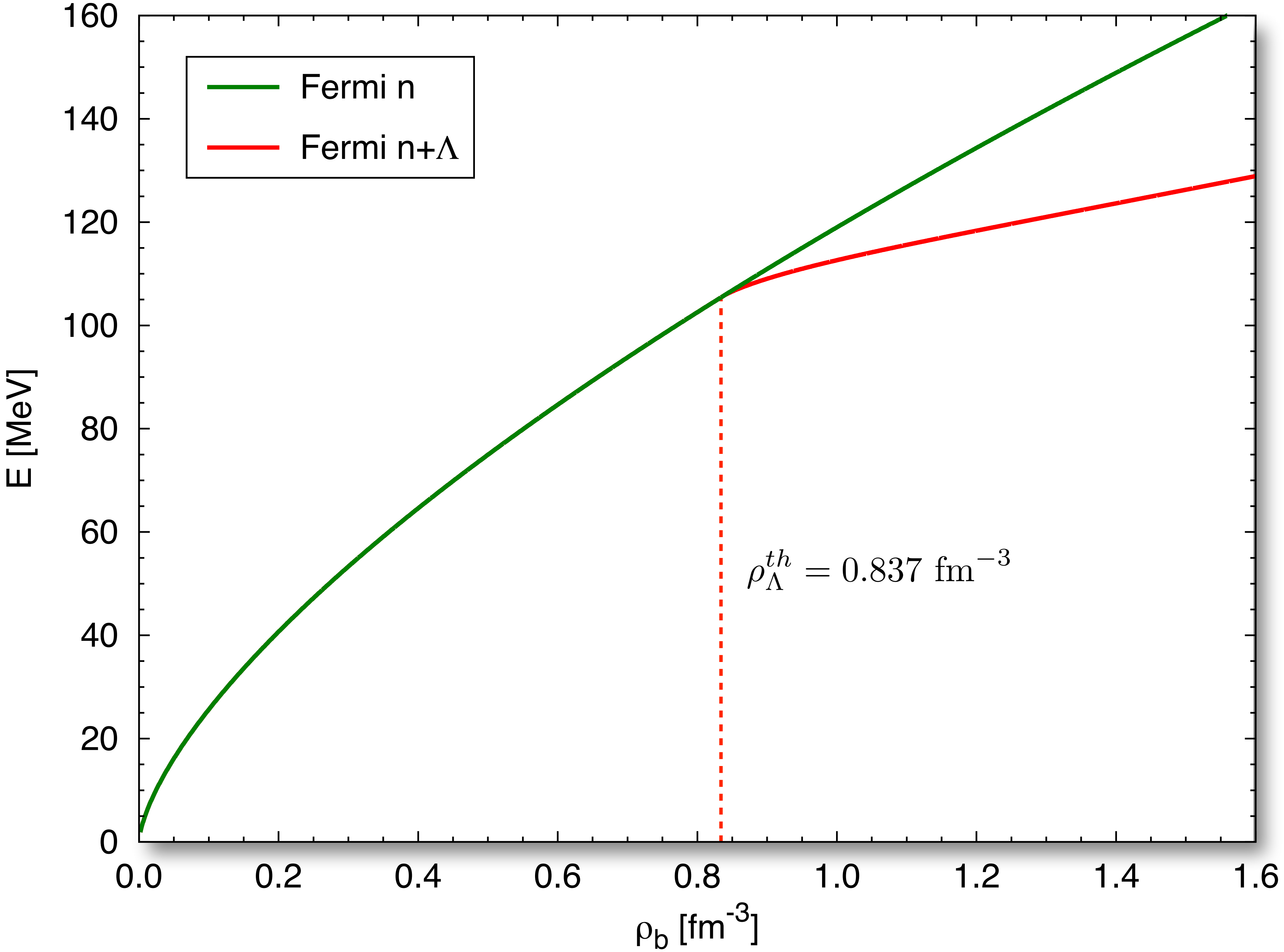
$$\mu_\Lambda(\rho_\Lambda) = \mu_n(\rho_n)$$

has solution!

⇨

$$\begin{cases} \rho_\Lambda^{th} \quad @ \quad x_\Lambda \to 0 \\ x_\Lambda(\rho_\Lambda) \end{cases}$$

The strange QMC project: hypermatter



The strange QMC project: hypermatter

- ✓ realistic case: interacting particles

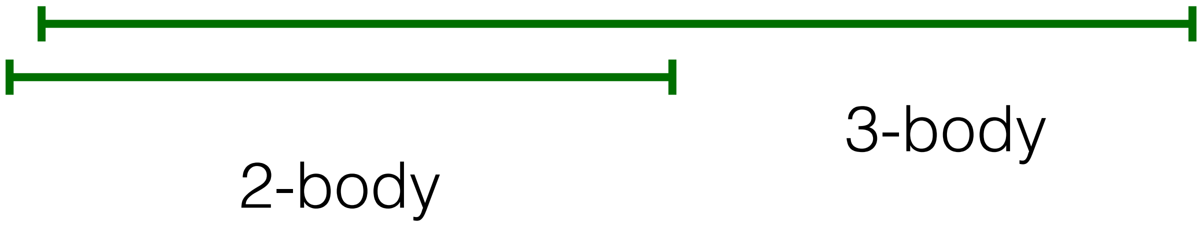
$$E_{\mathrm{HNM}}(\rho_b, x_\Lambda) = \Big[E_{\mathrm{PNM}}((1-x_\Lambda)\rho_b) + m_n\Big](1-x_\Lambda) \qquad n, nn$$

$$+ \Big[E_\Lambda^F(x_\Lambda\rho_b) + m_\Lambda\Big]x_\Lambda + f(\rho_b, x_\Lambda) \qquad \Lambda, n\Lambda, \cancel{\Lambda\Lambda}$$

Problem: limitation in x_Λ due to simulation box

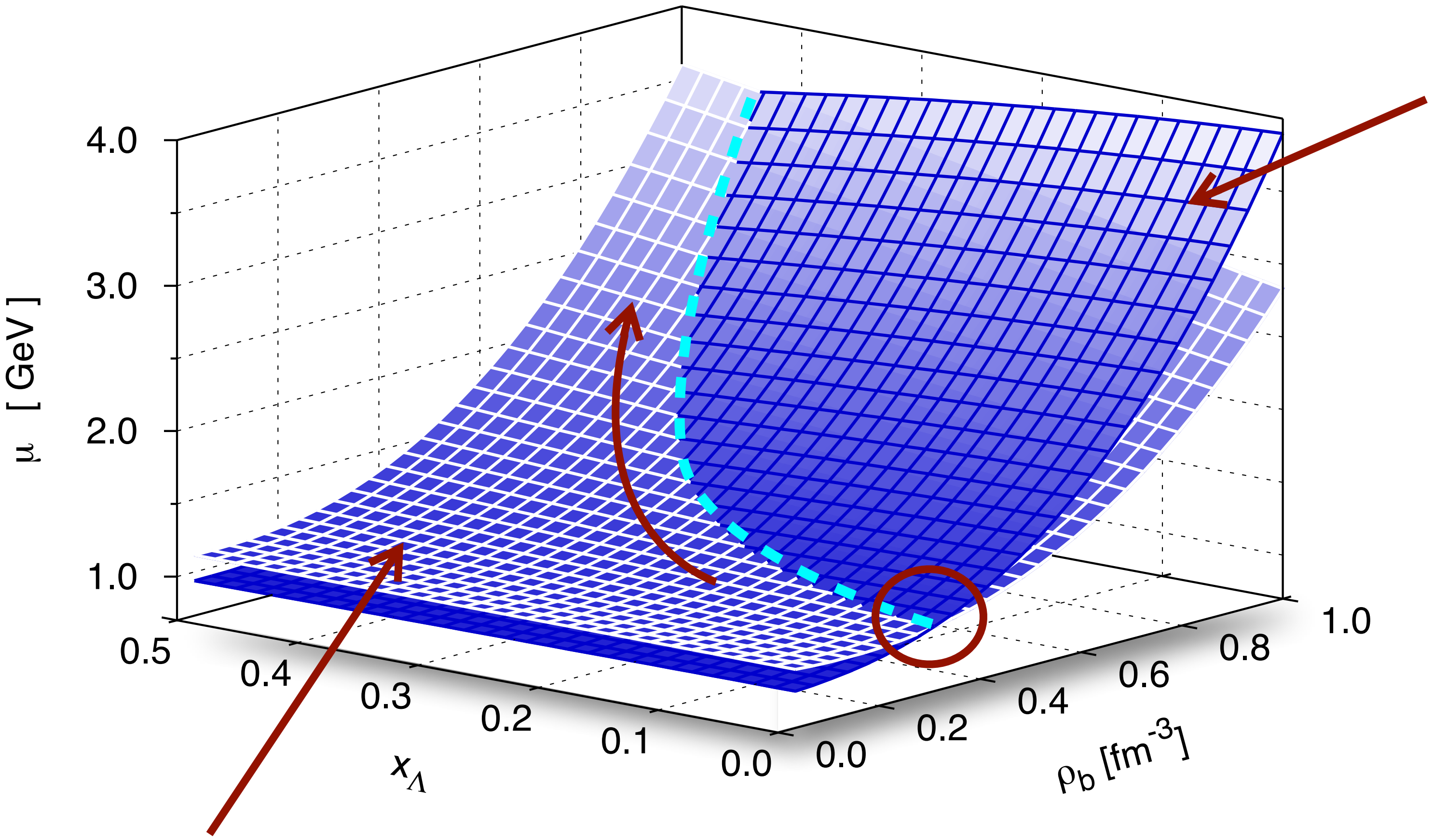
$$f(\rho_b, x_\Lambda) = \Delta E_{\mathrm{MC}}(@\, N_n) \longrightarrow \text{same volume}$$

cluster expansion $\dfrac{\rho_\Lambda\rho_n}{\rho_b}\,,\quad \dfrac{\rho_\Lambda\rho_n\rho_n}{\rho_b}\,,\quad \dfrac{\rho_\Lambda\rho_n\rho_n\rho_n}{\rho_b}$



The strange QMC project: hypermatter

$$\begin{cases} \mu_n(\rho_b, x_\Lambda) = E_{\text{PNM}}(\rho_n) + \rho_n \dfrac{\partial E_{\text{PNM}}}{\partial \rho_n} + m_n + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_n} \\ \mu_\Lambda(\rho_b, x_\Lambda) = \quad E_\Lambda^F(\rho_\Lambda) \quad + \quad \rho_\Lambda \dfrac{\partial E_\Lambda^F}{\partial \rho_\Lambda} \quad + m_\Lambda + f(\rho_b, x_\Lambda) + \rho_b \dfrac{\partial f}{\partial \rho_\Lambda} \end{cases}$$



$\mu_n(\rho_b, x_\Lambda)$

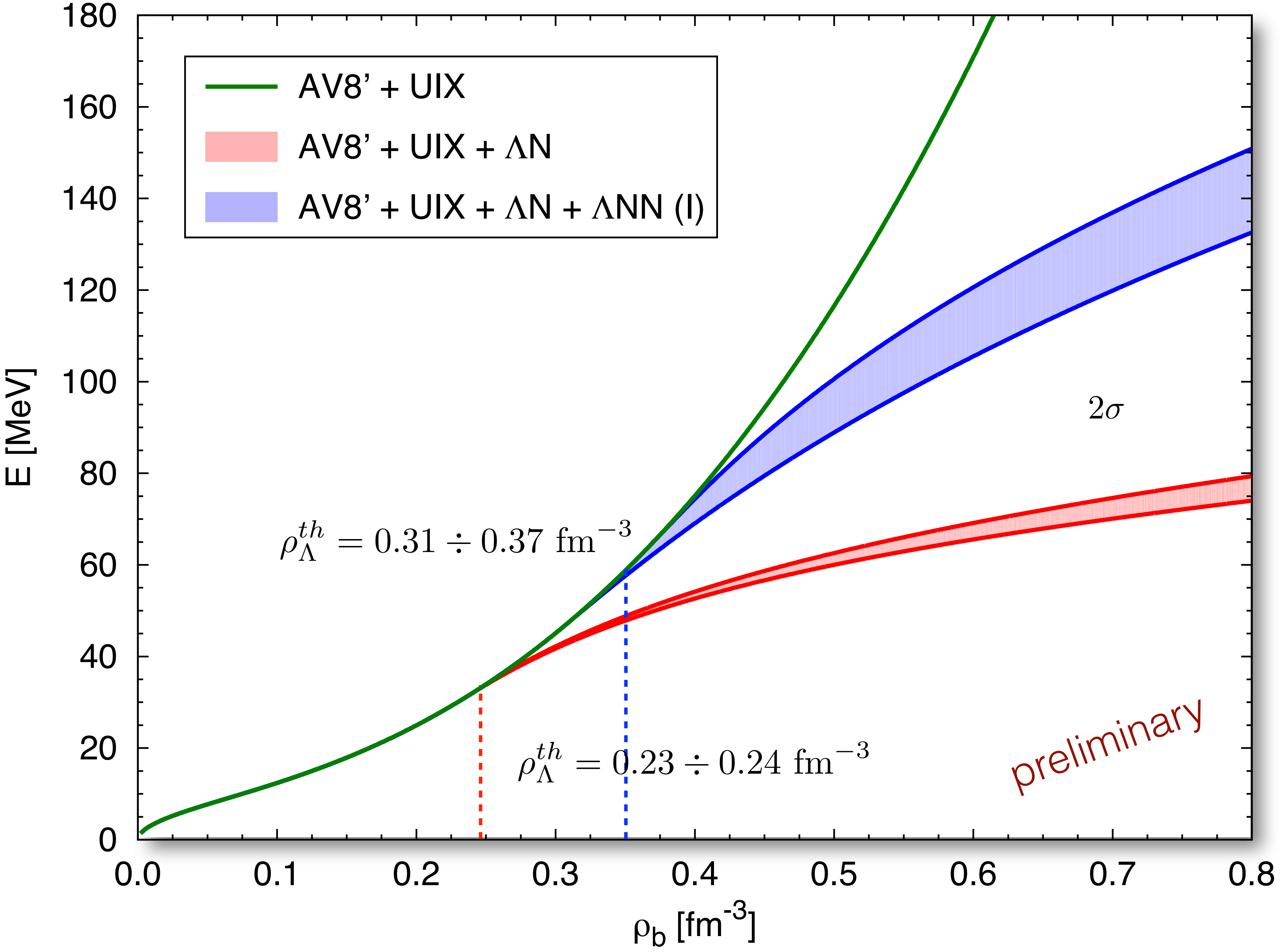
$\mu_\Lambda(\rho_b, x_\Lambda)$

equilibrium condition:

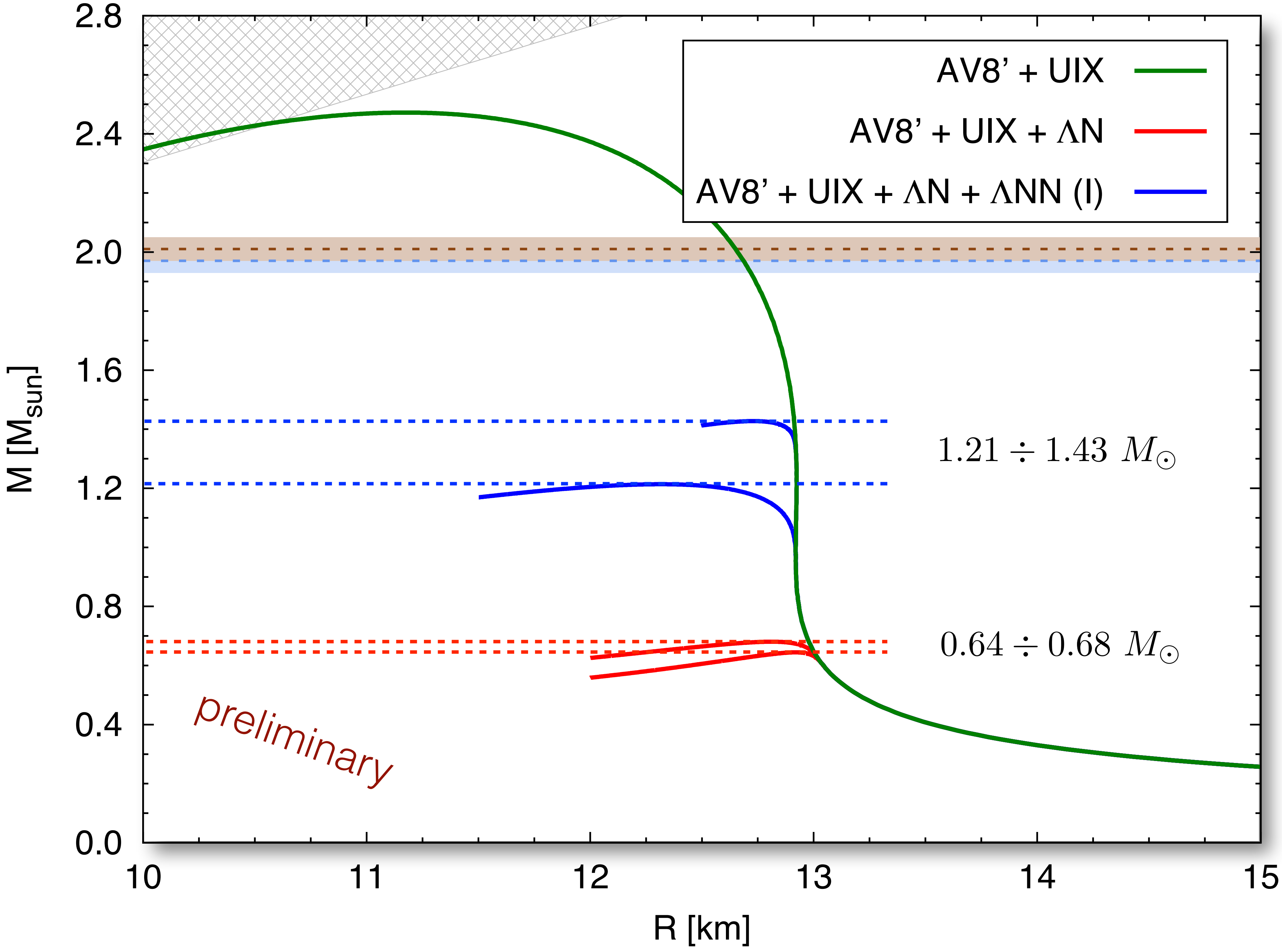
$$\mu_\Lambda(\rho_b, x_\Lambda) = \mu_n(\rho_b, x_\Lambda)$$

$$\rho_\Lambda^{th} \quad @ \quad x_\Lambda \to 0$$

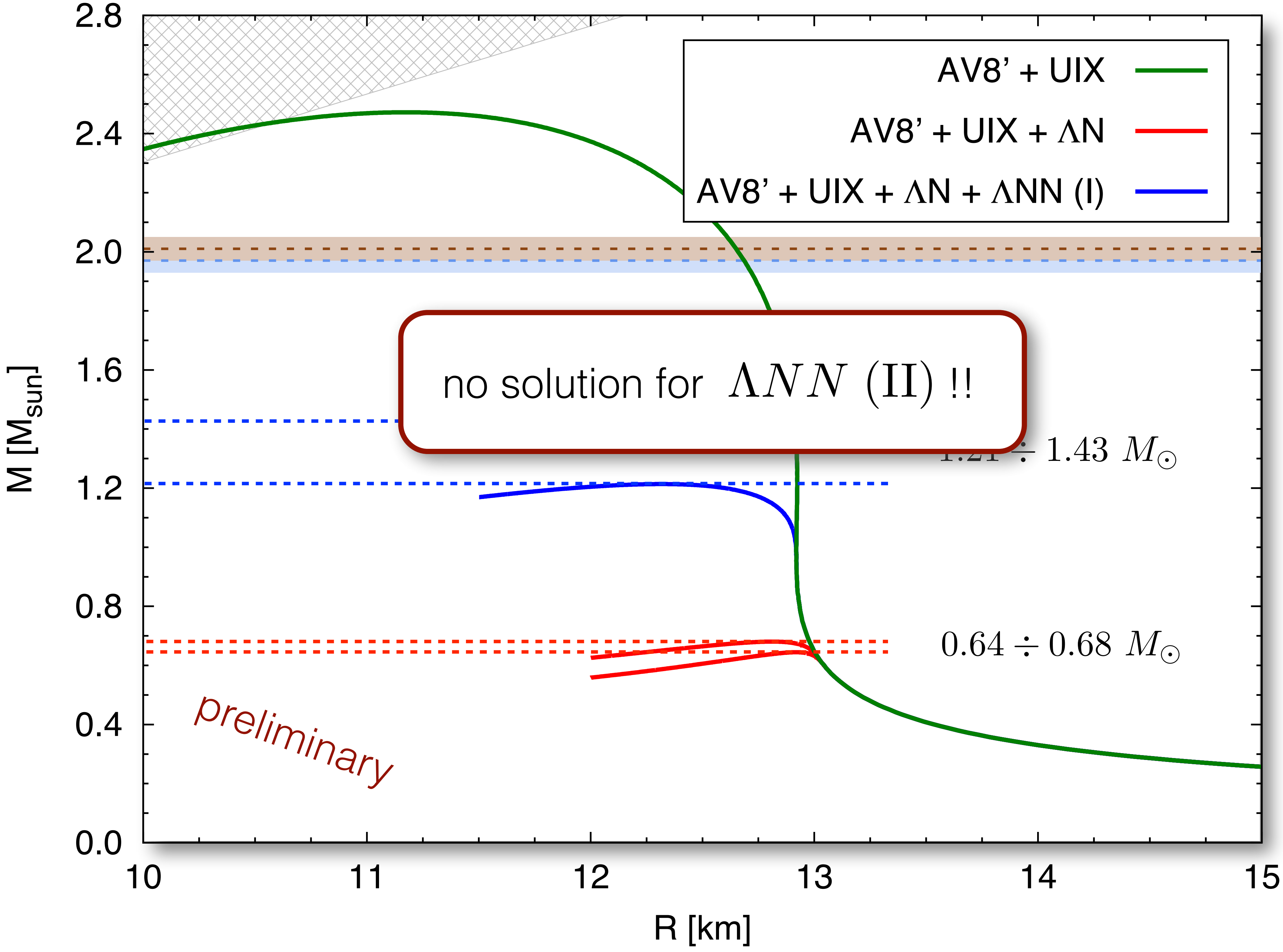
The strange QMC project: hypermatter



The strange QMC project: hypermatter



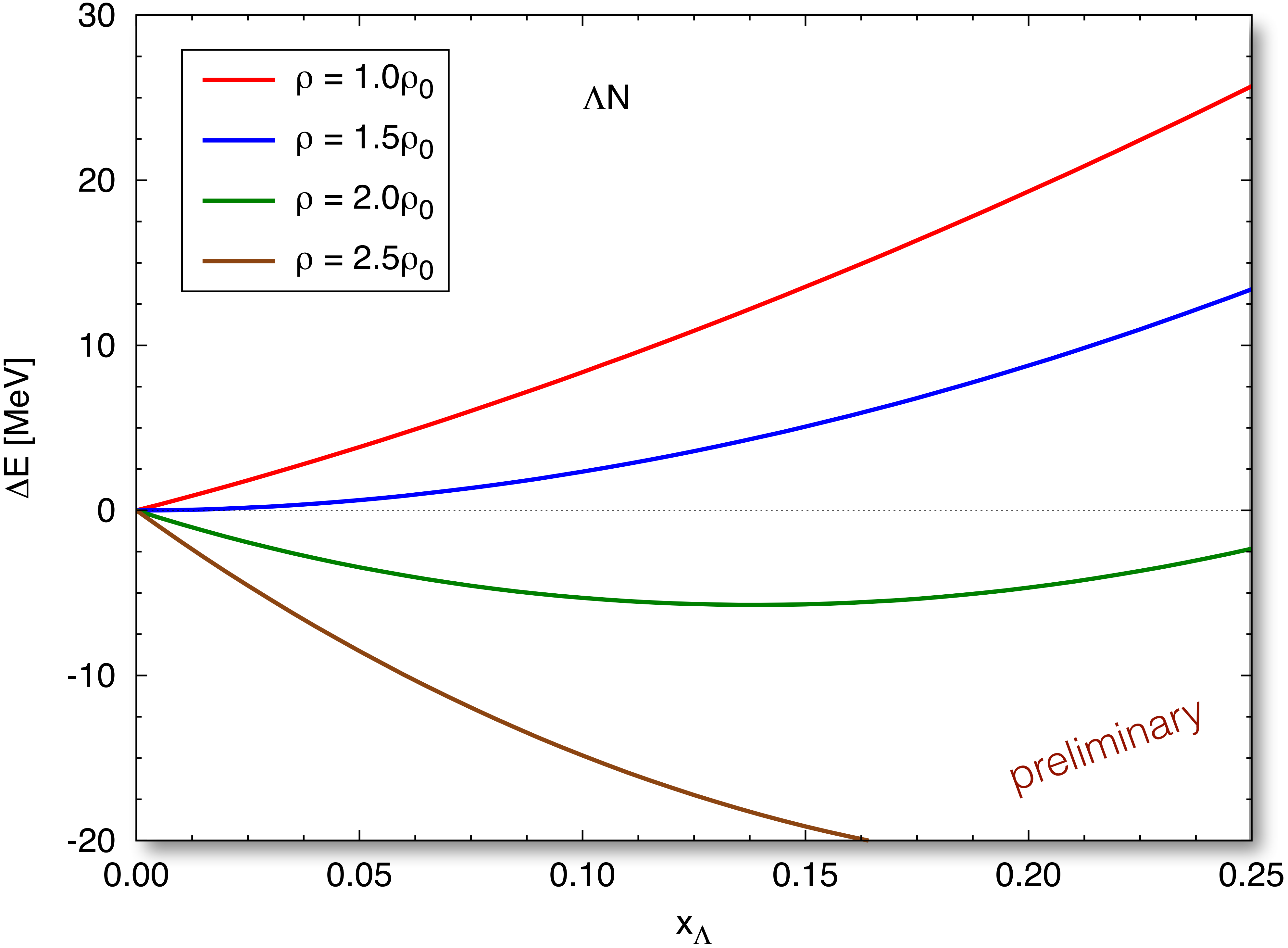
The strange QMC project: hypermatter



no solution for $\Lambda N N$ (II) !!

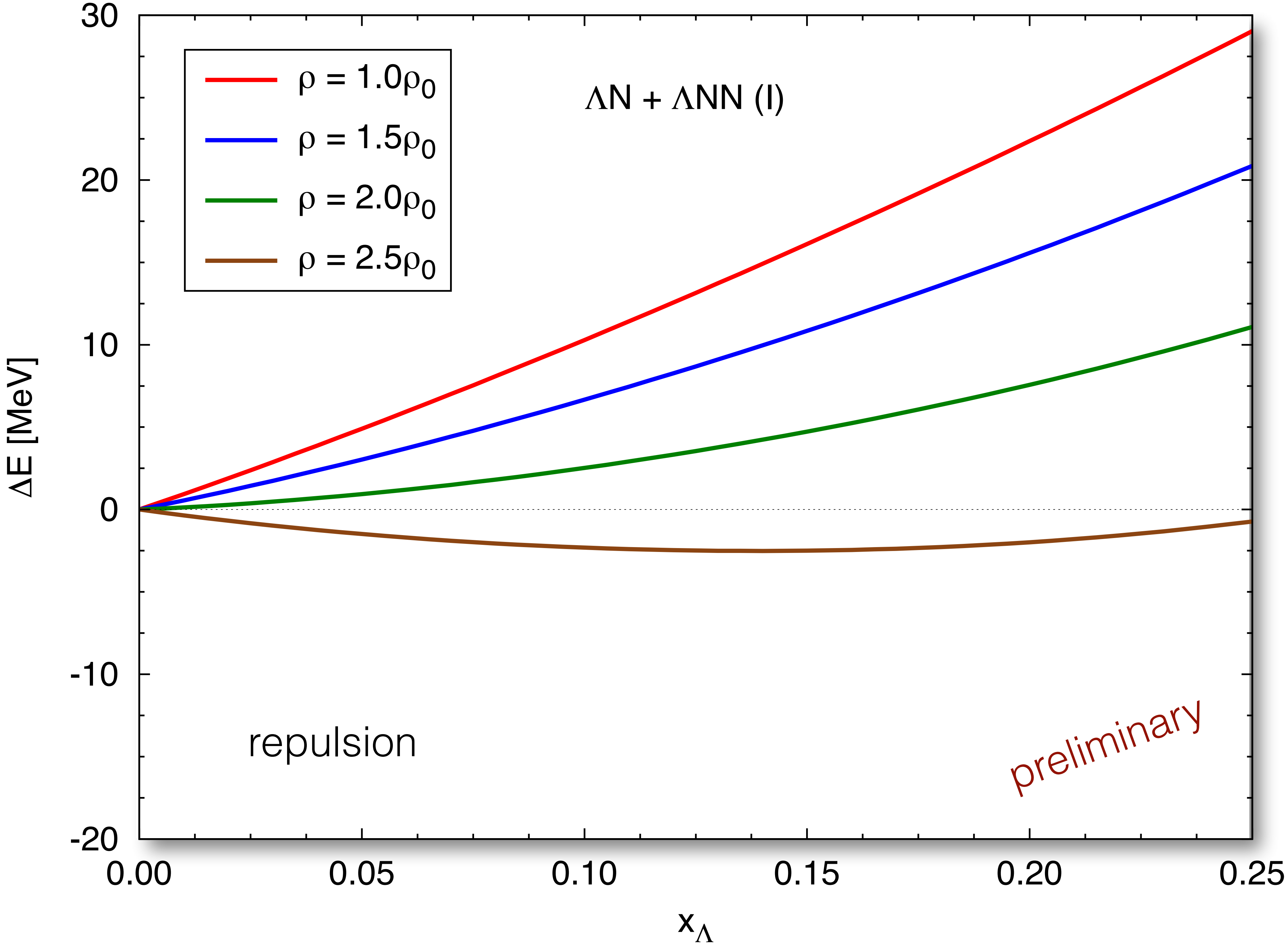
The strange QMC project: hypermatter

$$\Delta E = E_{\mathrm{HNM}}(\rho_b, x_\Lambda) - E_{\mathrm{PNM}}(\rho_b)$$



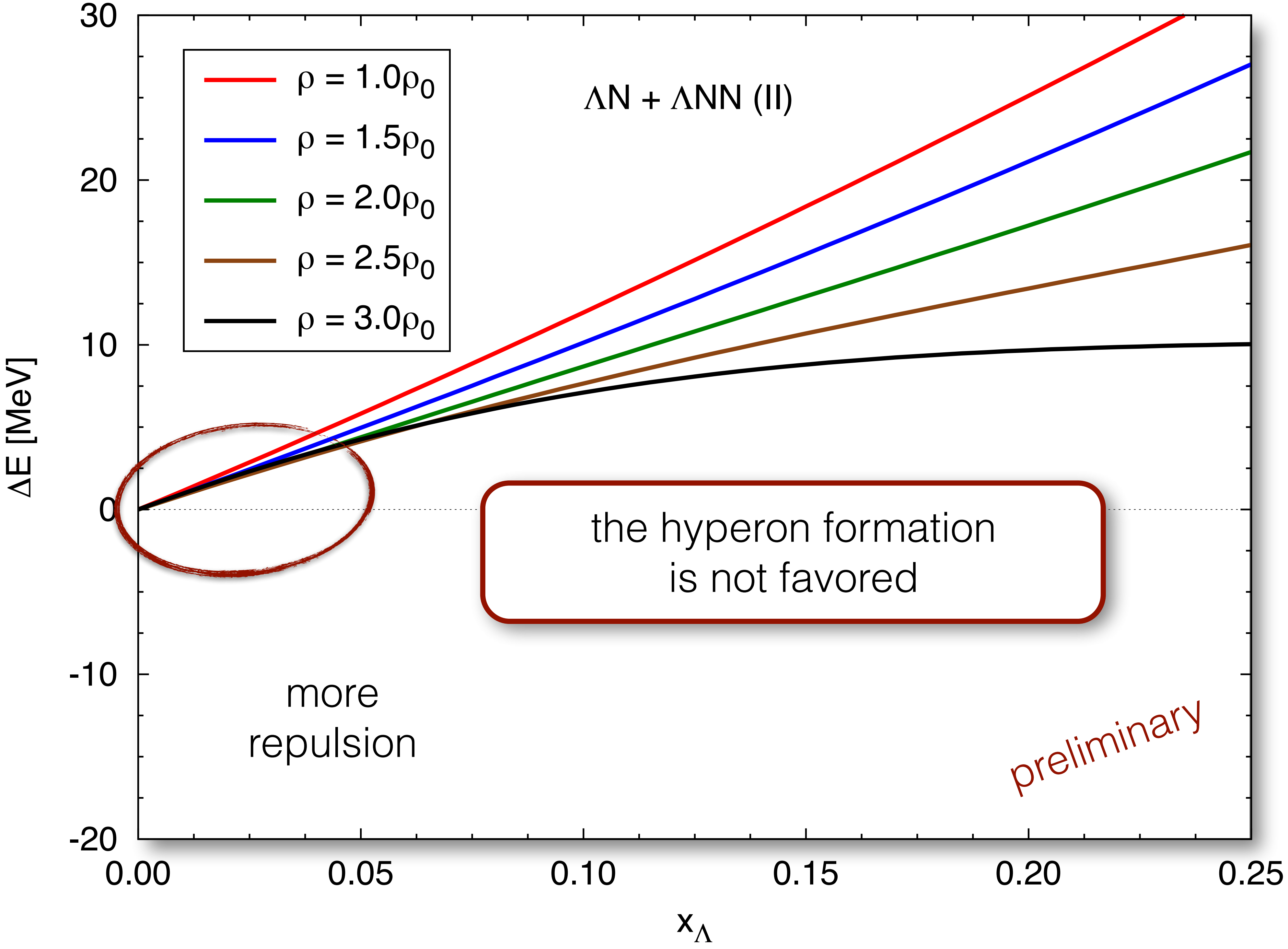
The strange QMC project: hypermatter

$$\Delta E = E_{\mathrm{HNM}}(\rho_b, x_\Lambda) - E_{\mathrm{PNM}}(\rho_b)$$



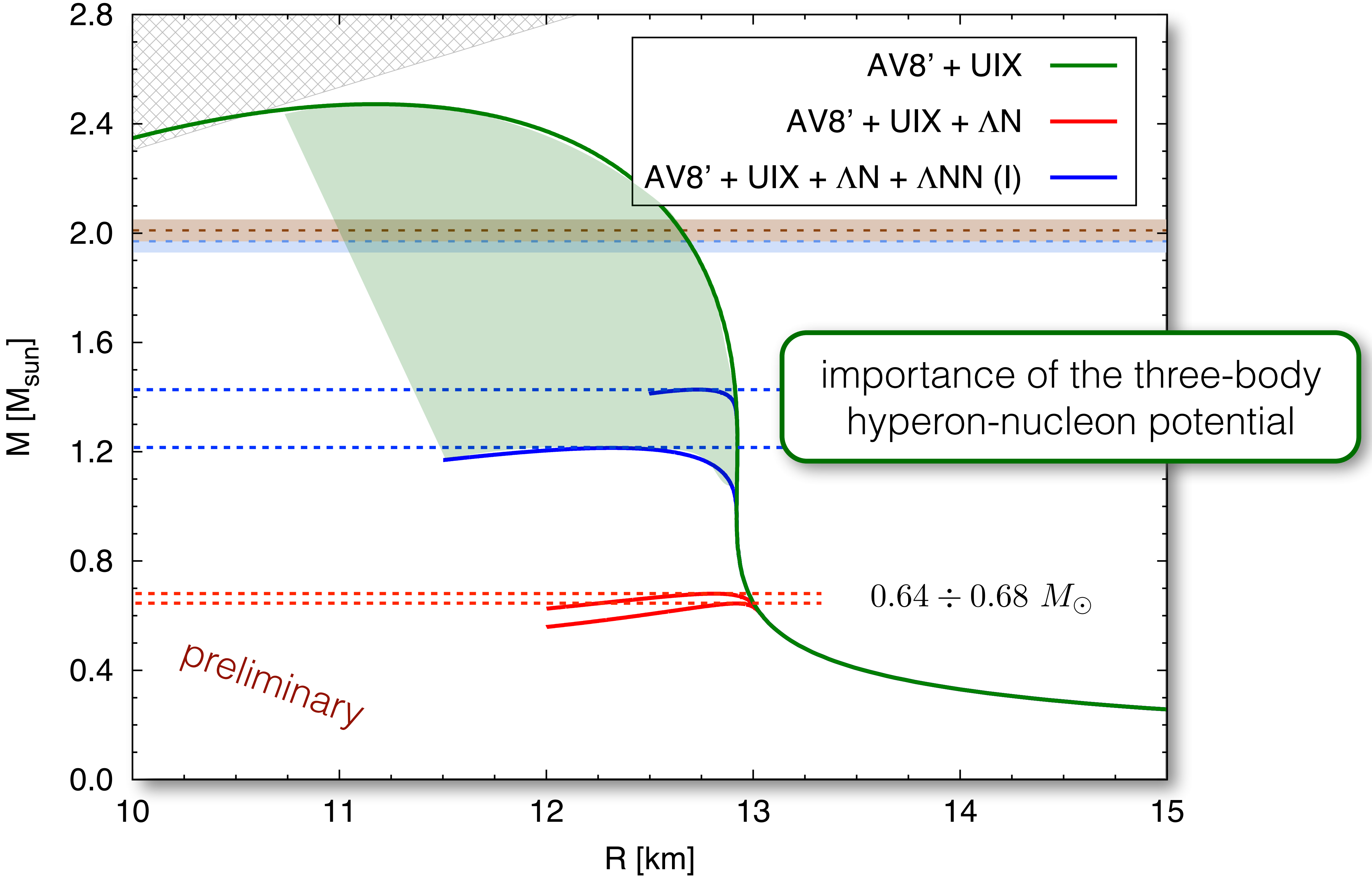
The strange QMC project: hypermatter

$$\Delta E = E_{\mathrm{HNM}}(\rho_b, x_\Lambda) - E_{\mathrm{PNM}}(\rho_b)$$



the hyperon formation is not favored

The strange QMC project: hypermatter



importance of the three-body hyperon-nucleon potential

Conclusions

- ✓ Method:
  - development of Quantum Monte Carlo framework for finite and infinite nuclear systems including strangeness

- ✓ Hypernuclei:
  - hypernuclei are overbound by a two-body hyperon-nucleon interaction alone
  - the inclusion of a three-body force provides the necessary repulsion to reproduce the ground state physics of medium-heavy hypernuclei
  - the three-body potential refitted on the separation energy of two selected hypernuclei correctly reproduces the experimental data up to A=91

- ✓ Hypermatter: (preliminary)
  - very soft EoS and low maximum mass by including a two-body hyperon-nucleon interaction alone
  - important repulsive contribution of the three-body force: chance to reconcile hyperons appearance and observational limits on neutron star masses

Future

- ✓ Method:
  - inclusion of nucleon-nucleon spin-orbit term, three-nucleon force
  - TABC for neutron and hyper-neutron matter
  - nuclear and hyper-nuclear matter
- ✓ Hypernuclei and hypermatter: need of better interactions
  - projection of the present hyperon-nucleon interaction on isospin channels: refit of the potential
  - use of different potentials, maybe chiral interactions

→ more experimental data needed

Thank you!

Backup: hyperon-nucleon interaction

- ✓ 2-body interaction

$$v_{\lambda i} = v_0(r_{\lambda i}) + \frac{1}{4} v_\sigma T_\pi^2(r_{\lambda i})\, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_i$$

charge symmetric

$$v_{\lambda i}^{CSB} = C_\tau\, T_\pi^2\,(r_{\lambda i})\, \tau_i^z$$

charge symmetry breaking

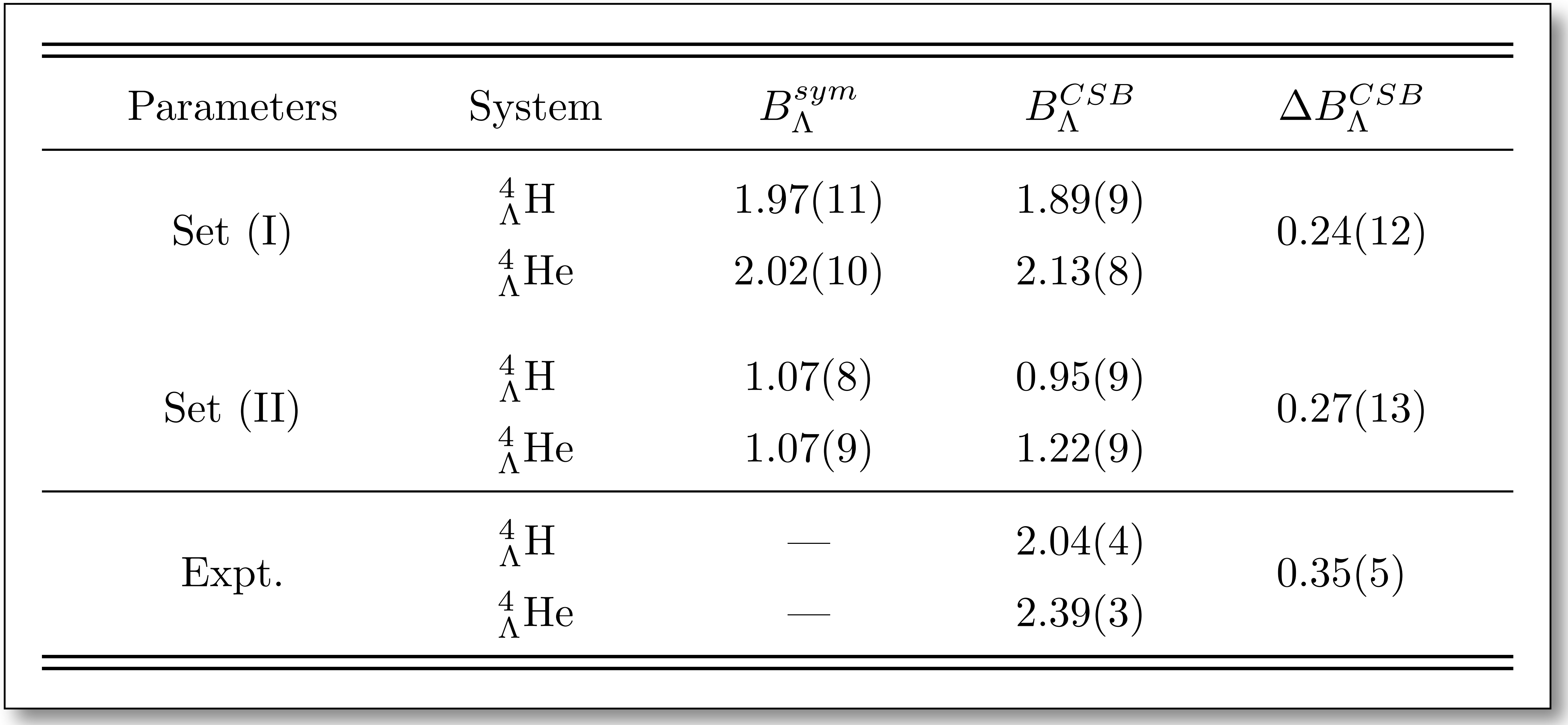
- ✓ 3-body interaction

$$v_{\lambda ij} = v_{\lambda ij}^{2\pi,P} + v_{\lambda ij}^{2\pi,S} + v_{\lambda ij}^{D}$$

$$\begin{cases} v_{\lambda ij}^{2\pi,P} = -\dfrac{C_P}{6}\Big\{X_{i\lambda}\,,X_{\lambda j}\Big\}\,\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{2\pi,S} = C_S\, Z\,(r_{\lambda i})\, Z\,(r_{\lambda j})\;\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\lambda}\, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\lambda}\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\lambda ij}^{D} = W_D\, T_\pi^2\,(r_{\lambda i})\, T_\pi^2\,(r_{\lambda j})\Big[1 + \dfrac{1}{6}\boldsymbol{\sigma}_\lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Big] \end{cases}$$

Backup: hyperon-nucleon interaction

CSB effect



Parameters	System	B_Λ^{sym}	B_Λ^{CSB}	ΔB_Λ^{CSB}
Set (I)	${}^4_\Lambda\mathrm{H}$	1.97(11)	1.89(9)	0.24(12)
	${}^4_\Lambda\mathrm{He}$	2.02(10)	2.13(8)	
Set (II)	${}^4_\Lambda\mathrm{H}$	1.07(8)	0.95(9)	0.27(13)
	${}^4_\Lambda\mathrm{He}$	1.07(9)	1.22(9)	
Expt.	${}^4_\Lambda\mathrm{H}$	—	2.04(4)	0.35(5)
	${}^4_\Lambda\mathrm{He}$	—	2.39(3)	

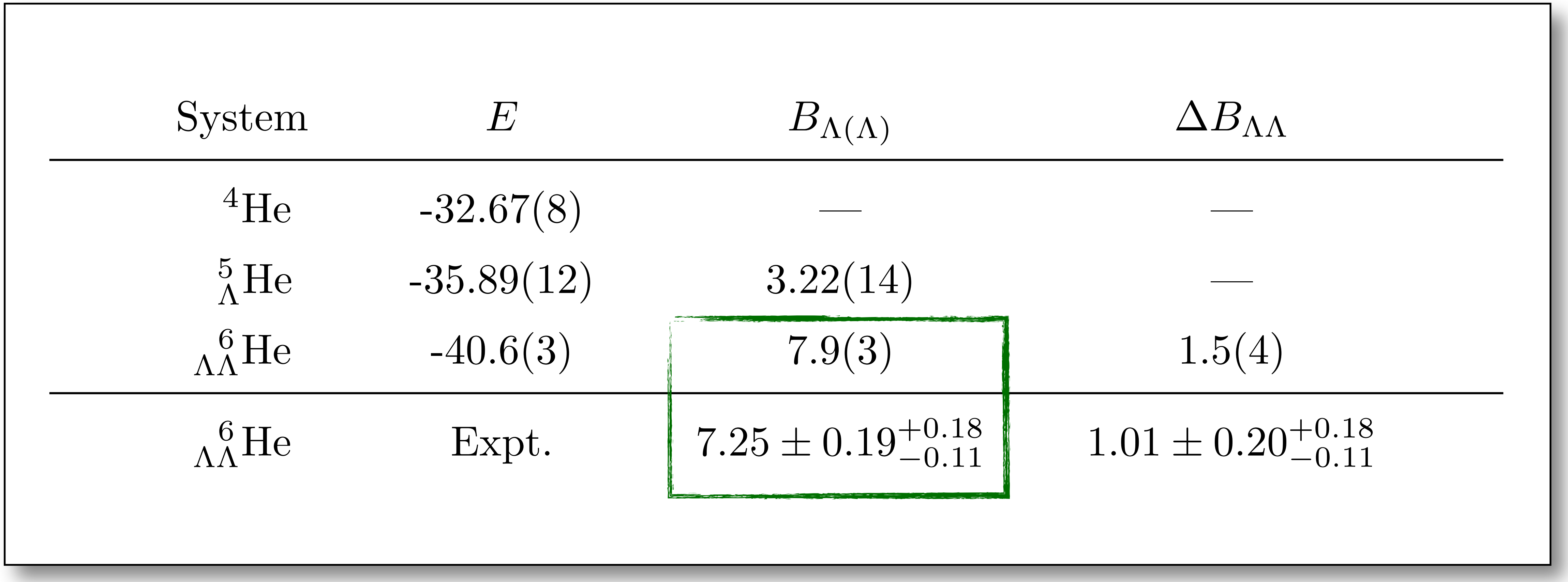
D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

Backup: hyperon-hyperon interaction

$$v_{\lambda\mu} = \sum_{k=1}^{3}\left(v_0^{(k)} + v_\sigma^{(k)}\,\boldsymbol{\sigma}_\lambda\cdot\boldsymbol{\sigma}_\mu\right)\mathrm{e}^{-\mu^{(k)} r_{\lambda\mu}^2}$$

E. Hiyama, et al., Phys. Rev. C 66, 024007 (2002)

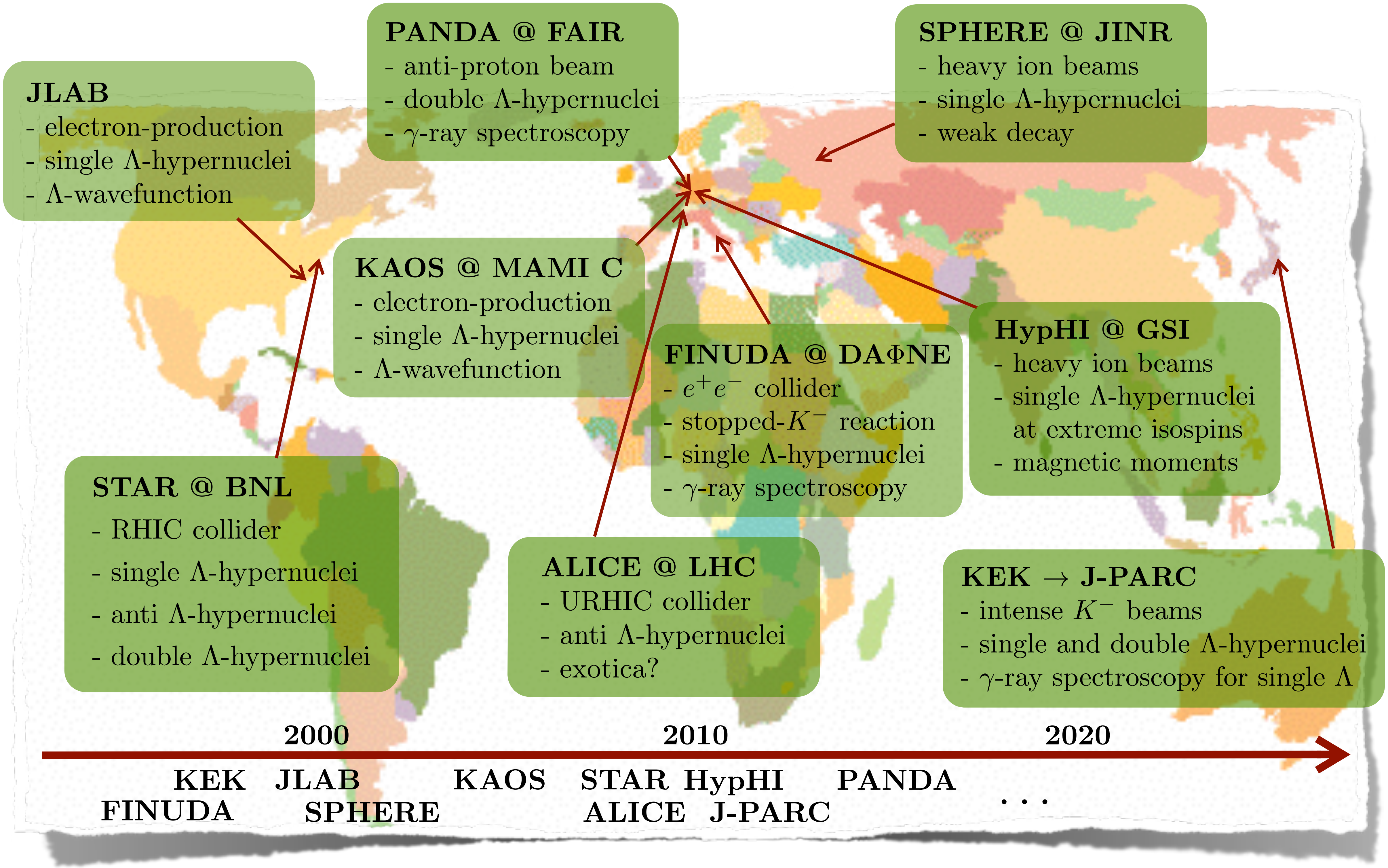
double Λ hypernuclei



System	E	$B_{\Lambda(\Lambda)}$	$\Delta B_{\Lambda\Lambda}$
^{4}He	-32.67(8)	—	—
$^5_\Lambda$He	-35.89(12)	3.22(14)	—
$^{\;6}_{\Lambda\Lambda}$He	-40.6(3)	7.9(3)	1.5(4)
$^{\;6}_{\Lambda\Lambda}$He	Expt.	$7.25 \pm 0.19^{+0.18}_{-0.11}$	$1.01 \pm 0.20^{+0.18}_{-0.11}$

D. L., F. Pederiva, S. Gandolfi, Phys. Rev. C 89, 014314 (2014)

Backup: hypernuclei experimental status



PANDA @ FAIR
- anti-proton beam
- double Λ-hypernuclei
- γ-ray spectroscopy

SPHERE @ JINR
- heavy ion beams
- single Λ-hypernuclei
- weak decay

JLAB
- electron-production
- single Λ-hypernuclei
- Λ-wavefunction

KAOS @ MAMI C
- electron-production
- single Λ-hypernuclei
- Λ-wavefunction

HypHI @ GSI
- heavy ion beams
- single Λ-hypernuclei at extreme isospins
- magnetic moments

FINUDA @ DAΦNE
- e^+e^- collider
- stopped-K^- reaction
- single Λ-hypernuclei
- γ-ray spectroscopy

STAR @ BNL
- RHIC collider
- single Λ-hypernuclei
- anti Λ-hypernuclei
- double Λ-hypernuclei

ALICE @ LHC
- URHIC collider
- anti Λ-hypernuclei
- exotica?

KEK → J-PARC
- intense K^- beams
- single and double Λ-hypernuclei
- γ-ray spectroscopy for single Λ

Adapted from: J. Pochodzalla, Acta Phys. Polon. B 42, 833– 842 (2011)

Backup: the method

$$\psi_T(R,S) = \prod_{\lambda i} f_c^{\Lambda N}(r_{\lambda i})\,\psi_T^N(R_N,S_N)\,\psi_T^\Lambda(R_\Lambda,S_\Lambda)$$

$$\begin{cases} \psi_T^\kappa(R_\kappa,S_\kappa) = \prod_{i<j} f_c^{\kappa\kappa}(r_{ij})\,\Phi_\kappa(R_\kappa,S_\kappa) \\[2ex] \Phi_\kappa(R_\kappa,S_\kappa) = \mathcal{A}\left[\prod_{i=1}^{\mathcal{N}_\kappa}\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right] = \det\left\{\varphi_\epsilon^\kappa(\boldsymbol{r}_i,s_i)\right\} \end{cases} \qquad \kappa = N,\Lambda$$

s.p. orbitals plane waves

$$s_i = \begin{pmatrix} a_i \\ b_i \\ c_i \\ d_i \end{pmatrix}_i = a_i|p\uparrow\rangle_i + b_i|p\downarrow\rangle_i + c_i|n\uparrow\rangle_i + d_i|n\downarrow\rangle_i$$

$$s_\lambda = \begin{pmatrix} u_\lambda \\ v_\lambda \end{pmatrix}_\lambda = u_\lambda|\Lambda\uparrow\rangle_\lambda + v_\lambda|\Lambda\downarrow\rangle_\lambda$$

Backup: the method

$$
\begin{aligned}
V_{NN}^{SD} + V_{\Lambda N}^{SD} &= \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N} \lambda_n^{[\sigma]} \left(\mathcal{O}_n^{[\sigma]}\right)^2 \\
&+ \frac{1}{2}\sum_{n=1}^{3\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\tau]} \left(\mathcal{O}_{n\alpha}^{[\sigma\tau]}\right)^2 \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N}\sum_{\alpha=1}^{3} \lambda_n^{[\tau]} \left(\mathcal{O}_{n\alpha}^{[\tau]}\right)^2 \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} \lambda_n^{[\sigma\Lambda]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda]}\right)^2 \\
&+ \frac{1}{2}\sum_{n=1}^{\mathcal{N}_N\mathcal{N}_\Lambda}\sum_{\alpha=1}^{3} B_n^{[\sigma]} \left(\mathcal{O}_{n\alpha}^{[\sigma\Lambda N]}\right)^2 \\
&+ \sum_{i=1}^{\mathcal{N}_N} B_i^{[\tau]}\,\tau_i^z
\end{aligned}
$$

$A_{i\alpha,j\beta}^{[\sigma]}$

$A_{i\alpha,j\beta}^{[\sigma\tau]}$

$A_{ij}^{[\tau]}$

$C_{\lambda\mu}^{[\sigma]}$

diagonalization:

λ_n eigenvalues

ψ_n eigenvectors

$\mathcal{O}_n = \sigma_n\,\psi_n$

direct calculation