Neutron matter based on chiral effective field theory interactions

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Main points

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1. Chiral effective field theory: Epelbaum *et al.*, PPNP (2006) and RMP (2009)
   - Systematic basis for nuclear forces, naturally includes many-body forces
   - Very successful in calculations of nuclei and nuclear matter

2. Neutron matter calculations with chiral EFT: IT, Krüger, Hebeler, Schwenk, PRL (2013)
   - Constraints on equation of state
   - Constraints on astrophysical observables

3. Quantum Monte Carlo calculations with chiral EFT interactions
   - Very precise for strongly interacting systems
   - Need of local interactions (only depend on $r = r_i - r_j \leftrightarrow q$)
   - Several sources of nonlocality in chiral EFT
   - Can be removed to N^2LO

     Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)

Motivation

Physics of neutron matter used in a wide variety of applications and density regimes:

J. Hester (ASU) et al., CXC, HST, NASA

Universal properties at low densities:

- Ultracold atoms

Nuclear densities:

- Neutron-rich nuclei

Very high densities:

- Neutron stars

To understand these phenomena $\rightarrow$ better understanding of neutron matter

Motivation

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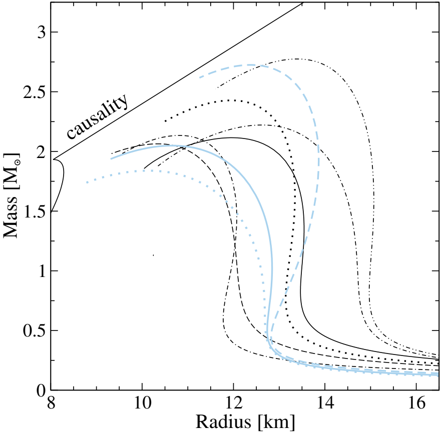
J. Hester (ASU) et al., CXC, HST, NASA

For model equations of state:

- Pressure is correlated with neutron star radius
- Pressure variation by a factor of 6 at saturation density (unrealistic) Hebeler, Lattimer, Pethick, Schwenk, PRL (2010)

$\Rightarrow$ Sizeable radius range for neutron stars

Motivation



For model equations of state:

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Chiral Effective Field Theory for Nuclear Forces

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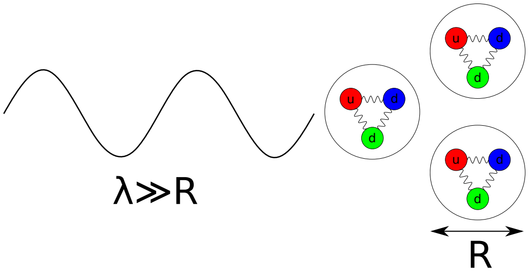
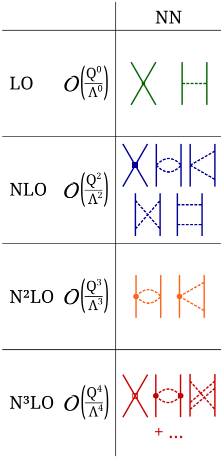
Basic principle of effective field theory:



At low energies (long wavelength) details not resolved!

- Choose relevant degrees of freedom for low-energy processes
- Systematic expansion of interaction terms constrained by symmetries

Chiral effective field theory for nuclear forces



Separation of scales:
low momenta $q \ll$ breakdown scale Λ_B

Write most general Lagrangian and expand in powers of $(q/\Lambda_B)^n$

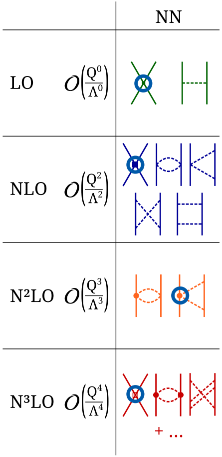
n=0: leading order (LO),
n=2: next-to-leading order (NLO), ...

expansion parameter $\approx 1/3$

Systematic: can work to desired accuracy and obtain error estimates

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Hammer, Kaiser, Machleidt, Meißner,...

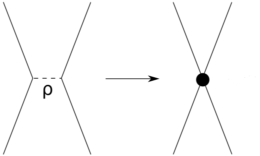
Chiral effective field theory for nuclear forces



Explicit degrees of freedom:
pions and nucleons

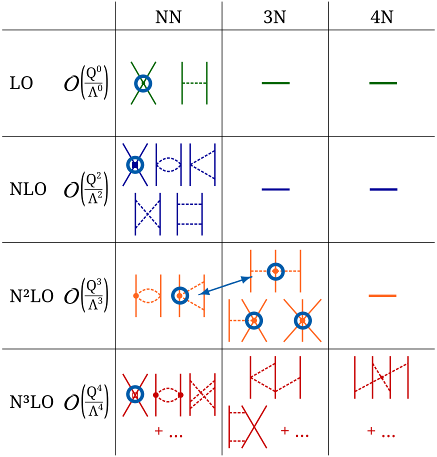
Long-range physics explicit,
short-range physics expanded
in general operator basis

High-momentum physics absorbed into
short-range couplings, fit to experiment



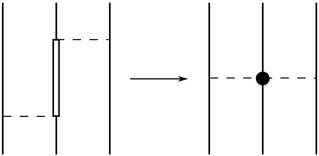
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Hammer, Kaiser, Machleidt, Meißner,...

Chiral effective field theory for nuclear forces



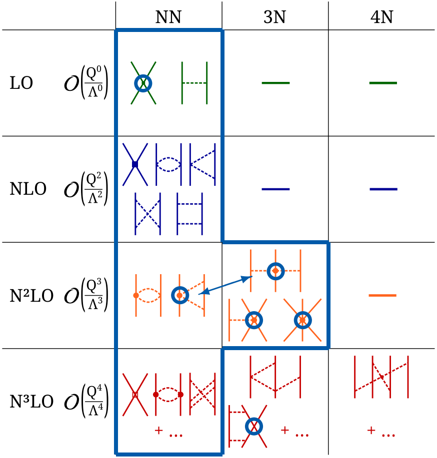
Many-body forces are crucial

Consistent interactions:
same couplings for NN and many-body sector



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Hammer, Kaiser, Machleidt, Meißner,...

Chiral effective field theory for nuclear forces



Many-body forces are crucial

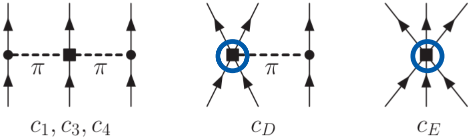
Consistent interactions:
same couplings for NN and many-body sector

In many calculations only N^2LO 3N forces included

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Hammer, Kaiser, Machleidt, Meissner,...

3N Interactions at N^2LO

3N forces: only two new couplings:



Hebeler *et al.*, PRC (2010)

c_D and c_E terms vanish in neutron matter for symmetric regulator
$\rightarrow$ neutron matter exciting lab system

Only long-range two-pion exchange contributes for neutrons,
depends on c_1 and c_3: Krebs *et al.*, PRC (2012)

$$N^2LO: c_1 = -(0.37 - 0.81)\,\text{GeV}^{-1} \text{ and } c_3 = -(2.71 - 3.40)\,\text{GeV}^{-1}$$
$$N^3LO: c_1 = -(0.75 - 1.13)\,\text{GeV}^{-1} \text{ and } c_3 = -(4.77 - 5.51)\,\text{GeV}^{-1}$$

Impact of 3N forces - neutron-rich nuclei

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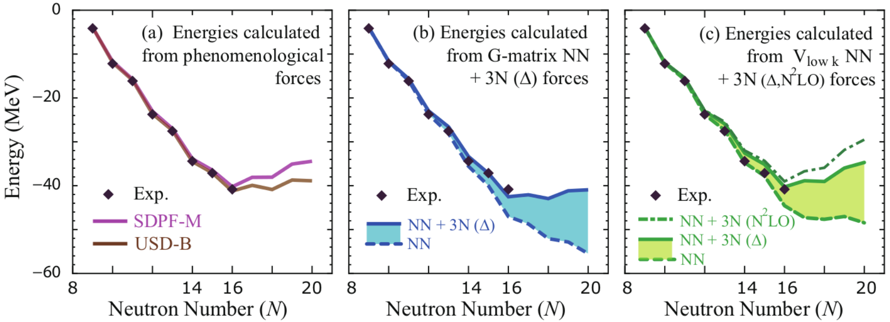
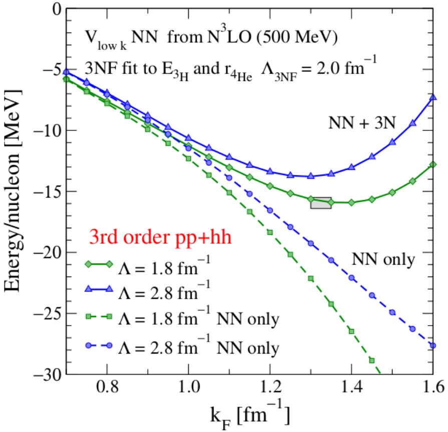


Shell model: Otsuka *et al.*, PRL (2010)

NN **+ 3N** forces: give correct physics of neutron-rich nuclei (oxygen dripline)

see also Hagen *et al.*, PRL (2012), Hergert *et al.*, PRL (2013)

Impact of 3N forces - nuclear matter



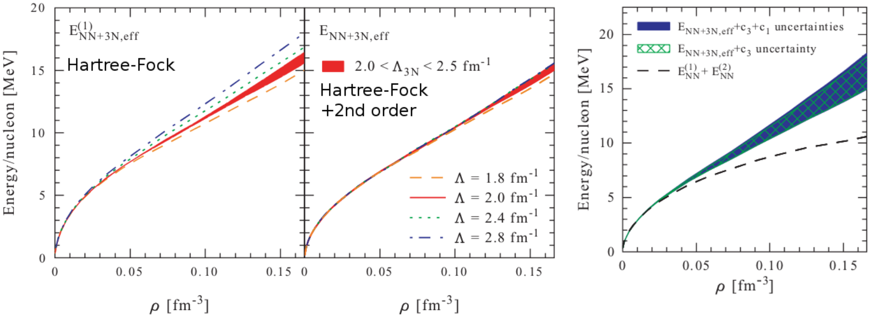
Chiral EFT constrains
nuclear-matter energy per particle

MBPT: Hebeler *et al.*, PRC (2011)

Couplings c_D and c_E fitted to ^3H and ^4He properties

NN **+ 3N** forces: give correct saturation with theoretical uncertainties for symmetric nuclear matter

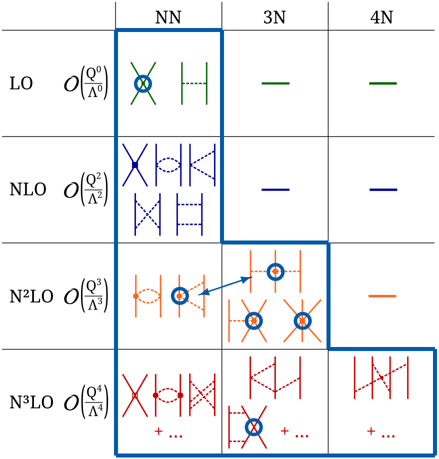
Impact of 3N forces - neutron matter



MBPT: Hebeler *et al.*, PRC (2010)

NN **+ 3N** forces: Uncertainties in many-body forces larger than many-body calculational uncertainties!

Chiral effective field theory for nuclear forces



Recently: first complete N^3LO neutron matter calculation

IT, Krüger, Hebeler, Schwenk, PRL 2013

In neutron matter:

- simpler, only certain parts of the many-body forces contribute
- chiral 3- and 4-neutron forces are predicted to N^3LO

3N Interactions at N^{3}LO

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$$
\begin{aligned}
& 8(-1+z)(1+z)M_\pi^4\left(3zq_1^3+\left(-1+10z^2\right)q_1^2q_3+3z\left(1+2z^2\right)q_1q_3^2+\left(1+2z^2\right)q_3^3\right)+ \\
& 2M_\pi^2q_2^2\left(z\left(-3+z^2\right)q_1^3+\left(3-9z^2\right)q_1^2q_3-z\left(5+z^2\right)q_1q_3^2+\left(-3-3z^2+4z^4\right)q_3^3\right)\right)+ \\
& \frac{zg_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2\left(zq_1+q_3\right)\right)}{128F^6\pi q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^2\left(4M_\pi^2+q_3^2\right)},
\end{aligned}
$$

$$
\begin{aligned}
R_4 = & \frac{A\left(q_2\right)g_A^6q_2^2\left(-2z^2q_1^2q_3+\left(1+z^2\right)q_3^3+2M_\pi^2\left(2zq_1+\left(1+z^2\right)q_3\right)\right)}{128F^6\pi\left(-1+z^2\right)^2q_1^2q_3^3}+ \\
& \frac{A\left(q_1\right)g_A^6\left(-2M_\pi^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+2zq_3^2\right)+q_3\left(2z^2q_1^3+2z^3q_1^2q_3+\left(1-4z^2+z^4\right)q_1q_3^2-2zq_3^3\right)\right)}{128F^6\pi\left(-1+z^2\right)^2q_1q_3^3}- \\
& \frac{A\left(q_3\right)g_A^6}{128F^6\pi\left(-1+z^2\right)^2q_1^2q_3^2}\left(2M_\pi^2\left(-z^2\left(-3+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)+\right. \\
& q_3\left(-\left(z+z^3\right)q_1^3-\left(1-z^2+2z^4\right)q_1^2q_3+z\left(1+z^2\right)q_1q_3^2+\left(1+z^2\right)q_3^3\right)\right)- \\
& \frac{I(4:0,-q_1,q_3;0)g_A^6}{32F^6\left(-1+z^2\right)^2q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^2}\left(q_2^4q_3\left(\left(z+z^3\right)q_1^2+\left(-1+z^2\right)^2q_1q_3-2zq_3^2\right)+\right. \\
& 8(-1+z)(1+z)M_\pi^4\left(3z^2q_1^3+9z^3q_1^2q_3+\left(-2+9z^2+2z^4\right)q_1q_3^2+z\left(2+z^2\right)q_3^3\right)+ \\
& 2M_\pi^2q_2^2\left(z^2\left(-3+z^2\right)q_1^3+\left(2z-8z^3\right)q_1^2q_3+\left(4+5z^2\left(-3+z^2\right)\right)q_1q_3^2+2z\left(-3+z^2+z^4\right)q_3^3\right)\right)+ \\
& \frac{zg_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2\left(zq_1+q_3\right)\right)}{128F^6\pi q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^2\left(4M_\pi^2+q_3^2\right)},
\end{aligned}
$$

$$
\begin{aligned}
R_5 = & \frac{A\left(q_2\right)g_A^6q_2^2\left(-4M_\pi^2\left(q_1+zq_3\right)+q_3\left(2zq_1^2+\left(-1+z^2\right)q_1q_3-2zq_3^2\right)\right)}{128F^6\pi\left(-1+z^2\right)^2q_1q_3^4}- \\
& \frac{A\left(q_3\right)g_A^6}{128F^6\pi\left(-1+z^2\right)^2q_1q_3^3}\left(2M_\pi^2\left(z\left(-3+z^2\right)q_1^2-2\left(1+z^2\right)q_1q_3-2zq_3^2\right)+q_3\left(\left(1+z^2\right)q_1^3+2z^3q_1^2q_3-\right.\right. \\
& \left(1+z^2\right)q_1q_3^2-2zq_3^3\right)\right)+ \\
& \frac{A\left(q_1\right)g_A^6\left(2M_\pi^2\left(2q_1^2+4zq_1q_3+\left(1+z^2\right)q_3^2\right)+q_3\left(-2zq_1^3+\left(1-3z^2\right)q_1^2q_3+2zq_1q_3^2+\left(1+z^2\right)q_3^3\right)\right)}{128F^6\pi\left(-1+z^2\right)^2q_3^4}+ \\
& \frac{I(4:0,-q_1,q_3;0)g_A^6}{32F^6\left(-1+z^2\right)^2\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^3}\left(q_2^4q_3\left(\left(1+z^2\right)q_1^2+z\left(-1+z^2\right)q_1q_3-\left(1+z^2\right)q_3^2\right)+\right. \\
& 8(-1+z)(1+z)M_\pi^4\left(3zq_1^3+\left(-1+10z^2\right)q_1^2q_3+3z\left(1+2z^2\right)q_1q_3^2+\left(1+2z^2\right)q_3^3\right)+ \\
& 2M_\pi^2q_2^2\left(z\left(-3+z^2\right)q_1^3+\left(3-9z^2\right)q_1^2q_3-z\left(5+z^2\right)q_1q_3^2+\left(-3-3z^2+4z^4\right)q_3^3\right)\right)- \\
& \frac{g_A^6M_\pi\left(2M_\pi^2+q_3^2\right)\left(q_2^2q_3+4M_\pi^2\left(zq_1+q_3\right)\right)}{128F^6\pi\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^3\left(4M_\pi^2+q_3^2\right)},
\end{aligned}
$$

$$
\begin{aligned}
R_6 = & \frac{A\left(q_2\right)g_A^6\left(2M_\pi^2+q_2^2\right)}{128F^6\pi}+\frac{A\left(q_1\right)g_A^6\left(2z\left(M_\pi^2+q_1^2\right)q_3+q_1\left(8M_\pi^2+3q_1^2+q_3^2\right)\right)}{128F^6\pi q_1}+ \\
& \frac{A\left(q_3\right)g_A^6\left(2zq_1\left(M_\pi^2+q_3^2\right)+q_3\left(8M_\pi^2+q_1^2+3q_3^2\right)\right)}{128F^6\pi q_3}- \\
& \frac{g_A^6M_\pi}{128F^6\pi q_1\left(4M_\pi^2+q_1^2\right)\left(4\left(-1+z^2\right)M_\pi^2-q_2^2\right)q_3\left(4M_\pi^2+q_3^2\right)}\left(\left(5+z^2\right)q_1^3q_2^2q_3^3+8M_\pi^6\left(z\left(-3+4z^2\right)q_1^2+\right.\right. \\
& 2\left(19-18z^2\right)q_1q_3+z\left(-3+4z^2\right)q_3^2\right)+2M_\pi^4\left(4z\left(-1+z^2\right)q_1^4+\left(77-36z^2\right)q_1^3q_3+2z\left(33+8z^2\right)q_1^2q_3^2+\right. \\
& \left(77-36z^2\right)q_1q_3^3+4z\left(-1+z^2\right)q_3^4\right)+2M_\pi^2q_1q_3\left(\left(10+z^2\right)q_1^4+2z\left(9+2z^2\right)q_1^3q_3+\left(29-7z^2\right)q_1^2q_3^2+\right. \\
& 2z\left(9+2z^2\right)q_1q_3^3+\left(10+z^2\right)q_3^4\right)\right)- \\
& \frac{I(4:0,-q_1,q_3;0)g_A^6\left(2M_\pi^2+q_2^2\right)}{32F^6q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3}\left(q_1q_2^2q_3\left(q_1^2+zq_1q_3+q_3^2\right)+4M_\pi^4\left(zq_1^2-2\left(-2+z^2\right)q_1q_3+zq_3^2\right)+\right. \\
& 2M_\pi^2\left(4q_1q_3\left(q_1^2+q_3^2\right)+z\left(q_1^4+6q_1^2q_3^2+q_3^4\right)\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
R_7 = & \frac{3g_A^6M_\pi\left(2M_\pi^2+q_2^2\right)}{256F^6\pi q_1^2\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)}-\frac{3A\left(q_3\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(\left(1+z^2\right)q_1+2zq_3\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^3}- \\
& \frac{3A\left(q_1\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(2zq_1+\left(1+z^2\right)q_3\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^2q_3}+\frac{3A\left(q_2\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+2zq_3^2\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^3q_3}+ \\
& \frac{3I(4:0,-q_1,q_3;0)g_A^6\left(2M_\pi^2+q_2^2\right)}{64F^6\left(-1+z^2\right)^2q_1^2\left(4\left(-1+z^2\right)M_\pi^2-q_2^2\right)}\left(-q_2^2\left(\left(1+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)+\right. \\
& 4\left(-1+z^2\right)M_\pi^2\left(\left(1+2z^2\right)q_1^2+2z\left(2+z^2\right)q_1q_3+\left(1+2z^2\right)q_3^2\right)\right),
\end{aligned}
$$

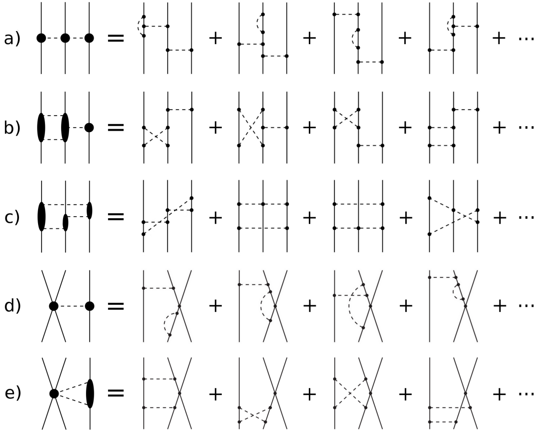
$$
\begin{aligned}
R_8 = & -\frac{3zg_A^6M_\pi\left(2M_\pi^2+q_2^2\right)}{256F^6\pi q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3}+\frac{3zA\left(q_3\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(\left(1+z^2\right)q_1+2zq_3\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^2q_3}+ \\
& \frac{3zA\left(q_1\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(2zq_1+\left(1+z^2\right)q_3\right)}{256F^6\pi\left(-1+z^2\right)^2q_1q_3^2}-\frac{3zA\left(q_2\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(2zq_1^2+\left(1+3z^2\right)q_1q_3+2zq_3^2\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^2q_3^2}- \\
& \frac{3I(4:0,-q_1,q_3;0)zg_A^6\left(2M_\pi^2+q_2^2\right)}{64F^6\left(-1+z^2\right)^2q_1\left(4\left(-1+z^2\right)M_\pi^2-q_2^2\right)q_3}\left(-q_2^2\left(\left(1+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)+\right. \\
& 4\left(-1+z^2\right)M_\pi^2\left(\left(1+2z^2\right)q_1^2+2z\left(2+z^2\right)q_1q_3+\left(1+2z^2\right)q_3^2\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
R_9 = & -\frac{3A\left(q_2\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(\left(1+z^2\right)q_1^2+z\left(3+z^2\right)q_1q_3+\left(1+z^2\right)q_3^2\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^2q_3^2}+ \\
& \frac{3A\left(q_1\right)g_A^6\left(\left(1+z^2\right)q_1^3+2z\left(2+z^2\right)q_1^2q_3-z^2\left(-7+z^2\right)q_1q_3^2+2zq_3^3+2M_\pi^2\left(\left(1+z^2\right)q_1+2zq_3\right)\right)}{256F^6\pi\left(-1+z^2\right)^2q_1q_3^2}+ \\
& \frac{3A\left(q_3\right)g_A^6\left(2zq_1^3-z^2\left(-7+z^2\right)q_1^2q_3+2z\left(2+z^2\right)q_1q_3^2+\left(1+z^2\right)q_3^3+2M_\pi^2\left(2zq_1+\left(1+z^2\right)q_3\right)\right)}{256F^6\pi\left(-1+z^2\right)^2q_1^2q_3}+ \\
& \frac{3I(4:0,-q_1,q_3;0)zg_A^6\left(2M_\pi^2+q_2^2\right)}{64F^6\left(-1+z^2\right)^2q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3}\left(q_2^2\left(-2q_1^2+z\left(-5+z^2\right)q_1q_3-2q_3^2\right)+\right. \\
& 4\left(-1+z^2\right)M_\pi^2\left(\left(2+z^2\right)q_1^2+6zq_1q_3+\left(2+z^2\right)q_3^2\right)\right)-\frac{3zg_A^6M_\pi\left(2M_\pi^2+q_2^2\right)}{256F^6\pi q_1\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3},
\end{aligned}
$$

$$
\begin{aligned}
R_{10} = & \frac{3\left(-1+z^2\right)g_A^6M_\pi\left(2M_\pi^2+q_2^2\right)}{256F^6\pi\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)}+\frac{3A\left(q_2\right)g_A^6\left(2M_\pi^2+q_2^2\right)\left(zq_1+q_3\right)\left(q_1+zq_3\right)}{256F^6\pi\left(-1+z^2\right)q_1q_3}- \\
& \frac{3A\left(q_1\right)g_A^6\left(zq_1^3+\left(1+2z^2\right)q_1^2q_3-z\left(-4+z^2\right)q_1q_3^2+q_3^3+2M_\pi^2\left(zq_1+q_3\right)\right)}{256F^6\pi\left(-1+z^2\right)q_3}- \\
& \frac{3A\left(q_3\right)g_A^6\left(q_1^3-z\left(-4+z^2\right)q_1^2q_3+\left(1+2z^2\right)q_1q_3^2+zq_3^3+2M_\pi^2\left(q_1+zq_3\right)\right)}{256F^6\pi\left(-1+z^2\right)q_1}+ \\
& \frac{3I(4:0,-q_1,q_3;0)g_A^6\left(2M_\pi^2+q_2^2\right)}{64F^6\left(-1+z^2\right)\left(4\left(-1+z^2\right)M_\pi^2-q_2^2\right)}\left(-q_2^2\left(q_1^2-z\left(-3+z^2\right)q_1q_3+q_3^2\right)+\right. \\
& 4\left(-1+z^2\right)M_\pi^2\left(\left(1+z^2\right)q_1^2+4zq_1q_3+\left(1+z^2\right)q_3^2\right)\right),
\end{aligned}
$$

$$
\begin{aligned}
R_{11} = & -\frac{A\left(q_2\right)g_A^6q_2^2\left(4M_\pi^2+q_1^2+q_3^2\right)}{256F^6\pi\left(-1+z^2\right)q_1^2q_3^2}+ \\
& \frac{A\left(q_3\right)g_A^6\left(2M_\pi^2\left(\left(-1+z^2\right)q_1^2+2zq_1q_3+2q_3^2\right)+q_3\left(zq_1^3+\left(-1+2z^2\right)q_1^2q_3+zq_1q_3^2+q_3^3\right)\right)}{256F^6\pi\left(-1+z^2\right)q_1^2q_3^2}+ \\
& \frac{A\left(q_1\right)g_A^6\left(2M_\pi^2\left(2q_1^2+2zq_1q_3+\left(-1+z^2\right)q_3^2\right)+q_1\left(q_1^3+zq_1^2q_3+\left(-1+2z^2\right)q_1q_3^2+zq_3^3\right)\right)}{256F^6\pi\left(-1+z^2\right)q_1^2q_3^2}- \\
& \frac{I(4:0,-q_1,q_3;0)g_A^6q_2^2}{\left(64F^6\left(-1+z^2\right)q_1^2\left(-4\left(-1+z^2\right)M_\pi^2+q_2^2\right)q_3^2\right)}\left(-\left(2M_\pi^2+q_1^2\right)\left(2M_\pi^2+q_3^2\right)\left(4M_\pi^2+q_1^2+q_3^2\right)+\right. \\
& 2z^3q_1q_3\left(-4M_\pi^4+q_1^2q_3^2\right)+z^2\left(4M_\pi^2+q_1^2+q_3^2\right)\left(4M_\pi^4+3q_1^2q_3^2+2M_\pi^2\left(q_1^2+q_3^2\right)\right)+ \\
& zq_1q_3\left(8M_\pi^4+q_1^4+q_3^4+4M_\pi^2\left(q_1^2+q_3^2\right)\right)\right)-
\end{aligned}
$$

3N Interactions at N^{3}LO



2π exchange

2π-1π exchange

pion ring (involved)

1π-exchange contact
(vanishes)

2π-exchange contact
(2-body-contact C_T)

+relativistic corrections
(2-body-contacts C_T, C_S)

Bernard *et al.*, PRC (2008) & PRC (2011)

4N Interactions at N^3LO

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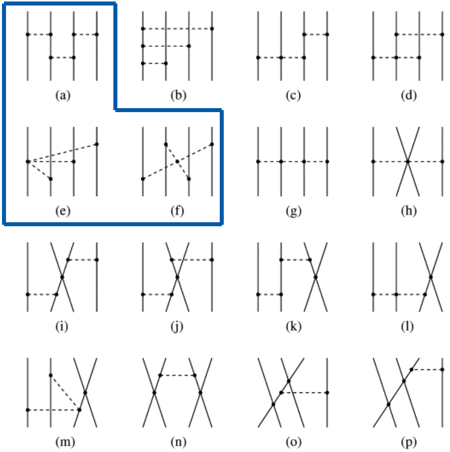

Epelbaum, PLB (2006)

Less involved than 3N forces (no loops)

In neutron matter only three diagrams contribute due to isospin structure

4N forces provide small contributions

Fiorilla *et al.*, NPA (2012)

Kaiser, EPJ A (2012)

McManus, Riska, PLB (1980)

Energy per Particle in Hartree-Fock Approximation

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Hartree-Fock is a very good approximation for the energy per particle for 3N forces at N^2LO [Hebeler *et al.*, PRC (2010)]

Expected to be even better at higher orders!

$$\frac{E}{N} = \frac{1}{\rho n!} \mathrm{tr}_{\sigma_1} \cdots \mathrm{tr}_{\sigma_n} \int \frac{d^3 k_1}{(2\pi)^3} \cdots \int \frac{d^3 k_n}{(2\pi)^3} f_R^2 n_{\mathbf{k}_1} \cdots n_{\mathbf{k}_n}$$
$$\times \langle 1 \cdots n | \mathcal{A}_n \sum_{i_1 \neq \ldots \neq i_n} V(i_1, \ldots, i_n) | 1 \cdots n \rangle$$

all exchange terms included

$$f_R = \exp\left[-\left(\frac{k_1^2 + \ldots + k_n^2 + \mathbf{k}_1 \cdot \mathbf{k}_2 + \ldots + \mathbf{k}_{n-1} \cdot \mathbf{k}_n}{n \Lambda_R^2} \right)^{n_{\mathrm{exp}}} \right], \quad n_{\mathrm{exp}} = 4$$

cutoff variation: $\Lambda_R = (2 - 2.5)\,\mathrm{fm}^{-1}$

Individual N^{3}LO many-body contributions

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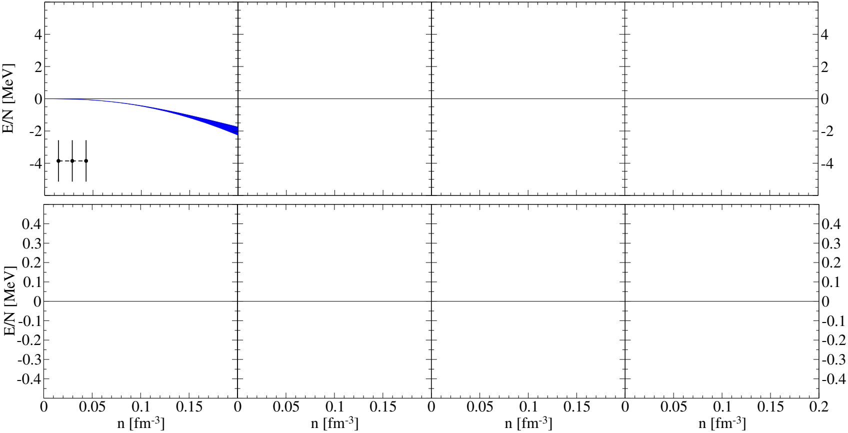
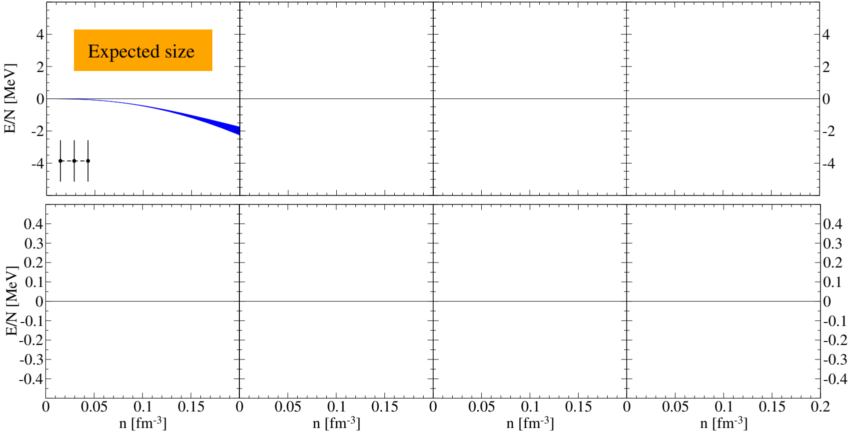


IT, Krüger, Hebeler, Schwenk, PRL (2013)

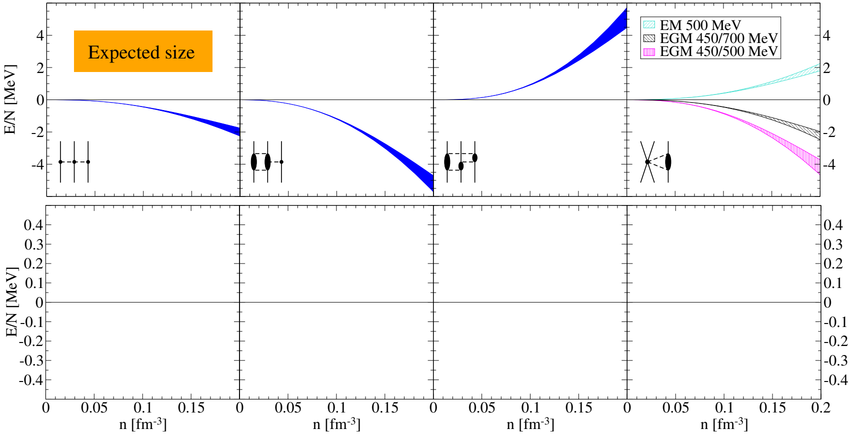
Individual N^3LO many-body contributions



IT, Krüger, Hebeler, Schwenk, PRL (2013)

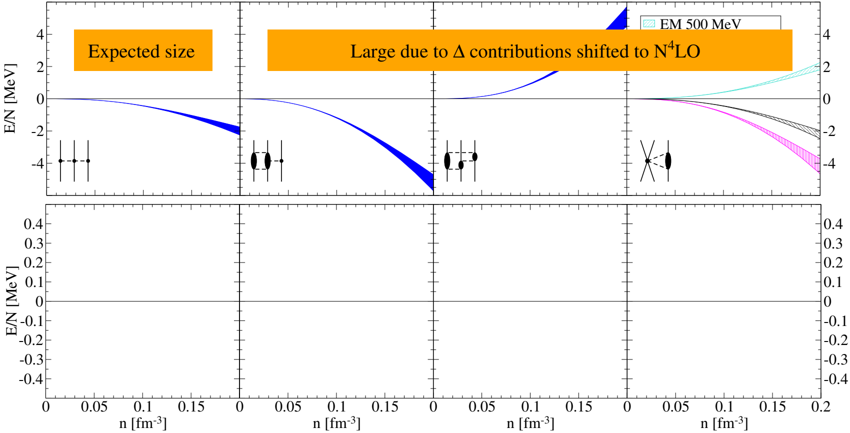
Individual N^{3}LO many-body contributions

TECHNISCHE UNIVERSITÄT DARMSTADT



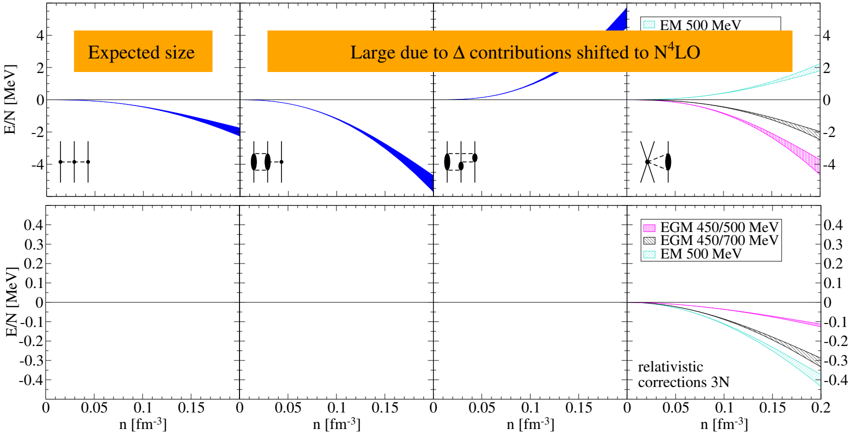
IT, Krüger, Hebeler, Schwenk, PRL (2013)

Individual N^3LO many-body contributions



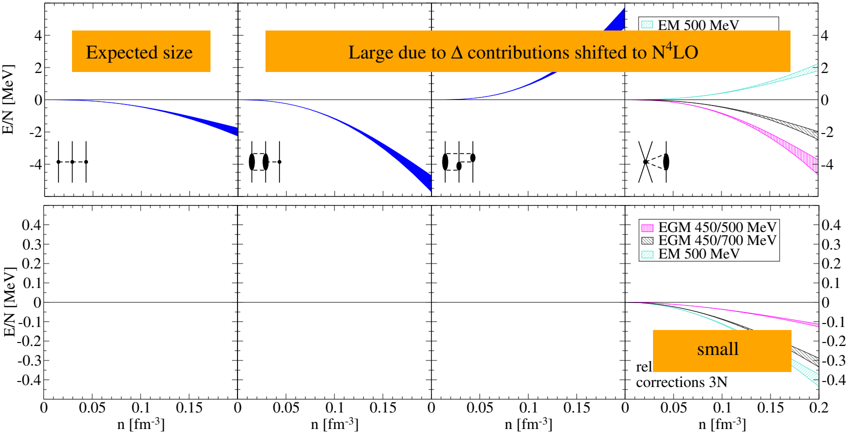
IT, Krüger, Hebeler, Schwenk, PRL (2013)

Individual N^3LO many-body contributions



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Individual N^{3}LO many-body contributions



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Individual N^3LO many-body contributions

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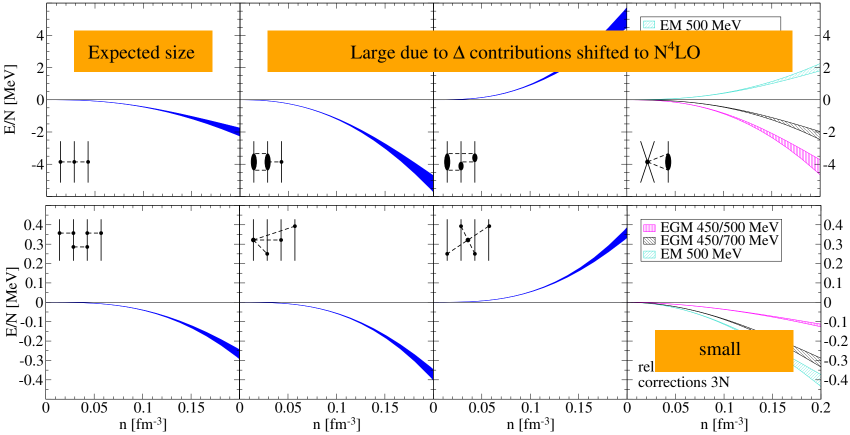
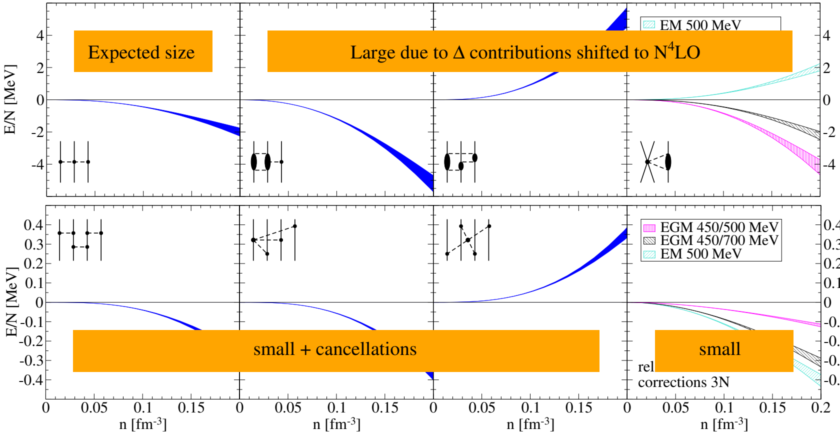


IT, Krüger, Hebeler, Schwenk, PRL (2013)

Individual N^3LO many-body contributions



IT, Krüger, Hebeler, Schwenk, PRL (2013)

Results: Individual 3N Contributions

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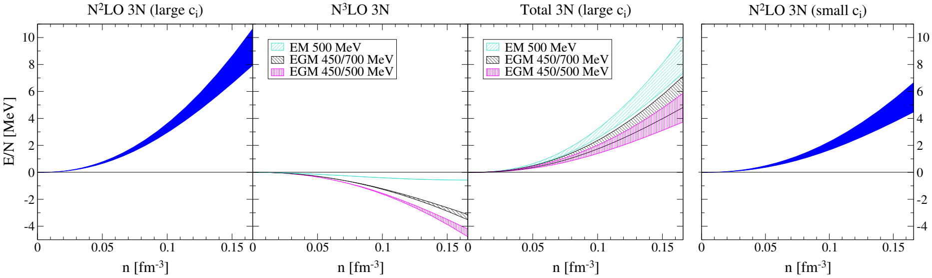
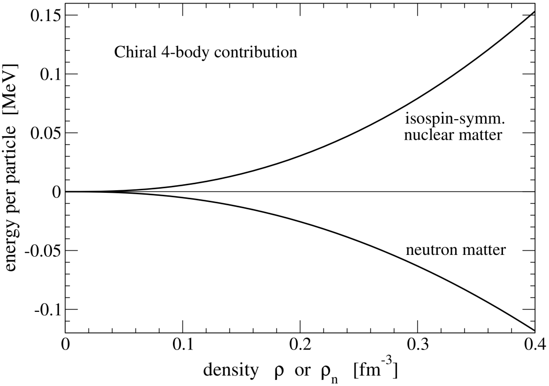
3N N^3LO result at n_0: $-(3.2-4.8)\,\text{MeV}/\text{N}$ for EGM potentials
$-0.5\,\text{MeV}/\text{N}$ for EM potential



Comparison: 4N Results with Fiorilla *et al.*



- 4NF in neutron matter studied by Fiorilla *et al.*
- but only last two diagrams:

$$\frac{E}{N}(\rho_0) = -16.2\,\text{keV}$$

- all topologies:

$$\frac{E}{N}(\rho_0) = -174 \pm 10\,\text{keV}$$

[S. Fiorilla, N. Kaiser, W. Weise, Nucl. Phys. A **880**, 65 (2012)]

Chiral EFT for neutron matter

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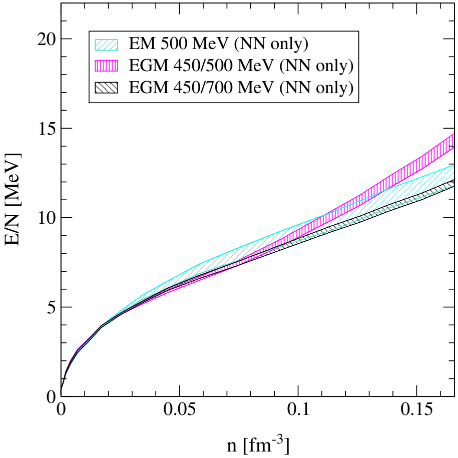
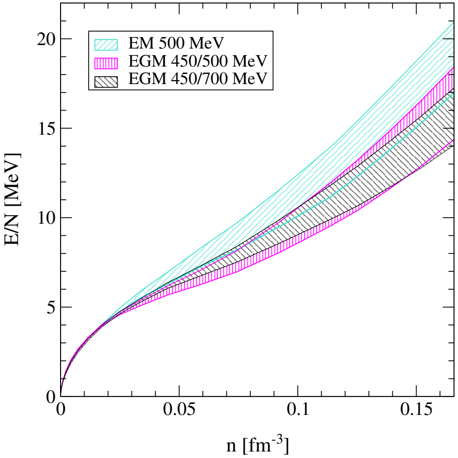


NN forces only:

- uncertainties due to many-body calculation small

Chiral EFT for neutron matter



NN forces only:

- uncertainties due to many-body calculation small

Many-body forces:

- have large impact on neutron-matter energy
- uncertainties dominated by 3N forces (large c_3)

Chiral EFT for neutron matter

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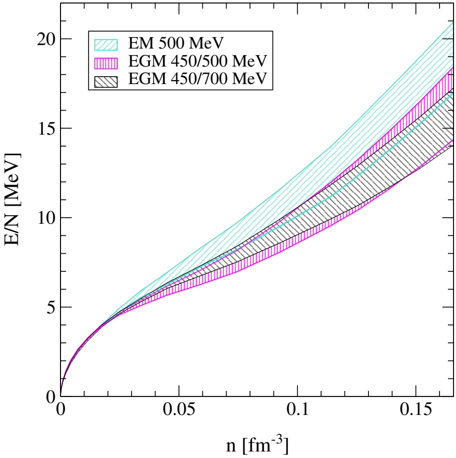

Bands include:

- $\Lambda = (2 - 2.5)\,\mathrm{fm}^{-1}$
- many-body uncertainties
- $c_1 = -(0.75 - 1.13)\,\mathrm{GeV}^{-1}$
  $c_3 = -(4.77 - 5.51)\,\mathrm{GeV}^{-1}$
  Krebs *et al.*, PRC (2012)

Final N^3LO result:

$$\frac{E}{N}(n_0) = (14.1 - 21.0)\,\mathrm{MeV}$$

Chiral EFT for neutron matter

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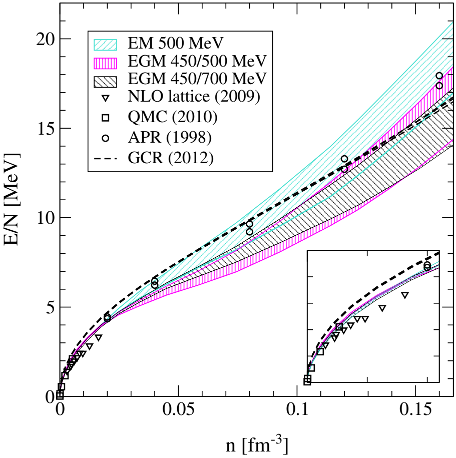

Universal properties at low densities

- agreement with Quantum Monte Carlo and NLO lattice calculations

  Gezerlis, Carlson, PRC (2010)

  Epelbaum *et al.*, EPJ A (2009)

Good agreement with other calculations at higher densities

- but in those no theoretical uncertainties

  Akmal *et al.*, PRC (1998)

  Gandolfi *et al.*, PRC (2012)

From N^2LO to N^3LO

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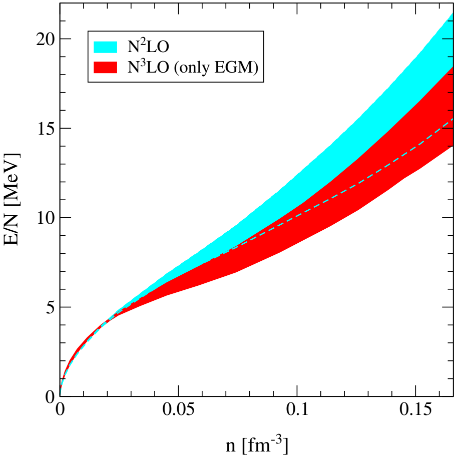

- Final N^2LO result:

$$\frac{E}{N}(n_0) = (15.5 - 21.4)\,\text{MeV}$$

- Final N^3LO result (EGM only):

$$\frac{E}{N}(n_0) = (14.1 - 18.4)\,\text{MeV}$$

- E/N reduced from N^2LO to N^3LO
- Theoretical uncertainty reduced

Chiral EFT for neutron matter

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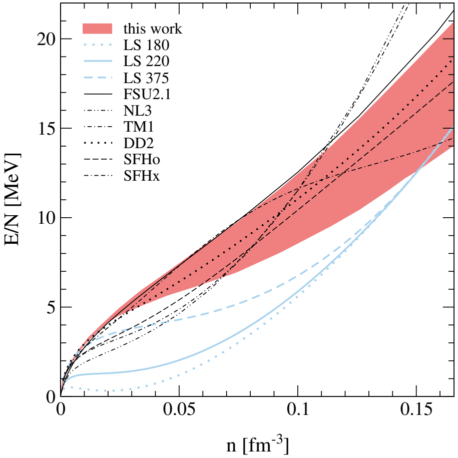

Lines from Hempel, Lattimer, G. Shen

Chiral EFT constrains neutron-matter energy per particle

N^3LO many-body forces add more density dependence

Rules out many model equations of state

Impact on Symmetry Energy

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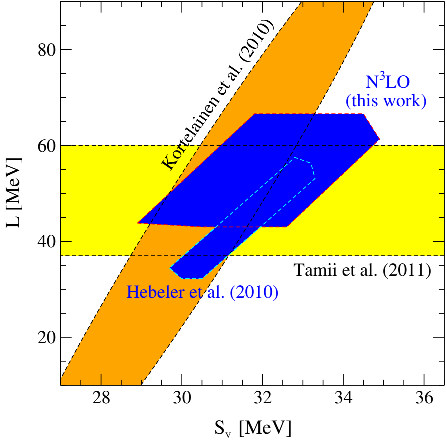


Neutron matter band puts constraints on symmetry energy and its density dependence

Hebeler *et al.*, PRL (2010)

- $S_V = 28.9 - 34.9$ MeV
- $L = 43.0 - 66.6$ MeV

Good agreement with experimental constraints

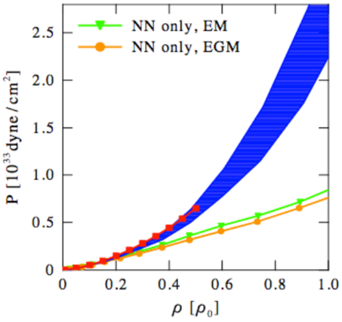
Dipole polarizability - Tamii *et al.*, PRL (2011)

Nuclear masses - Kortelainen *et al.*, PRC (2010)

Impact on Neutron Stars

Equation of state for neutron star matter: extend results to small $Y_{e,p}$

Hebeler et al., PRL (2010) and APJ (2013)



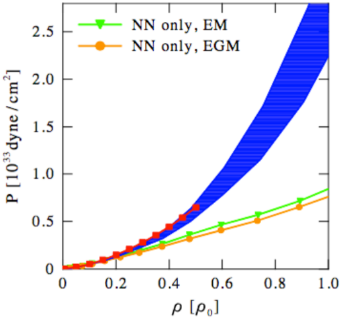
Agrees with standard crust EOS
after inclusion of many-body forces

Impact on Neutron Stars

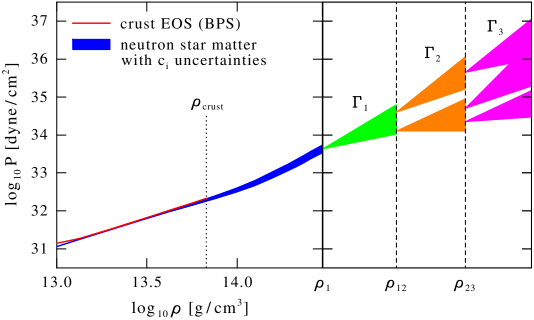
TECHNISCHE UNIVERSITÄT DARMSTADT

Equation of state for neutron star matter: extend results to small $Y_{e,p}$

Hebeler et al., PRL (2010) and APJ (2013)



Agrees with standard crust EOS after inclusion of many-body forces

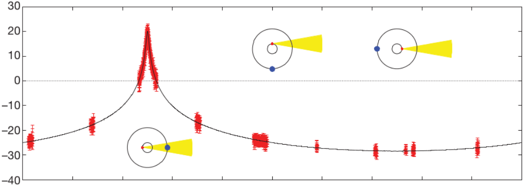


Extend to higher densities using polytropic expansion

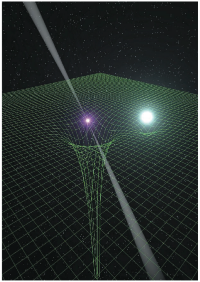
Impact on Neutron Stars

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Nature (2010)



A Massive Pulsar in a Compact Relativistic Binary

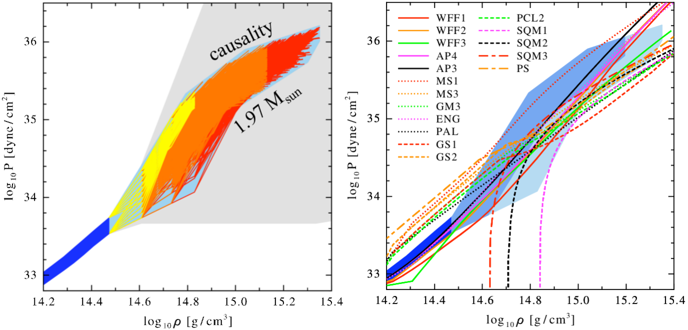
John Antoniadis,* Paulo C. C. Freire, Norbert Wex, Thomas M. Tauris, Ryan S. Lynch, Marten H. van Kerkwijk, Michael Kramer, Cees Bassa, Vik S. Dhillon, Thomas Driebe, Jason W. T. Hessels, Victoria M. Kaspi, Vladislav I. Kondratiev, Norbert Langer, Thomas R. Marsh, Maura A. McLaughlin, Timothy T. Pennucci, Scott M. Ransom, Ingrid H. Stairs, Joeri van Leeuwen, Joris P. W. Verbiest, David G. Whelan

$$M_{PSR} = 2.01 M_{\odot}$$

Science (2013)

Impact on Neutron Stars

Constrain resulting EOS: causality and observed 1.97 $M_\odot$ neutron star



Chiral EFT interactions provide strong constraints for EOS

Rule out many model equations of state

Impact on Neutron Stars

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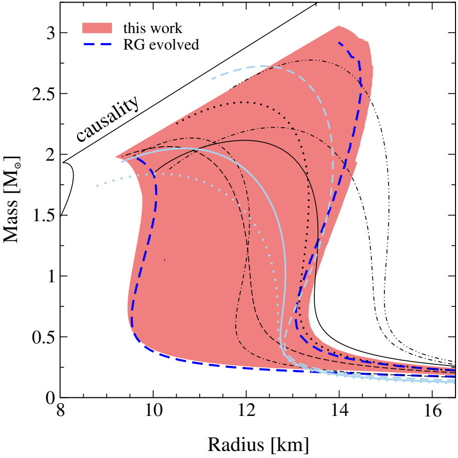


Radius for 1.4 $M_\odot$ neutron star:

- $R = 9.7 - 13.9\,\text{km}$

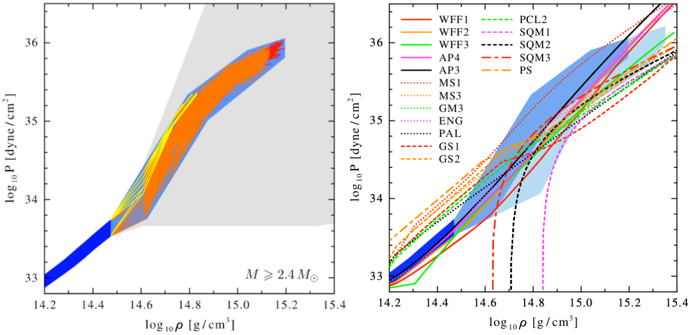
Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14km)

Uncertainties from many-body forces and polytropic expansion

Impact on Neutron Stars

If a 2.4 $M_\odot$ neutron star was observed:



Even stronger constraints on high-density equation of state

Impact on Neutron Stars

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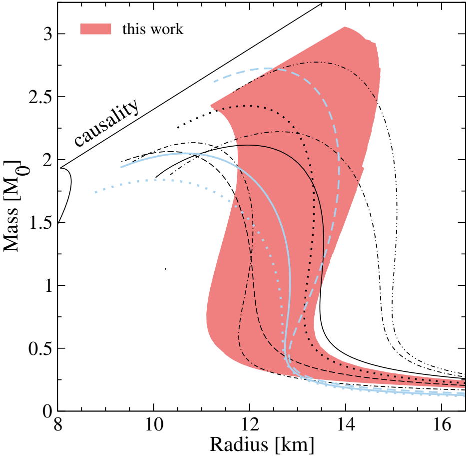


Radius for 1.4 $M_\odot$ neutron star:

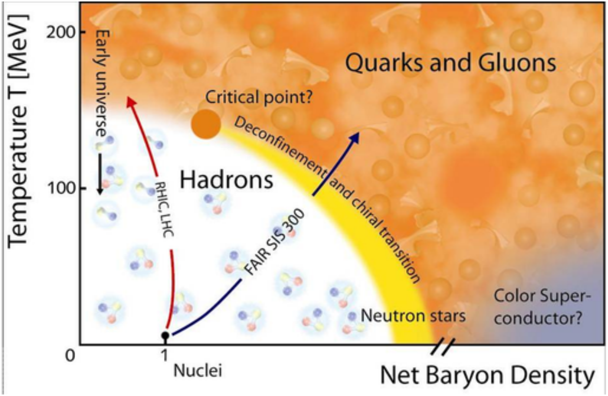
- $R = 11.5 - 13.9$ km

Maximum mass neutron star:

- $M_{max} \leq 3.05 M_\odot$ (14km)

Uncertainties from many-body forces and polytropic expansion

Chiral condensate



Chiral condensate is order parameter for chiral phase transition

Chiral condensate

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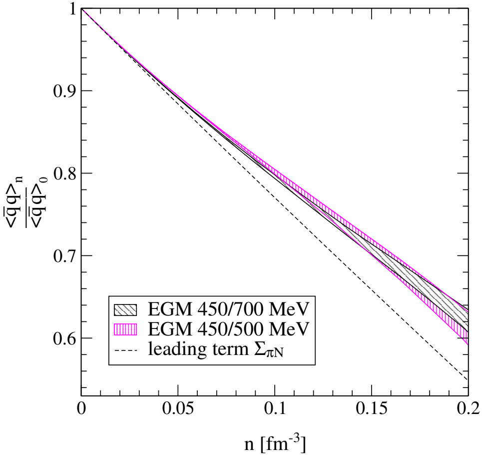


Krüger, IT,Hebeler, Friman, Schwenk, PLB (2013)

Include:

- Explicit m_π variation
- No m_π-dependent contacts

Constraints on chiral condensate:

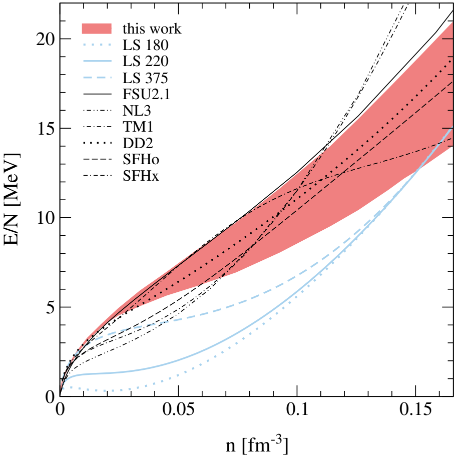
[W. Weise, Prog. Part. Nucl. Phys. (2012)]

- EGM 450/500: 66.8-69.3% at n_0
- EGM 450/700: 67.2-68.9% at n_0

Without interaction:

- 62.3% at n_0

Chiral EFT for neutron matter



Lines from Hempel, Lattimer, G. Shen

Neutron matter from chiral EFT:

- chiral EFT constrains neutron matter equation of state and astrophysical observables
  Krüger, IT, Hebeler, Schwenk, PRC (2013)

So far used in perturbative calculations

- need for nonperturbative benchmark

Perturbativeness of NN potentials

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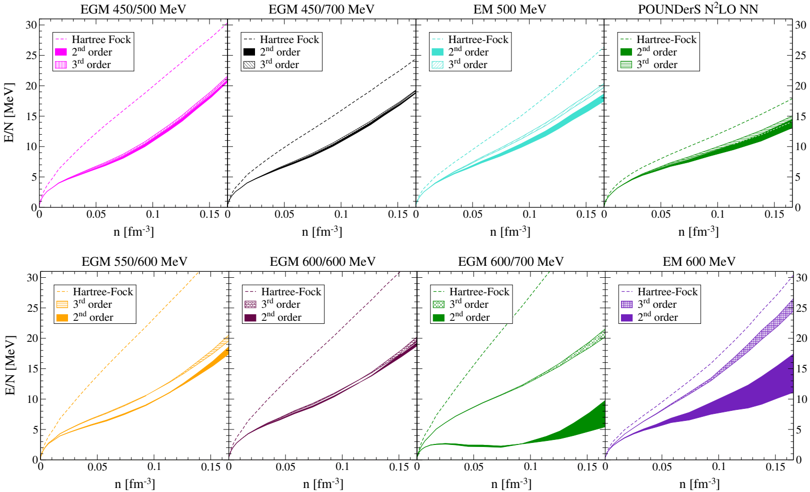

Krueger, IT, Hebeler, Schwenk, PRC 88 (2013)

Quantum Monte Carlo method

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Solve the Schrödinger equation

$$H \,|\, \Psi(R,\tau) \,\rangle = -\frac{\partial}{\partial \tau} \,|\, \Psi(R,\tau) \,\rangle \,, \qquad \tau = i \cdot t,$$

using the general solution

$$|\, \Psi(R,\tau) \,\rangle = \int \mathrm{d}^3 R' \, G(R,R',\tau) \,|\, \Psi(R',0) \,\rangle \,.$$

with the propagator

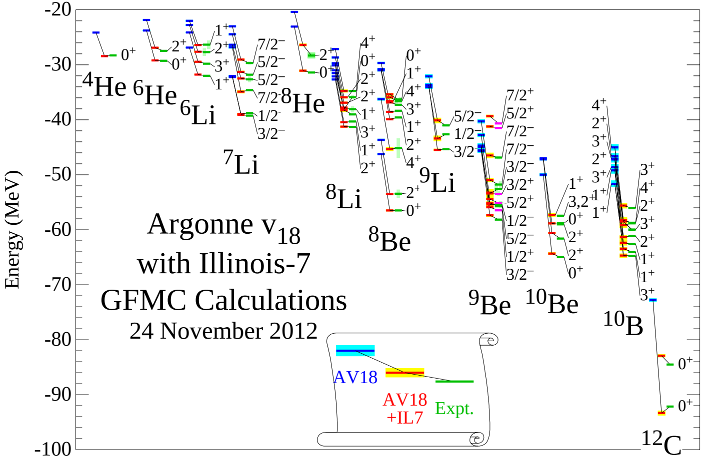
$$G(R,R',\tau) = \langle\, R \,|\, e^{-H\tau} \,|\, R' \,\rangle \approx \langle\, R \,|\, \left(e^{-\frac{V}{2}\Delta\tau} e^{-T\Delta\tau} e^{-\frac{V}{2}\Delta\tau} \right)^N \,|\, R' \,\rangle$$

$\rightarrow$ Analytically solvable only for local potentials:

$$\langle\, R \,|\, e^{-\sum \frac{p_i^2}{2m}\Delta\tau} \,|\, R' \,\rangle \, e^{-\frac{V(R)+V(R')}{2}\Delta\tau}$$

Carlson, Gandolfi, Gezerlis, PTEP (2012); Carlson, Schiavilla, RMP (1998)

Quantum Monte Carlo method

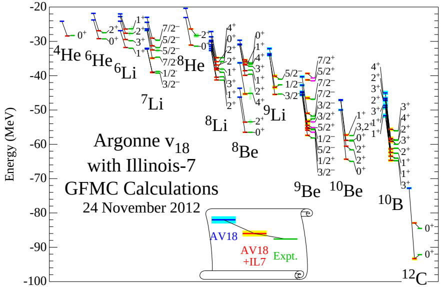


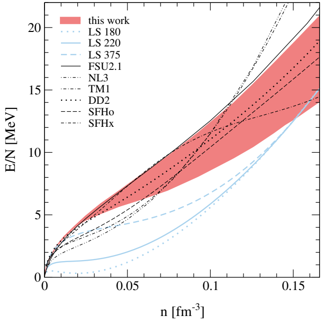
Carlson, Pieper, Wiringa et al. (2012)

Motivation

Chiral EFT:

- Systematic
- EFT includes nonlocal interactions





Quantum Monte Carlo:

- Nonperturbative
- Need: local interactions

→ Combination of these approaches would be powerful!

Sources of nonlocality

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Chiral EFT is momentum space expansion, two momenta:

- momentum transfer $q = p' - p$
- momentum transfer in exchange channel $k = (p' + p)/2$

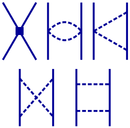
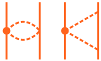
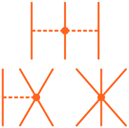
Locality means: $$\langle\, r' \mid \hat{V} \mid r \,\rangle = \begin{cases} V(r)\,\delta(r - r'), & \text{if local} \\ V(r', r), & \text{if nonlocal} \end{cases}$$

After Fourier transformation, $q \rightarrow r$ but $k \rightarrow$ derivatives $\rightarrow$ nonlocal

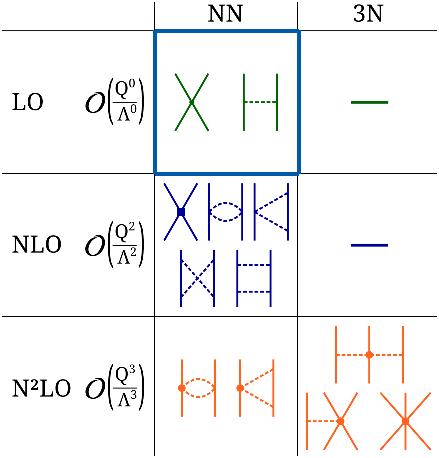
Two sources of nonlocality:

- usual regulator on relative momenta $f(p) = e^{-(p/\Lambda)^{2n}}$ and $f(p')$
- k-dependent contact interactions

Local chiral potential to N^2LO

		NN	3N
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$		—
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$		—
N^2LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$		

Local chiral potential to N^{2}LO



Leading order:

$$V^{(0)} = V^{(0)}_{\text{cont}} + V_{\text{OPE}}$$

Contact potential:

$$V^{(0)}_{\text{cont}} = \alpha_1 + \alpha_2\, \sigma_1 \cdot \sigma_2 + \alpha_3\, \tau_1 \cdot \tau_2 + \alpha_4\, \sigma_1 \cdot \sigma_2\, \tau_1 \cdot \tau_2$$

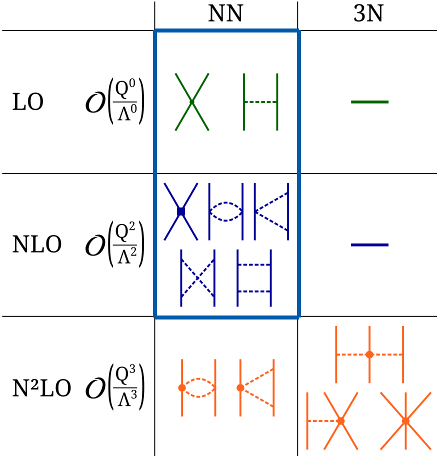
$\rightarrow$ only two independent (Pauli principle):

$$V^{(0)}_{\text{cont}} = C_S + C_T\, \sigma_1 \cdot \sigma_2$$

Regulate OPE in coordinate space:

$$V_{\text{OPE}}(r)\left(1 - e^{-(r/R_0)^4}\right)$$

Local chiral potential to N^2LO



Next-to-leading order:

$$V^{(2)} = V^{(2)}_{\text{cont}} + V^{(2)}_{\text{TPE}}$$

$\rightarrow$ Regulate TPE in coordinate space

$$\begin{aligned}
V^{(2)}_{\text{cont}} = {} & \gamma_1\, q^2 + \gamma_2\, q^2\, \sigma_1 \cdot \sigma_2 + \gamma_3\, q^2\, \tau_1 \cdot \tau_2 \\
& + \gamma_4\, q^2 \tau_1 \cdot \tau_2 \sigma_1 \cdot \sigma_2 \\
& + \gamma_5\, k^2 + \gamma_6\, k^2\, \sigma_1 \cdot \sigma_2 + \gamma_7\, k^2\, \tau_1 \cdot \tau_2 \\
& + \gamma_8\, k^2 \tau_1 \cdot \tau_2 \sigma_1 \cdot \sigma_2 \\
& + (\sigma_1 + \sigma_2)(\mathbf{q} \times \mathbf{k})(\gamma_9 + \gamma_{10}\, \tau_1 \cdot \tau_2) \\
& + (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})(\gamma_{11} + \gamma_{12}\, \tau_1 \cdot \tau_2) \\
& + (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})(\gamma_{13} + \gamma_{14}\, \tau_1 \cdot \tau_2)
\end{aligned}$$

Local chiral potential to N^{2}LO

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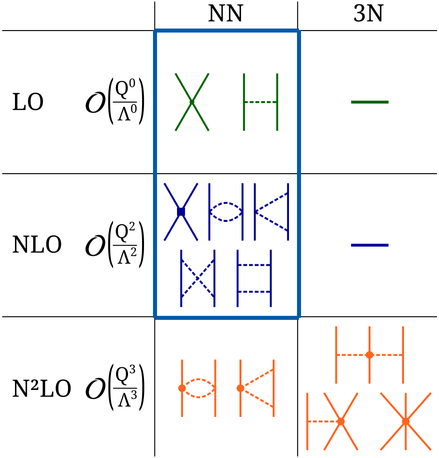

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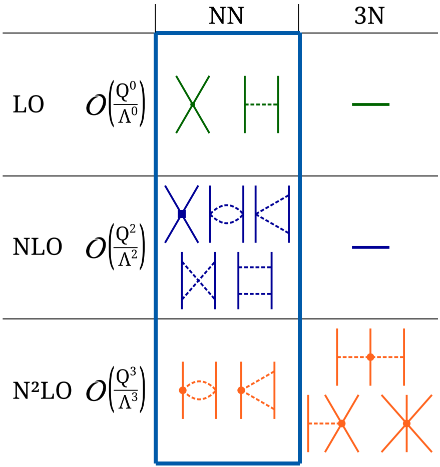
\end{aligned}$$

Local chiral potential to N^2LO



Next-to-next-to-leading order:

$$V^{(3)} = V^{(3)}_{\mathrm{TPE}} + V^{(3)}_{\mathrm{IB}}$$

Regulator on contact interactions:

$$\int \frac{d\mathbf{q}}{(2\pi)^3} C f_{\mathrm{local}}(q^2) e^{i\mathbf{q}\cdot\mathbf{r}} = C \frac{e^{-(r/R_0)^4}}{\pi \Gamma\left(\frac{3}{4}\right) R_0^3}$$

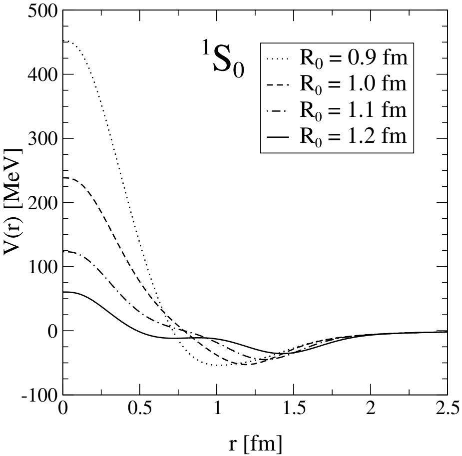
Variation of cutoff $R_0 = 1.0 - 1.2\,\mathrm{fm}$

$\approx (500 - 400\,\mathrm{MeV})$

$\Lambda_{\mathrm{SFR}} = 1000 - 1400\,\mathrm{MeV}$

Fitting of C_i to NN phase shifts

Potential in $^1S_0$ channel



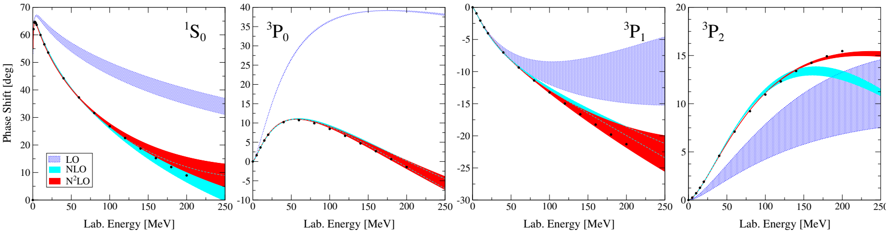
Note: potential is not observable!

Potential in the $^1S_0$ channel

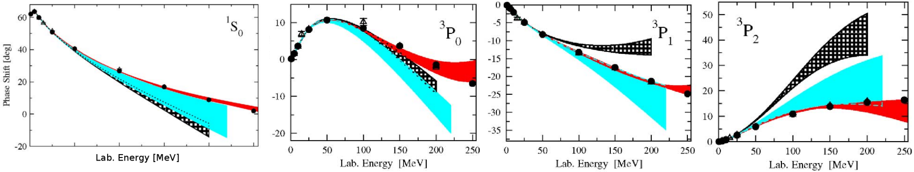
- neutron-neutron system
- softening of the potential when lowering the momentum space cutoff (increasing coordinate space cutoff)

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

Phase shifts

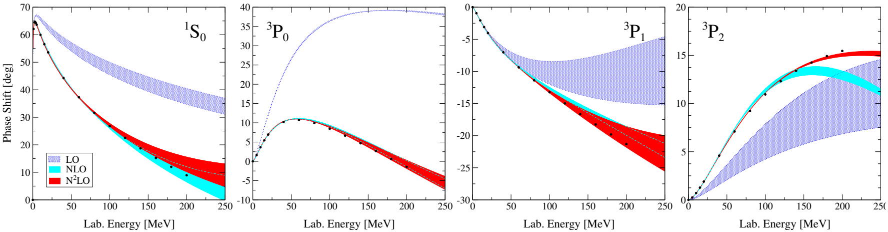


Comparison to EGM momentum space N^{2}LO potentials:

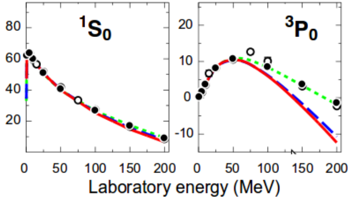


Epelbaum, Glöckle, Meißner, Nucl. Phys. A (2005)

Phase shifts



Comparison to POUNDERS N^{2}LO potential:



Ekström et al., PRL 110 (2013)

AFDMC results for neutron matter

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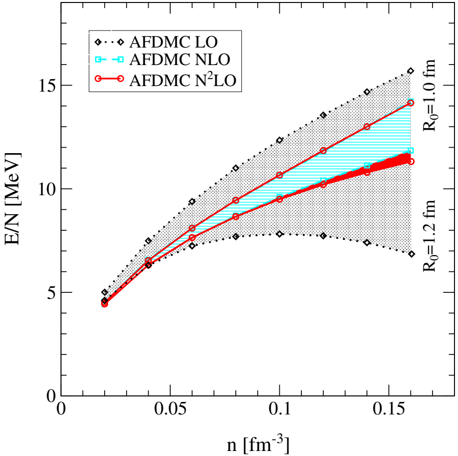

Auxiliary Field Diffusion Monte Carlo:

- so far only NN interaction
- statistical uncertainty smaller than points
- order-by-order convergence up to saturation density
- NLO $\approx$ N^2LO due to large c_i

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation, and PRL (2013)

AFDMC results for neutron matter

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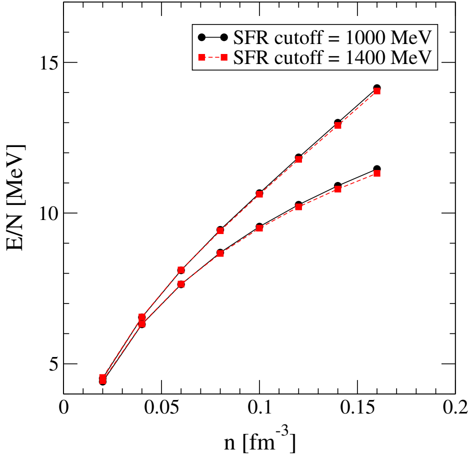

Auxiliary Field Diffusion Monte Carlo:

- Check influence of SRF cutoff: Variation 1.0 – 1.4 GeV
- Effect less than 0.2 MeV

→ SFR cutoff has negligible effect

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

MBPT results for neutron matter

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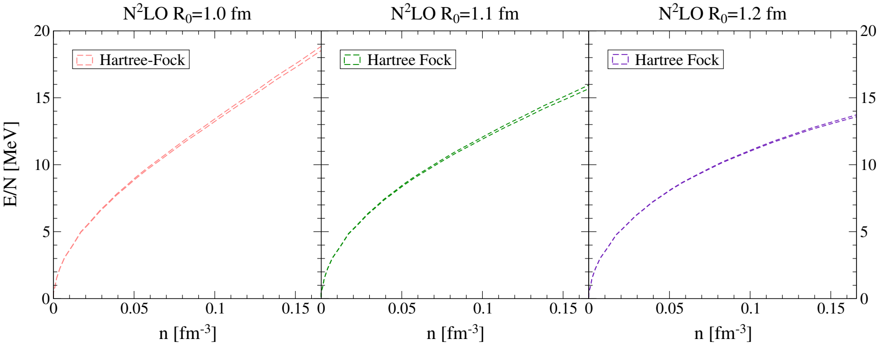
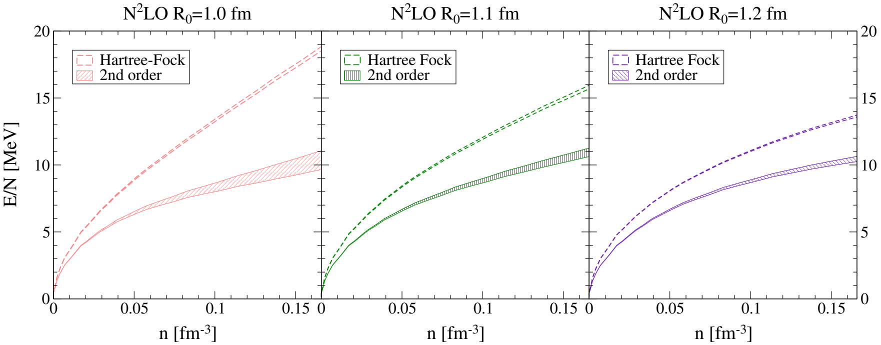


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

Many-body perturbation theory:

- Hartree-Fock

MBPT results for neutron matter

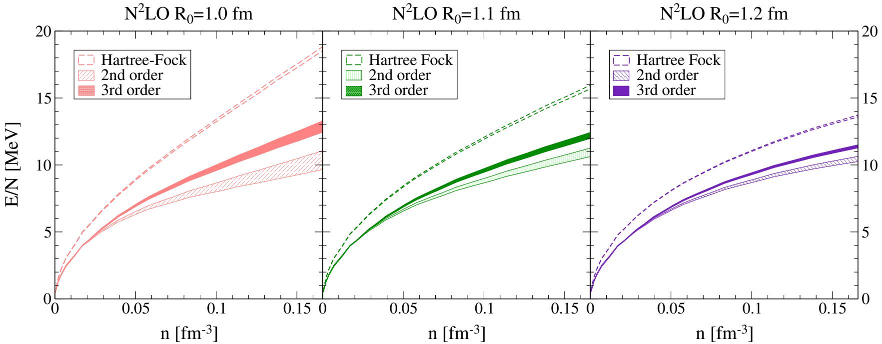


Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

Many-body perturbation theory:

- Hartree-Fock +2nd order
- Bands correspond to different single-particle spectra (free, HF)

MBPT results for neutron matter



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

Many-body perturbation theory:

- Hartree-Fock +2nd order +3rd order (pp+hh)
- Bands correspond to different single-particle spectra (free, HF)

MBPT results for neutron matter

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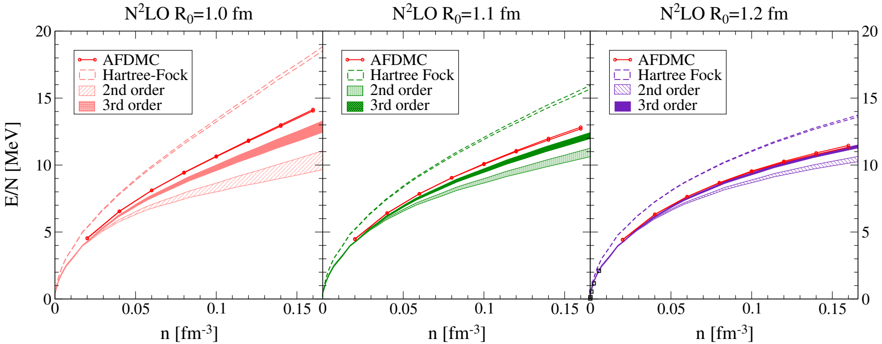

Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, in preparation

Many-body perturbation theory:

- excellent agreement with AFDMC for low-cutoff potentials (R_0=1.2 fm (400 MeV))
- validates perturbative calculations for those interactions

MBPT results for neutron matter

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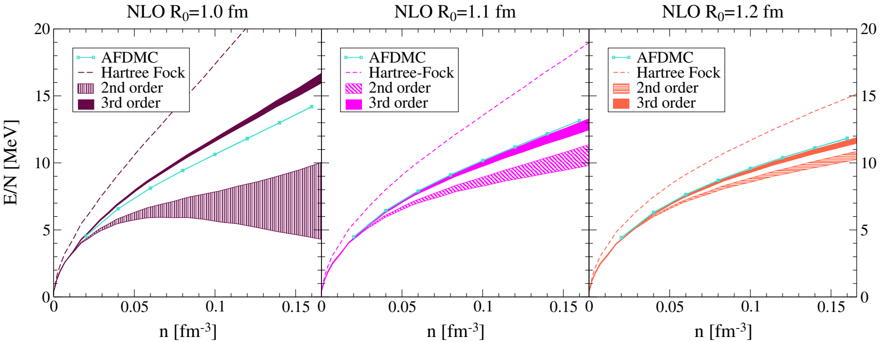

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Summary

Chiral effective field theory:

- Provides strong constraints for symmetry energy, neutron star EOS, chiral condensate
- Inclusion of many-body forces is frontier

First QMC calculation with chiral EFT interactions

- Local N^2LO chiral EFT potential
- Low-cutoff MBPT results in excellent agreement

Next (during the visit at LANL):

- Inclusion of leading 3N forces
- Use local potential in calculations of nuclei

Thanks

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A. Gezerlis

Forschungszentrum Jülich:
A. Nogga

European Research Council
Established by the European Commission

Thanks for your attention!