# Order, Chaos and Quasi Symmetries in a First-Order Quantum Phase Transition

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M. Macek, A. Leviatan, Phys. Rev. C **84** (2011) 041302(R)

A. Leviatan, M. Macek, Phys. Lett. B **714** (2012) 110

M. Macek, A. Leviatan, arXiv:1404.0604 [nucl-th]

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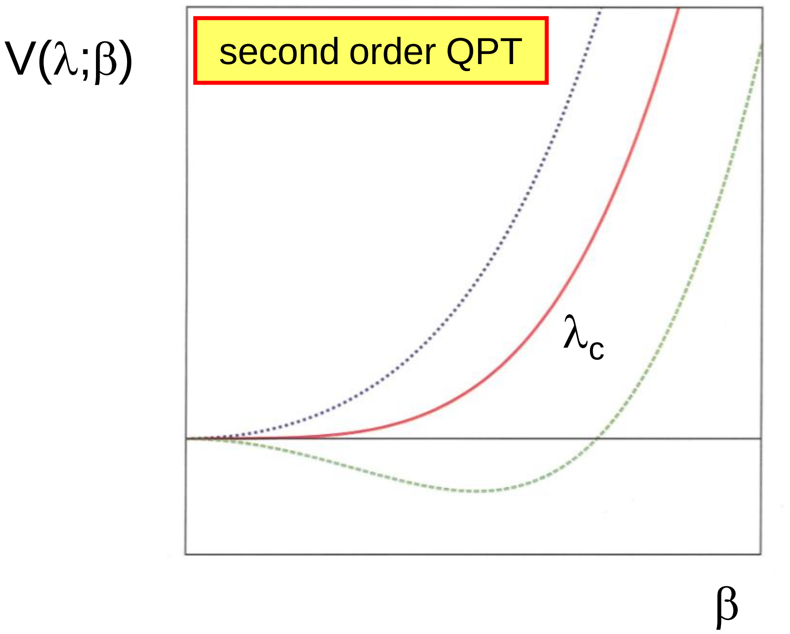
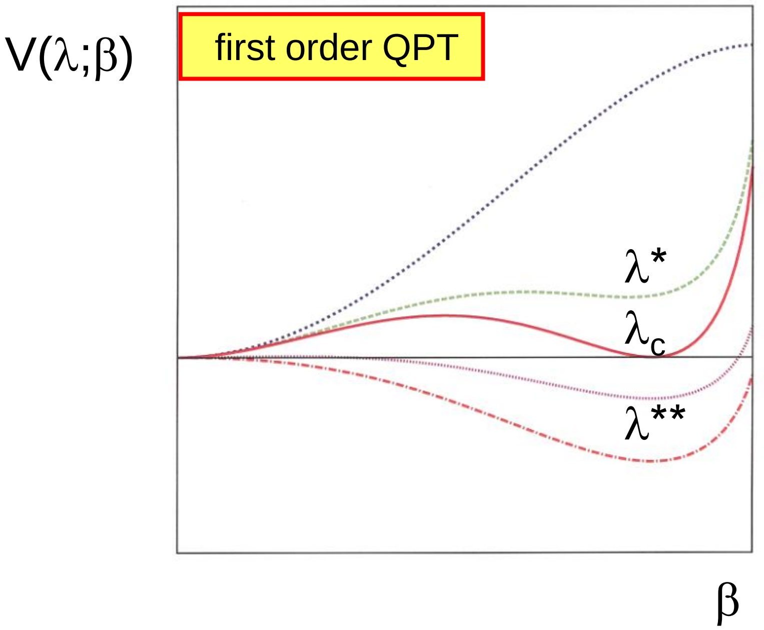
*Los Alamos, April 15, 2014*

# Quantum Phase Transition (QPT)

$H(\lambda)$ control parameter $\lambda$

$V(\lambda;\beta)$ Landau potential

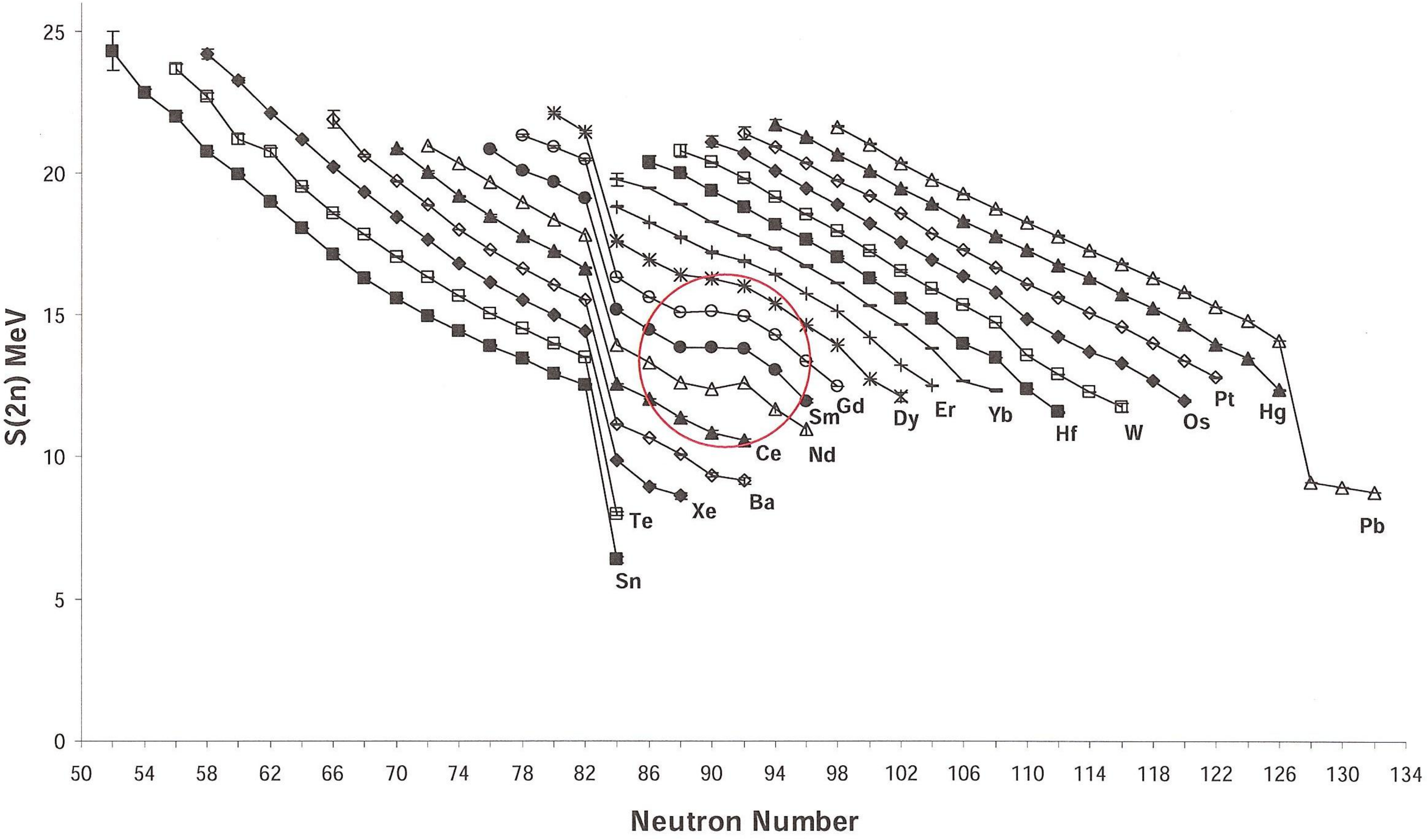




| | |
|---|---|
| $\lambda < \lambda_c$ | single minimum |
| $\lambda = \lambda_c$ | critical point |
| $\lambda > \lambda_c$ | single minimum |

| | |
|---|---|
| $\lambda < \lambda^*$ | single minimum |
| $\lambda = \lambda^*$ | spinodal point: 2nd min. appears |
| $\lambda = \lambda_c$ | critical point: two degenerate minima |
| $\lambda = \lambda^{**}$ | anti-spinodal point: 1st min. disappears |
| $\lambda > \lambda^{**}$ | single minimum |

$\lambda^* < \lambda < \lambda^{**}$ coexistence region



EXP $^{148}$Sm (spherical) $^{152}$Sm(critical) $^{154}$Sm(deformed)

- What is the nature of the dynamics (regularity v.s. chaos) in such circumstances ?

$$H(\lambda) = \lambda \mathbf{H_1} + (1- \lambda) \mathbf{H_2}$$

- Competing interactions
- Incompatible symmetries
- Evolution of order and chaos across the QPT
- Remaining regularity and persisting symmetries

Dicke model of quantum optics, 2nd order QPT (*Emary, Brandes, PRL, PRE 2003*)

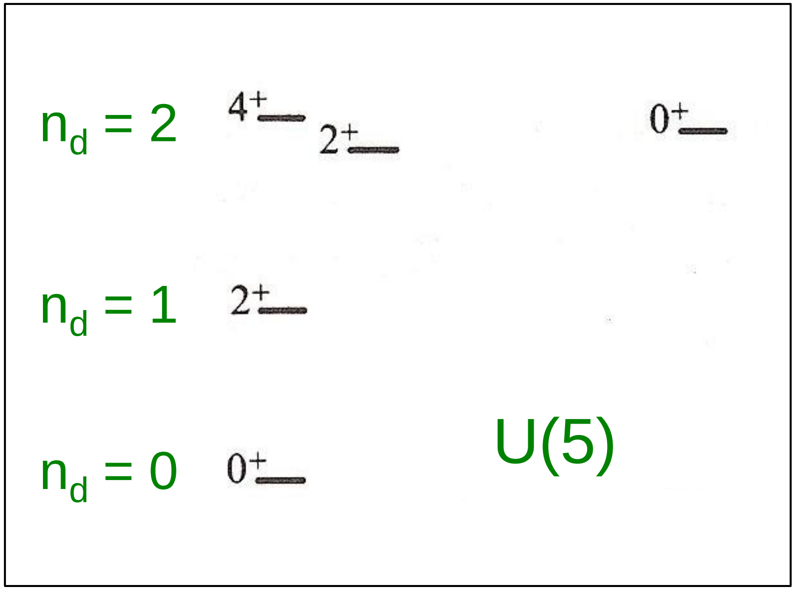
Interacting boson model (IBM) of nuclei, 1st order QPT (*this talk*)

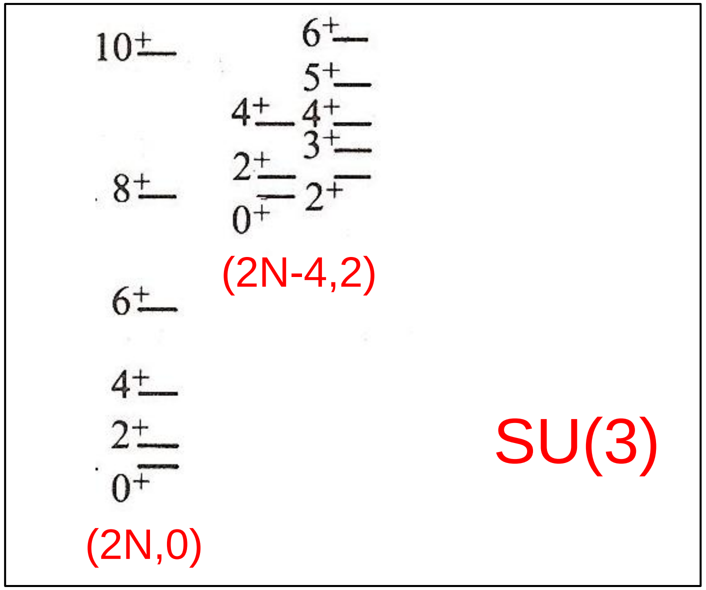
- IBM: s (L=0) , d (L=2) bosons, N conserved *(Arima, Iachello 75)*
- Spectrum generating algebra U(6)

$$H = \sum_{\alpha\beta} \epsilon_{\alpha\beta}\, \mathcal{G}_{\alpha\beta} + \sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta}\, \mathcal{G}_{\alpha\beta}\, \mathcal{G}_{\gamma\delta} \qquad \mathcal{G}_{\alpha\beta} = \{s^\dagger s, s^\dagger d_\mu, d^\dagger_\mu s, d^\dagger_\mu d_{\mu'}\}$$

- Dynamical symmetries

| | | |
|---|---|---|
| $U(6) \supset U(5) \supset O(5) \supset O(3)$ | $\vert [N]\ n_d\ \tau\ n_\Delta\ L \rangle$ | Spherical vibrator |
| $U(6) \supset SU(3) \supset O(3)$ | $\vert [N]\ (\lambda\ ,\mu\ )\ K\ L \rangle$ | Axial rotor |
| $U(6) \supset O(6) \supset O(5) \supset O(3)$ | $\vert [N]\ \sigma\ \tau\ n_\Delta\ L \rangle$ | $\gamma$-unstable rotor |





- Geometry

$$\boxed{V(\beta,\gamma) = \langle \beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle}$$

$$|\beta,\gamma;N\rangle = (N!)^{-1/2}[\,\Gamma_c^\dagger(\beta,\gamma)\,]^N|0\rangle$$

$$\Gamma_c^\dagger(\beta,\gamma) = \left[\beta\cos\gamma d_0^\dagger + \beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger)/\sqrt{2} + \sqrt{2-\beta^2}s^\dagger\right]/\sqrt{2}$$

global min: ($\beta_{eq}$ , $\gamma_{eq}$)

$\beta_{eq}$ = 0 spherical shape

$\beta_{eq}$ > 0, $\gamma_{eq}$ = 0, $\pi$/3, $\gamma$-indep. deformed shape

- Intrinsic collective resolution

$$\hat{H} = \hat{H}_{\text{int}} + \hat{H}_{\text{col}}$$

$$\hat{H}_{\text{int}}|\beta=\beta_{\text{eq}},\gamma=\gamma_{\text{eq}};N\rangle = 0$$

affects V($\beta$,$\gamma$)

rotation terms

- Geometry

$$\boxed{V(\beta,\gamma) = \langle \beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle}$$ ← **Landau potential**

$$|\beta,\gamma;N\rangle = (N!)^{-1/2}[\,\Gamma_c^\dagger(\beta,\gamma)\,]^N|0\rangle$$

$$\Gamma_c^\dagger(\beta,\gamma) = \left[\beta\cos\gamma d_0^\dagger + \beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger)/\sqrt{2} + \sqrt{2-\beta^2}s^\dagger\right]/\sqrt{2}$$

global min: $(\beta_{eq}\,,\,\gamma_{eq})$ ← **order parameters**

$\beta_{eq} = 0$ spherical shape

$\beta_{eq} > 0$, $\gamma_{eq} = 0, \pi/3$, $\gamma$-indep. deformed shape

- Intrinsic collective resolution

$$\hat{H} = \hat{H}_{\rm int} + \hat{H}_{\rm col}$$

$\hat{H}_{\rm int}|\beta=\beta_{\rm eq},\gamma=\gamma_{\rm eq};N\rangle = 0$ ↙ affects $V(\beta,\gamma)$ ↘ rotation terms

- **QPT** $H(\lambda) = \lambda\, H_{G1} + (1-\lambda)\, H_{G2}$

dynamical symmetries **$G_i$** = U(5), SU(3), O(6) ↔ phases [spherical, deformed: axial, $\gamma$-unstable]

- Geometry $$\boxed{V(\beta,\gamma) = \langle \beta,\gamma;N|\hat{H}|\beta,\gamma;N\rangle}$$ ⟵ **Landau potential**

$$|\beta,\gamma;N\rangle = (N!)^{-1/2}[\,\Gamma^\dagger_c(\beta,\gamma)\,]^N|0\rangle$$

$$\Gamma^\dagger_c(\beta,\gamma) = \left[\beta\cos\gamma d^\dagger_0 + \beta\sin\gamma(d^\dagger_2 + d^\dagger_{-2})/\sqrt{2} + \sqrt{2-\beta^2}s^\dagger\right]/\sqrt{2}$$

global min: $(\beta_{eq}\,,\,\gamma_{eq})$ ⟵ **order parameters**

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- Intrinsic collective resolution $$\hat{H} = \hat{H}_{\rm int} + \hat{H}_{\rm col}$$

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- **QPT** $$H(\lambda) = \lambda\, H_{G1} + (1-\lambda)\, H_{G2}$$

dynamical symmetries $\mathbf{G_i}$ = U(5), SU(3), O(6) ↔ phases [spherical, deformed: axial, $\gamma$-unstable]

exact DS: integrable **regular** dynamics

broken DS: non-integrable **chaotic** dynamics

# First-order QPT

Intrinsic Hamiltonian

$$\hat{H}_1(\rho)/\bar{h}_2 = 2(1-2\rho^2)\hat{n}_d(\hat{n}_d-1) + 2R^\dagger_2(\rho)\cdot\tilde{R}_2(\rho)$$

spherical

$$\hat{H}_2(\xi)/\bar{h}_2 = \xi P^\dagger_0 P_0 + P^\dagger_2\cdot\tilde{P}_2$$

deformed

control parameters

$$0 \le \rho \le \frac{1}{\sqrt{2}}$$

$$0 \le \xi \le 1$$

critical point

$$\rho_c = \frac{1}{\sqrt{2}} \qquad \xi_c = 0$$

$$\hat{n}_d = \sum_\mu d^\dagger_\mu d_\mu$$

$$R^\dagger_{2\mu}(\rho) = \sqrt{2}s^\dagger d^\dagger_\mu + \rho\sqrt{7}(d^\dagger d^\dagger)^{(2)}_\mu$$

$$P^\dagger_0 = d^\dagger\cdot d^\dagger - 2(s^\dagger)^2$$

$$P^\dagger_{2\mu} = 2s^\dagger d^\dagger_\mu + \sqrt{7}(d^\dagger d^\dagger)^{(2)}_\mu$$

$$\hat{H}_1(\rho=0)/\bar{h}_2 = 2\hat{n}_d(\hat{n}_d-1) + 4(\hat{N}-\hat{n}_d)\hat{n}_d$$

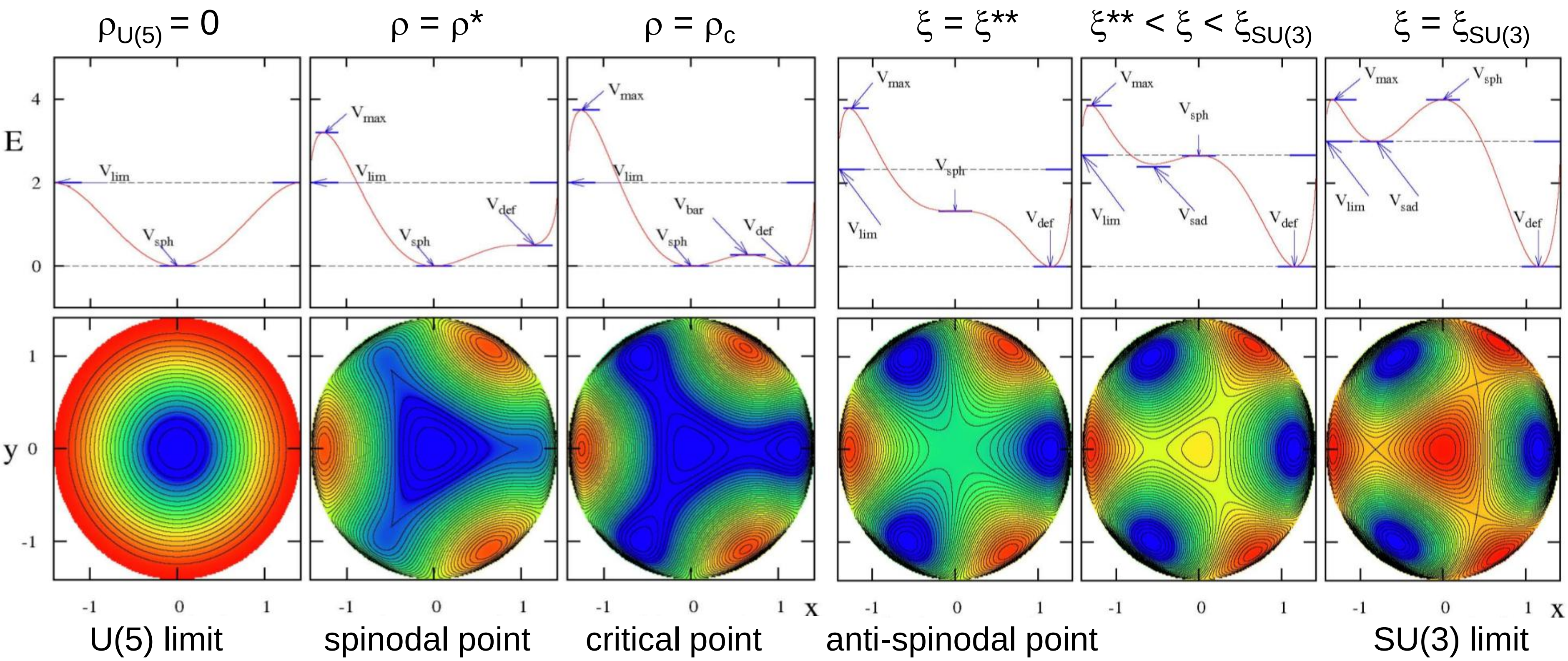
U(5) DS

$$\hat{H}_2(\xi=1)/\bar{h}_2 = -\hat{C}_{SU(3)} + 2\hat{N}(2\hat{N}+3)$$

SU(3) DS

$$\hat{H}^{\rm int}_{\rm cri} \equiv \hat{H}_1(\rho_c) = \hat{H}_2(\xi_c)$$

critical-point Hamiltonian

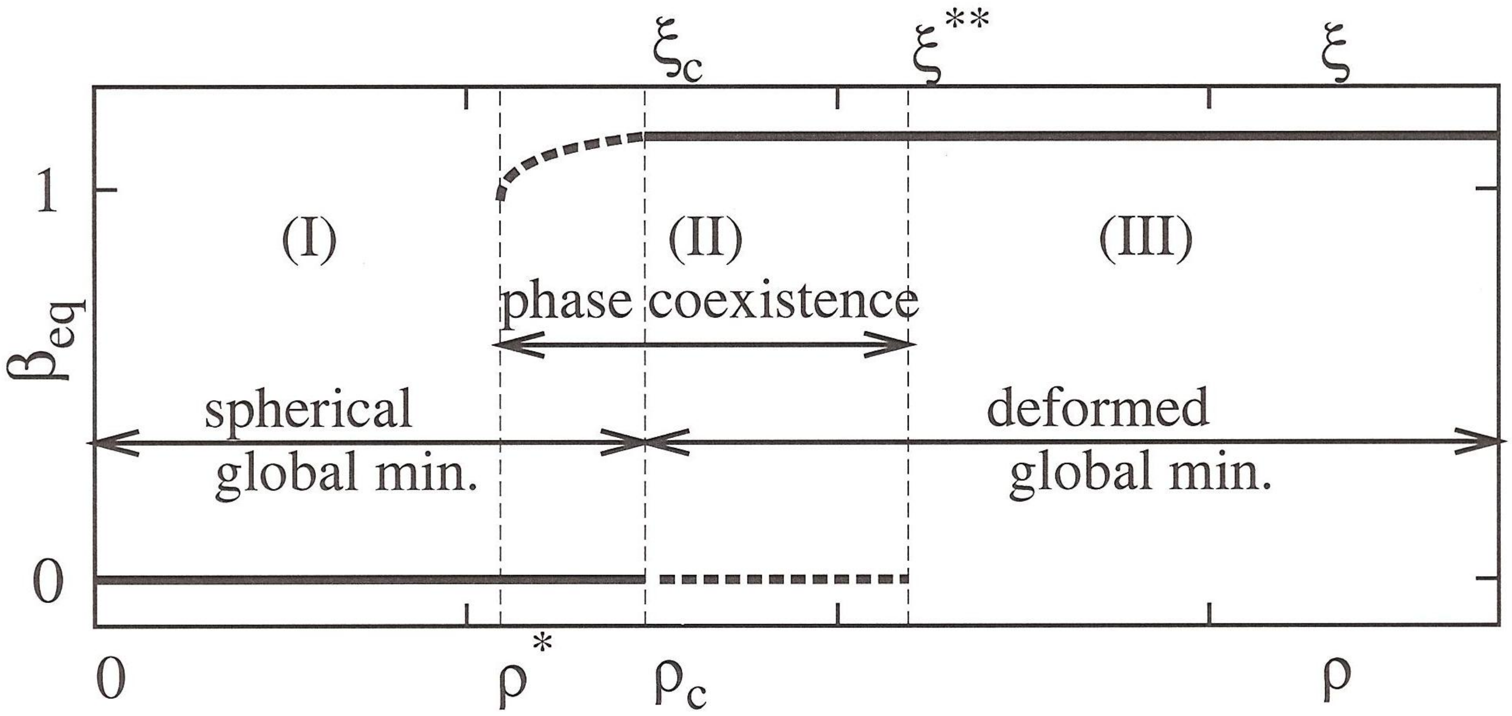


$\rho_{U(5)} = 0$ $\rho = \rho^*$ $\rho = \rho_c$ $\xi = \xi^{**}$ $\xi^{**} < \xi < \xi_{SU(3)}$ $\xi = \xi_{SU(3)}$

U(5) limit spinodal point critical point anti-spinodal point SU(3) limit

| potential | | phase |
|---|---|---|
| $V_1(\rho)/h_2 = 2\beta^2 - 2\rho\sqrt{2-\beta^2}\beta^3\cos 3\gamma - \frac{1}{2}\beta^4$ | $\beta_{eq} = 0$ | **spherical** |
| $V_2(\xi)/h_2 = 2(1-3\xi)\beta^2 - \sqrt{2(2-\beta^2)}\beta^3\cos 3\gamma + \frac{1}{4}(9\xi - 2)\beta^4 + 4\xi$ | $\beta_{eq} = \frac{2}{\sqrt{3}}$ $\gamma_{eq} = 0$ | **deformed** |

spinodal : $\rho^* = \frac{1}{2}$ critical : $\rho_c = \frac{1}{\sqrt{2}}$ $\xi_c = 0$ anti-spinodal: $\xi^{**} = \frac{1}{3}$



- Region I stable spherical phase $\rho \in [0\ ,\ \rho^*]$
- Region II phase coexistence $\rho \in (\rho^*, \rho_c]$ $\xi \in [\xi_c\ ,\ \xi^{**})$
- Region III stable deformed phase $\xi \in (\xi^{**}, 1]$

# Classical analysis

- Classical Hamiltonian: $s^\dagger \; d^\dagger_\mu$ → $\alpha_s^* \; \alpha_\mu^*$ , coherent states (N→ ∞)

  zero momenta: ⇒ classical potential V(β,γ)

- For L=0 classical Hamiltonian becomes two-dimensional

$\beta, \gamma, p_\beta, p_\gamma \leftrightarrow x = \beta\cos\gamma , y = \beta\sin\gamma, p_x , p_y$ V(β,γ) = V(x,y)

$$\mathcal{H}_1(\rho)/h_2 = \mathcal{H}^2_{d,0} + 2(1-\mathcal{H}_{d,0})\mathcal{H}_{d,0} + 2\rho^2 p_\gamma^2$$

$$+\rho\sqrt{2(1-\mathcal{H}_{d,0})}\left[(p_\gamma^2/\beta - \beta p_\beta^2 - \beta^3)\cos 3\gamma + 2p_\beta p_\gamma \sin 3\gamma\right]$$

$$\mathcal{H}_{d,0} \equiv (T+\beta^2)/2$$

$$T = p_\beta^2 + p_\gamma^2/\beta^2$$

$$\mathcal{H}_2(\xi)/h_2 = \mathcal{H}^2_{d,0} + 2(1-\mathcal{H}_{d,0})\mathcal{H}_{d,0} + p_\gamma^2$$

$$+\sqrt{1-\mathcal{H}_{d,0}}\left[(p_\gamma^2/\beta - \beta p_\beta^2 - \beta^3)\cos 3\gamma + 2p_\beta p_\gamma \sin 3\gamma\right]$$

$$+\xi\left[\beta^2 p_\beta^2 + \frac{1}{4}(\beta^2 - T)^2 - 2(1-\mathcal{H}_{d,0})(\beta^2 - T) + 4(1-\mathcal{H}_{d,0})^2\right]$$

# Classical analysis

- Classical Hamiltonian: $s^\dagger \ d_\mu^\dagger \rightarrow \alpha_s^* \ \alpha_\mu^*$ , coherent states ($N \rightarrow \infty$)

  zero momenta: $\Rightarrow$ classical potential $V(\beta,\gamma)$

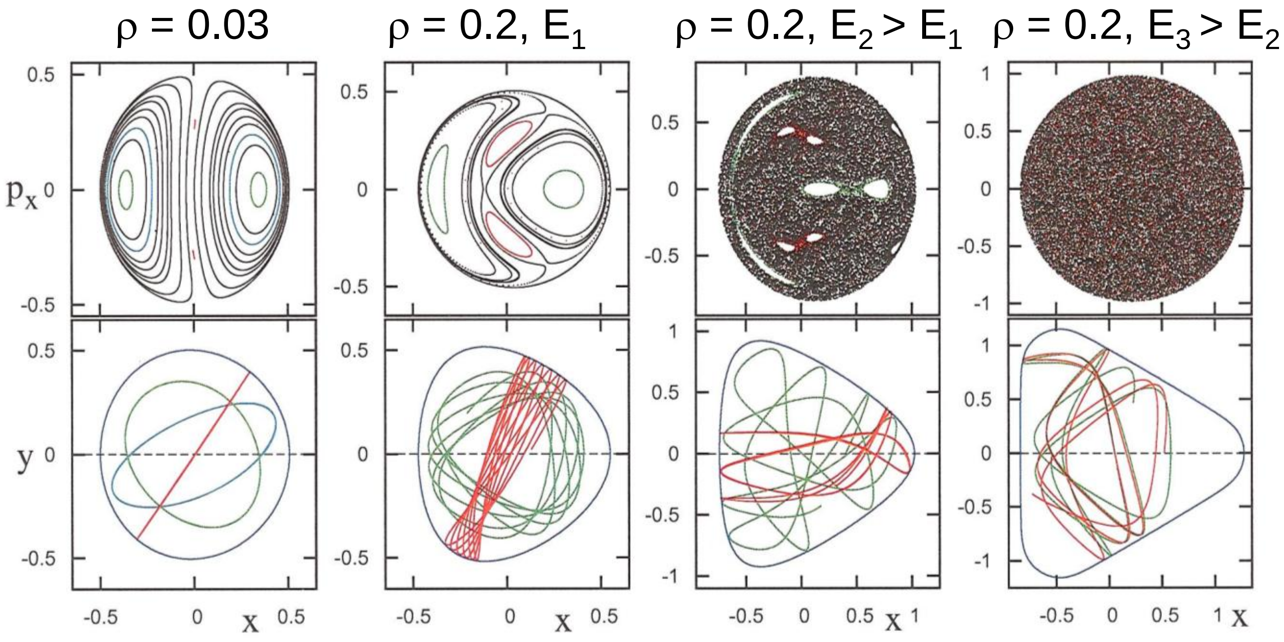
- For $L=0$ classical Hamiltonian becomes two-dimensional

$$\beta, \gamma, p_\beta, p_\gamma \quad \leftrightarrow \quad x = \beta\cos\gamma \text{ , } y = \beta\sin\gamma, \ p_x \text{ , } p_y \qquad V(\beta,\gamma) = V(x,y)$$

- Classical dynamics can be depicted conveniently via Poincare sections (y=0, fixed E)

Regular trajectories: bound to toroidal manifolds within the phase space
intersections with plane of section lie on 1D curves (ovals)

Chaotic trajectories: randomly cover kinematically accessible areas
of the section



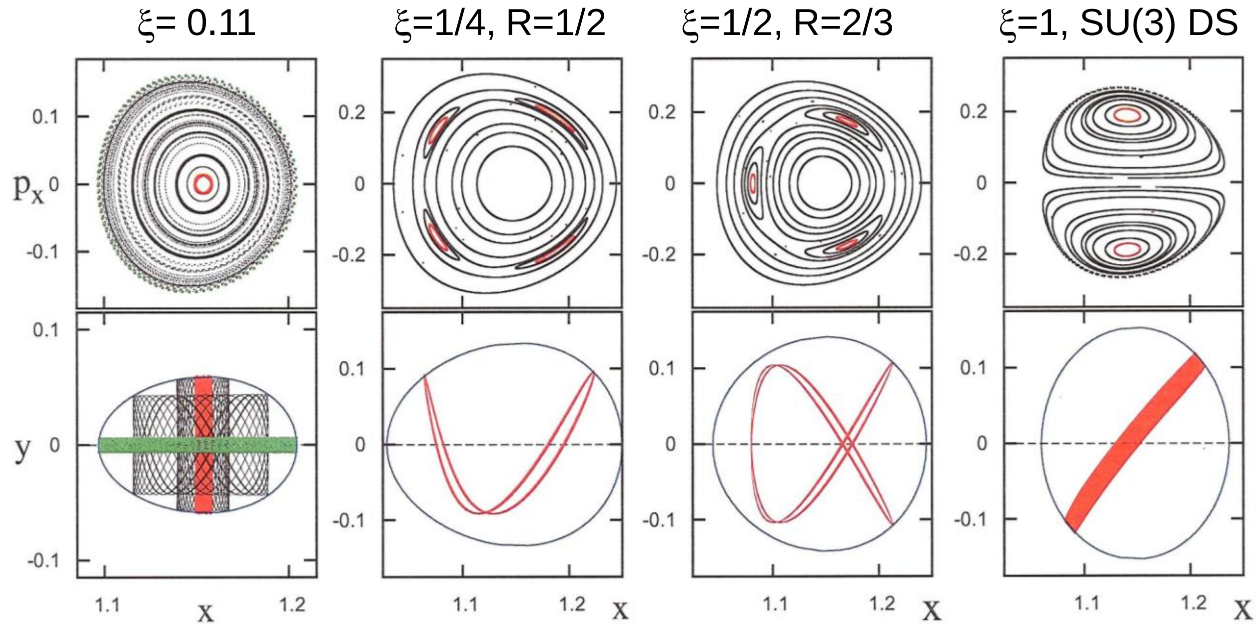
$\rho = 0.03$ $\rho = 0.2, E_1$ $\rho = 0.2, E_2 > E_1$ $\rho = 0.2, E_3 > E_2$

## dynamics near $\beta_{eq} = 0$

- $\rho > 0$:
  non-integrability due to O(5)-breaking term in $H_1(\rho)$

- Henon-Heiles system

$$V_1(\rho) \approx 2\beta^2 - 2\sqrt{2}\rho\beta^3\cos 3\gamma$$



$\xi = 0.11$ $\xi = 1/4, R = 1/2$ $\xi = 1/2, R = 2/3$ $\xi = 1$, SU(3) DS

## dynamics near $\beta_{eq} > 0$

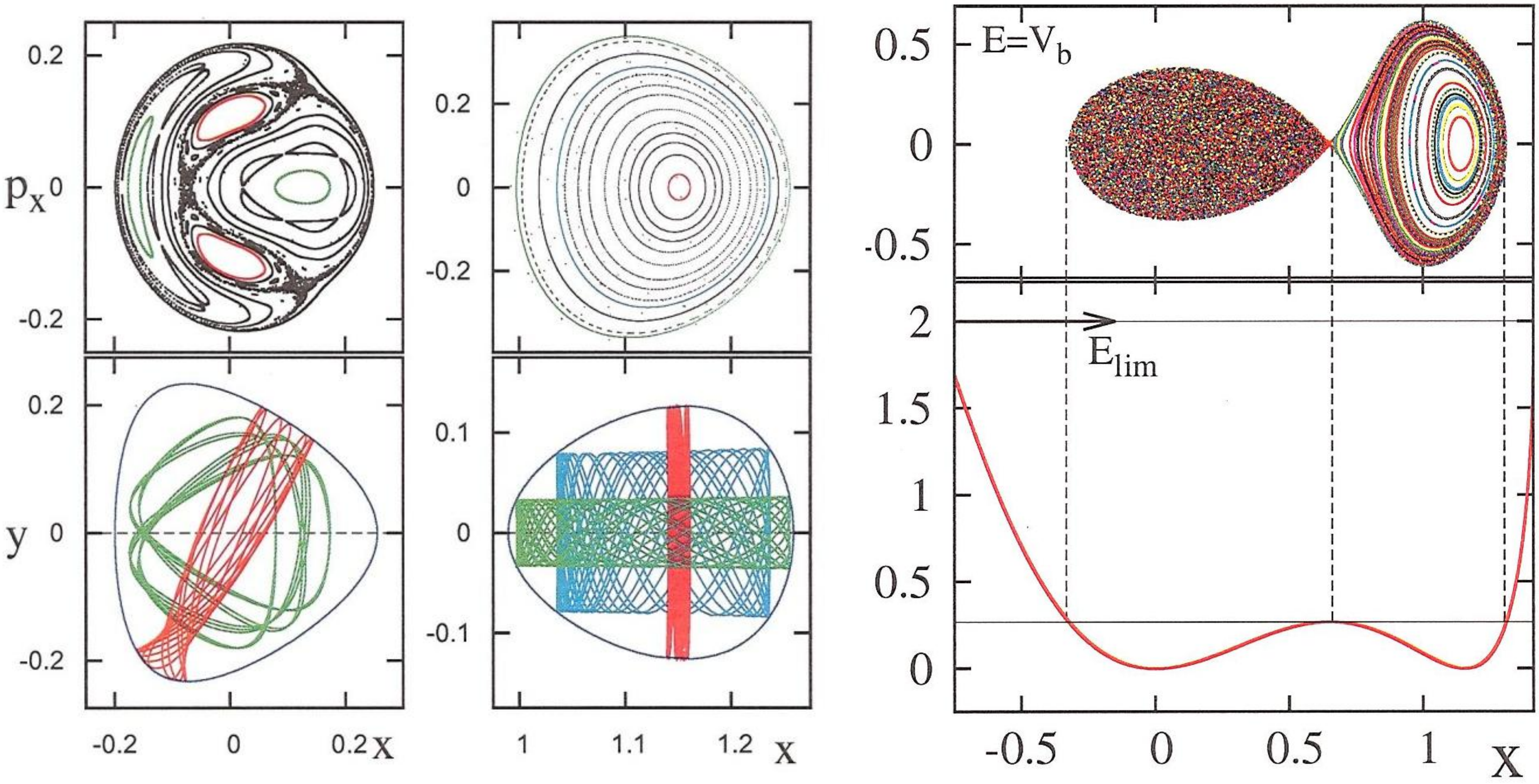
- $\xi < 1$:
  SU(3)-DS broken in $H_2(\xi)$ but dynamics remains robustly regular

- Basic simple form:
  single island of concentric loops

- Resonances at rational values of

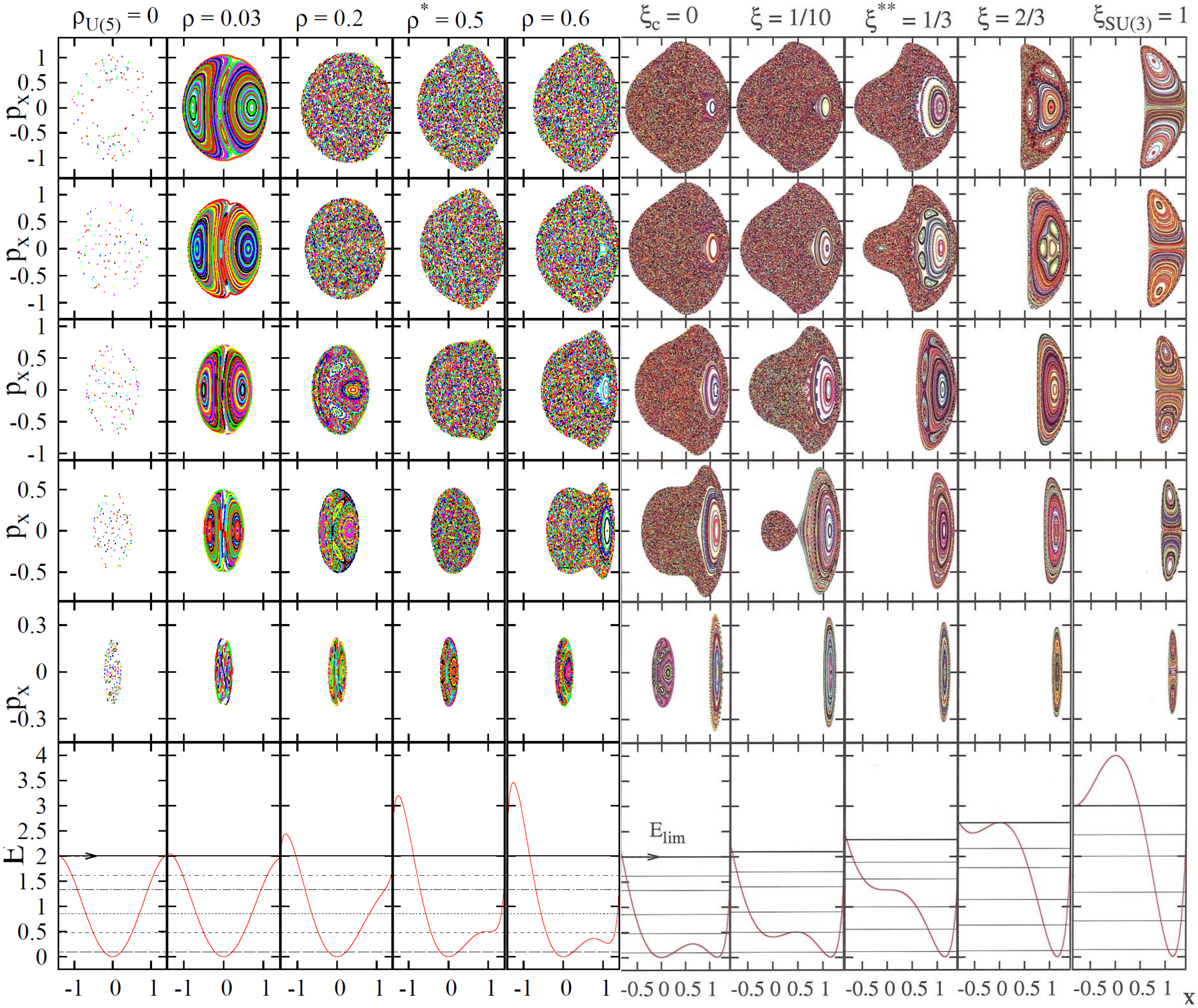
$$R = \frac{\epsilon_\beta}{\epsilon_\gamma} = \frac{1}{3}(2\xi + 1)$$

# classical dynamics in the coexistence region

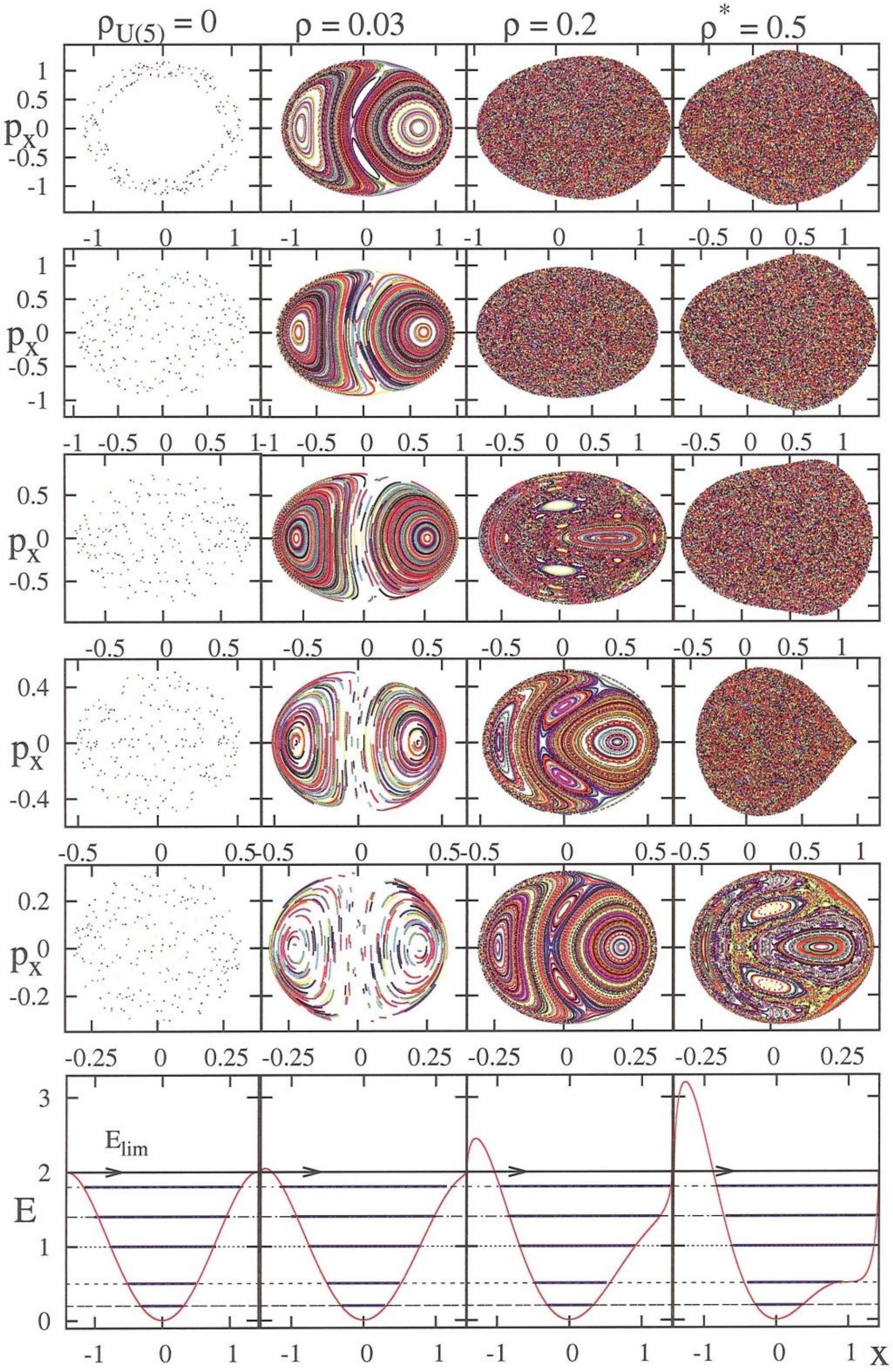


Both types of dynamics occur at the same energy in different regions of phase space

- Spherical well: HH-like chaotic motion
- Deformed well: regular dynamics


$\rho_{U(5)} = 0$
$\rho = 0.03$
$\rho = 0.2$
$\rho^* = 0.5$
$\rho = 0.6$
$\xi_c = 0$
$\xi = 1/10$
$\xi^{**} = 1/3$
$\xi = 2/3$
$\xi_{SU(3)} = 1$
$-p_x$
$p_x$
$-p_x$
E
$E_{lim}$
x
-1 0 1
-0.5 0 0.5 1
1 0.5 0 -0.5 -1
0.3 0 -0.3
4 3.5 3 2.5 2 1.5 1 0.5 0

# Region I: stable spherical phase



- ρ=0: anharmonic (quartic) oscillator

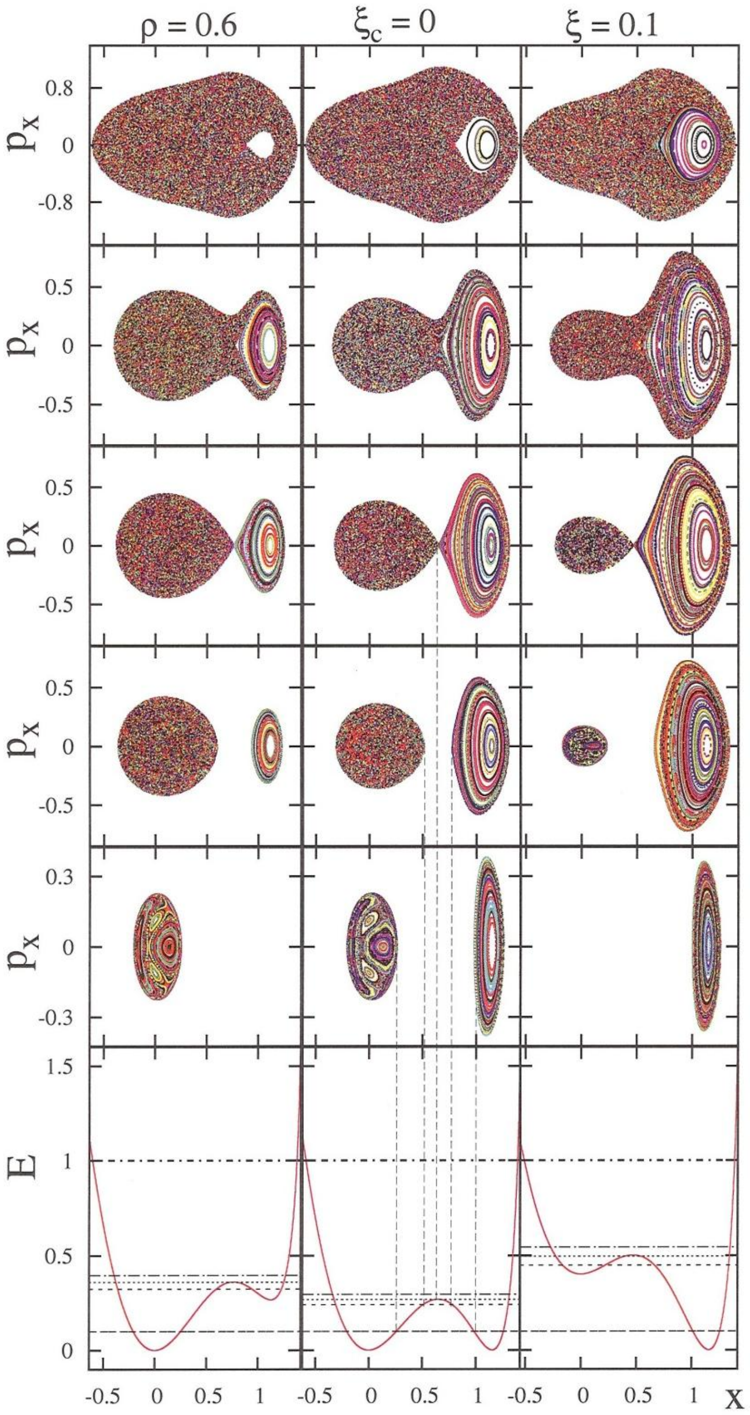
$$V_1(\rho = 0) \approx 2\beta^2 - \tfrac{1}{2}\beta^4$$

- small β: Henon-Heiles system
  regularity at low E
  marked onset of chaos at higher E

$$V_1(\rho) \approx 2\beta^2 - 2\sqrt{2}\rho\beta^3 \cos 3\gamma$$

- chaotic component maximizes at ρ*

## Region II: shape coexistence

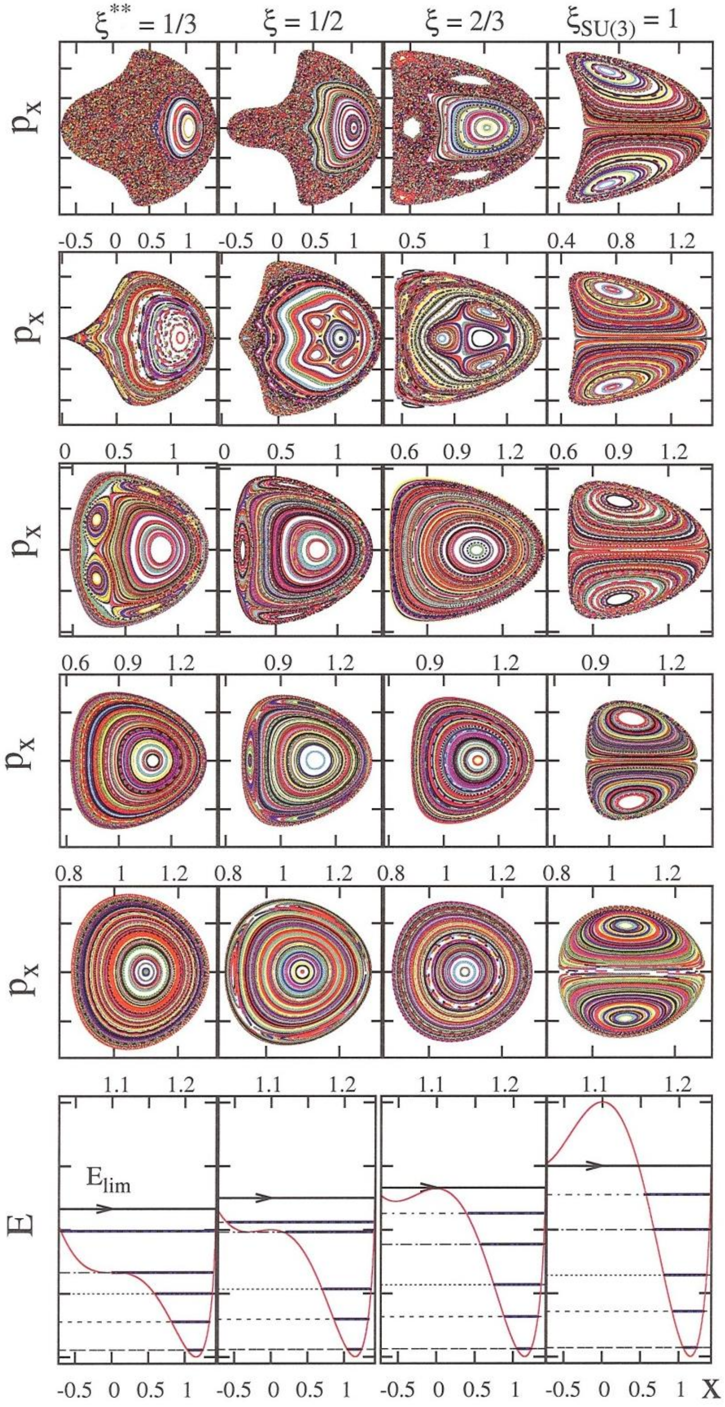


- dynamics changes in the coexistence region

as the local deformed min develops,
regular dynamics appears

regular island remains even at E > barrier!
well separated from chaotic environment

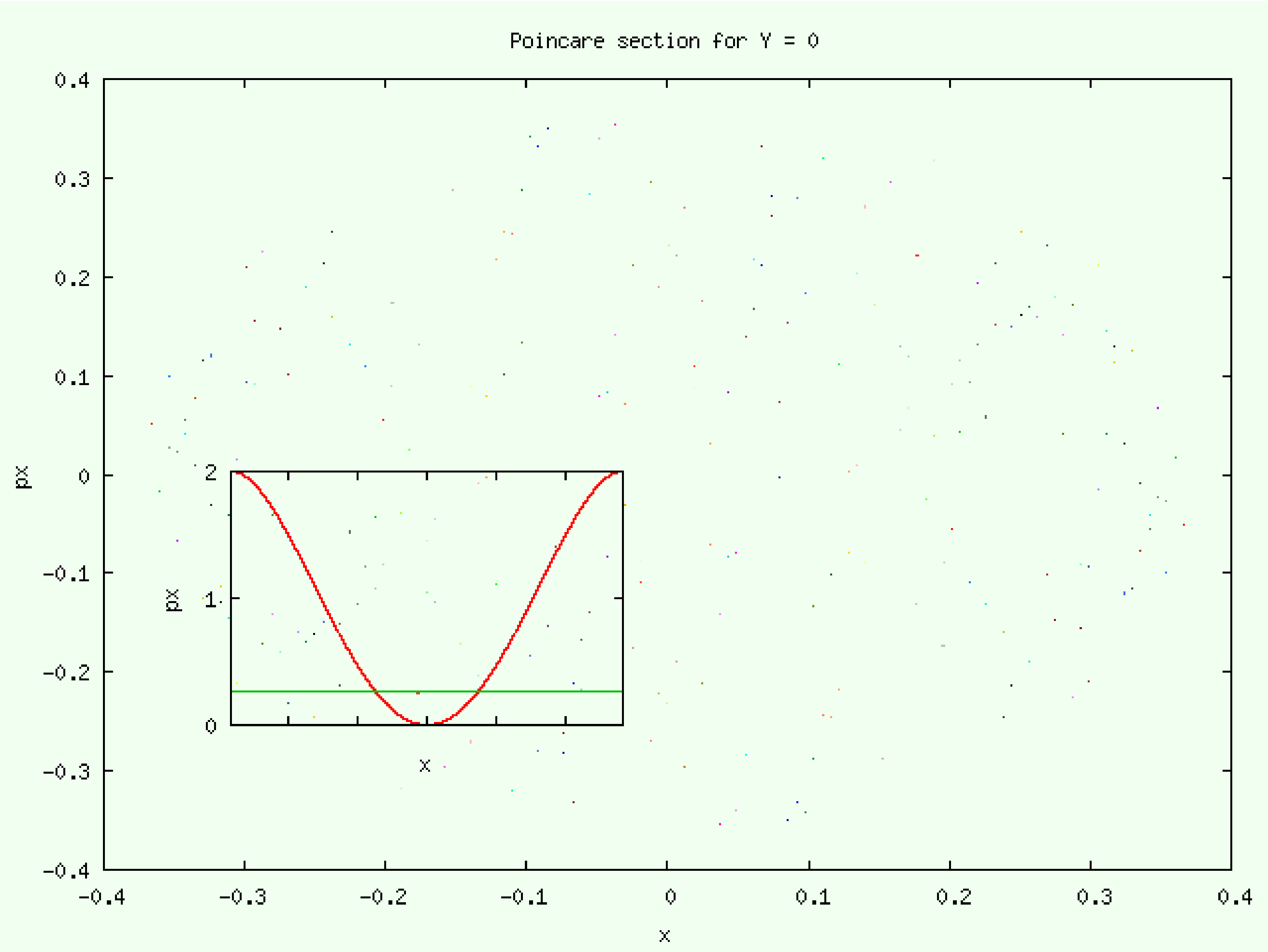
# Region III: stable deformed phase



- as $\xi$ increases,
  spherical min becomes shallower,
  HH dynamics diminishes & disappears at $\xi^{**}$

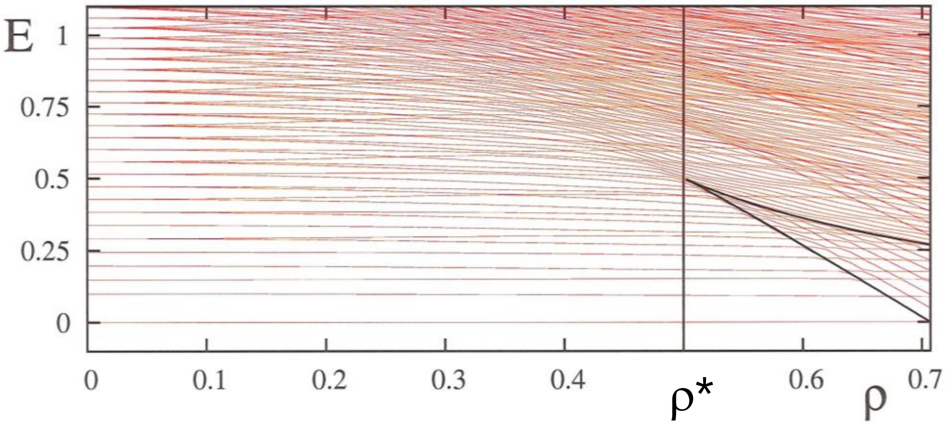
- regular motion prevails for $\xi > \xi^{**}$, where
  landscape changes: single → several islands

- dynamics is sensitive to local
  normal-model degeneracies

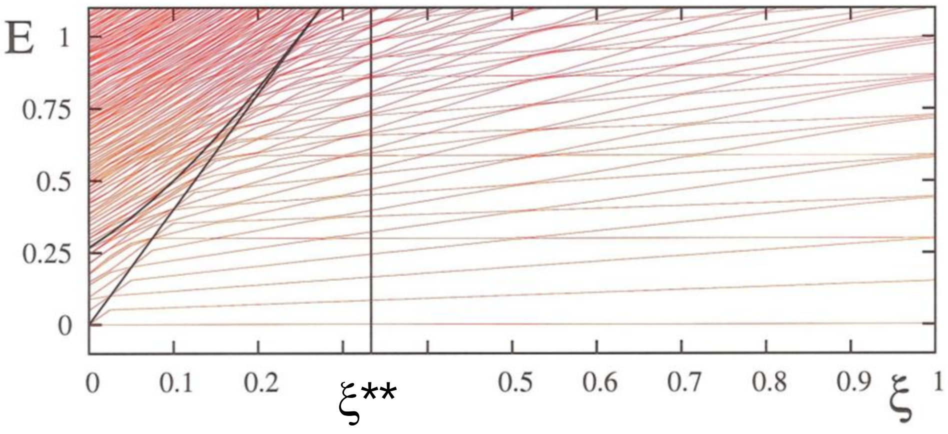

Poincare section for Y = 0
px
x
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
2
1
0
px
x

# Quantum spectrum L=0 states

spherical side $(0 \leq \rho \leq \rho_c)$



deformed side $(\xi_c \leq \xi \leq 1)$



normal modes $\epsilon = 4\bar{h}_2 N$

$$\epsilon_\beta = 4\bar{h}_2 N(2\xi + 1)$$

$$\epsilon_\gamma = 12\bar{h}_2 N$$

$$R = \frac{\epsilon_\beta}{\epsilon_\gamma} = \frac{1}{3}(2\xi + 1)$$

(avoided) level crossing
In classical chaotic regimes

β-γ resonances
bunching of levels

# Quantum analysis

Quantum manifestation of classical chaos

Mixed quantum systems: level statistics in-between Poisson (regular) and GOE (chaotic)
Such global measures of quantum chaos are insufficient for an inhomogeneous phase space

Need to distinguish between regular and irregular states in the same energy interval

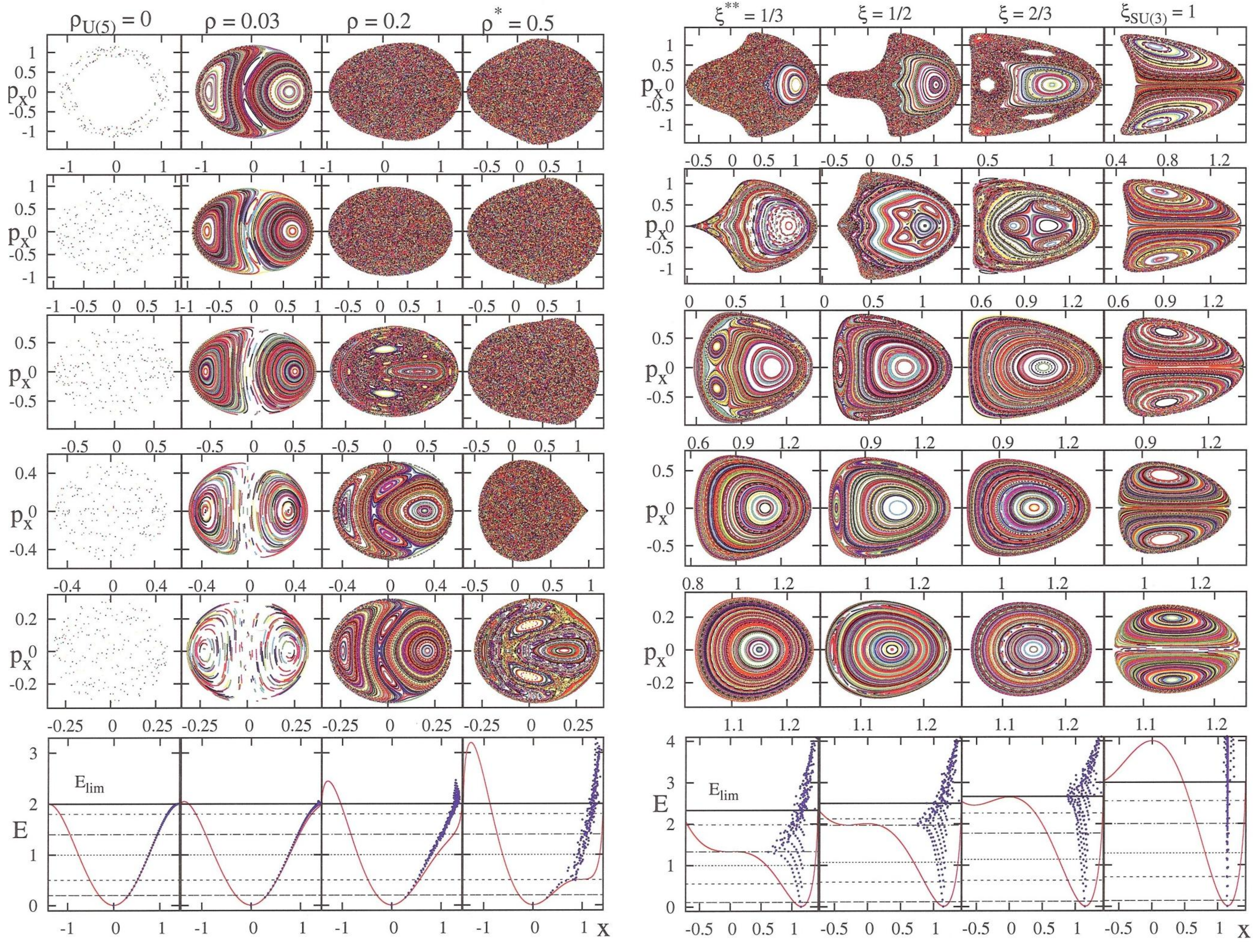
- Peres lattices $O_i = \langle i|\hat{O}|i\rangle$ $\left[\hat{O}, \hat{H}\right] \neq 0$ $\hat{H}|i\rangle = E_i|i\rangle$ $\{O_i, E_i\}$

A. Peres, Phys. Rev. Lett. **53**, 1711 (1984)

$$\{x_i, E_i\} \qquad x_i \equiv \sqrt{2\langle i|\hat{n}_d|i\rangle/N} \qquad \beta = x \leftrightarrow x_i$$

- Regular states: ordered pattern

- Irregular states: disordered meshes of points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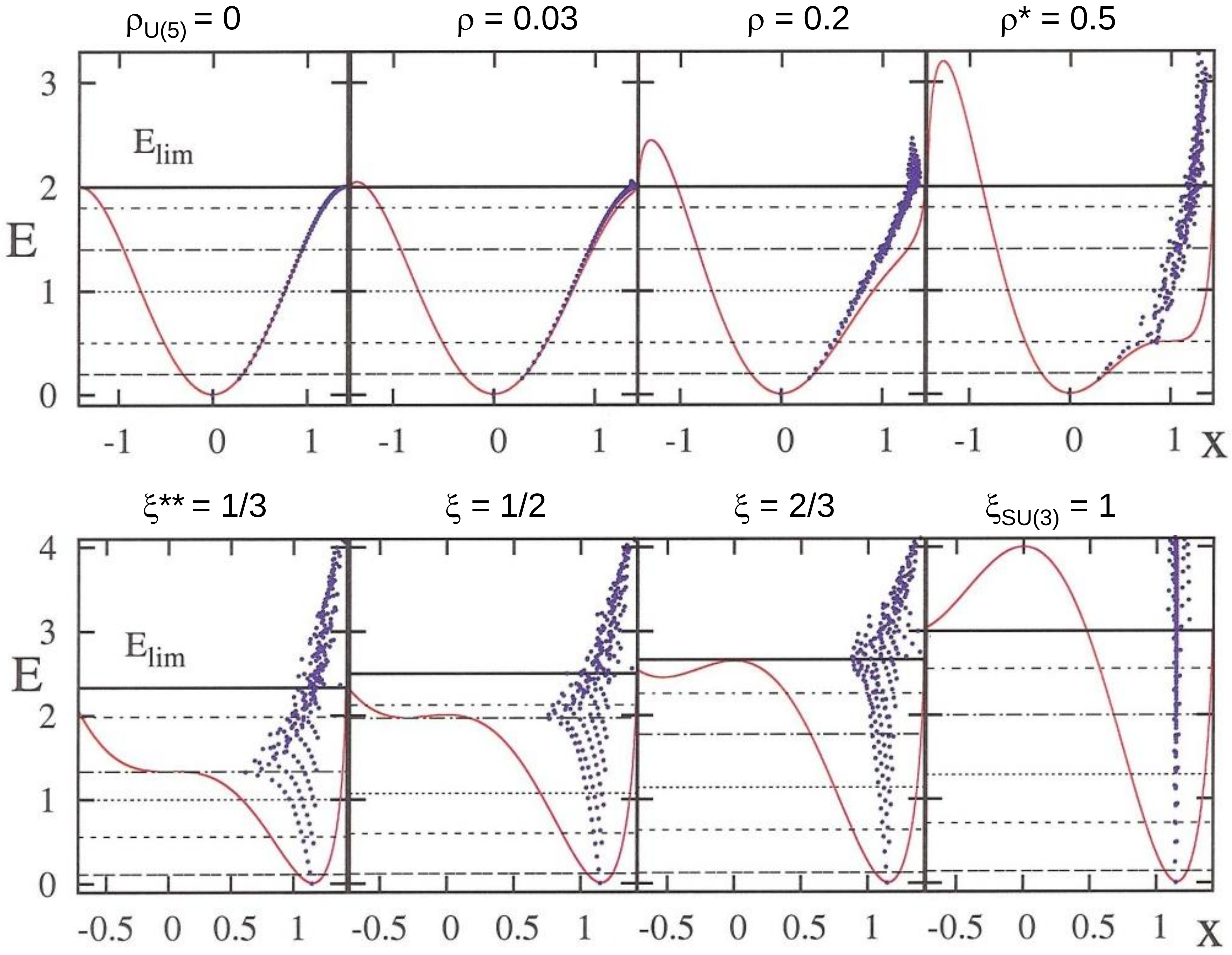
$\xi_{SU(3)} = 1$
$p_x$
$E_{lim}$
E
x


$\rho_{U(5)} = 0$
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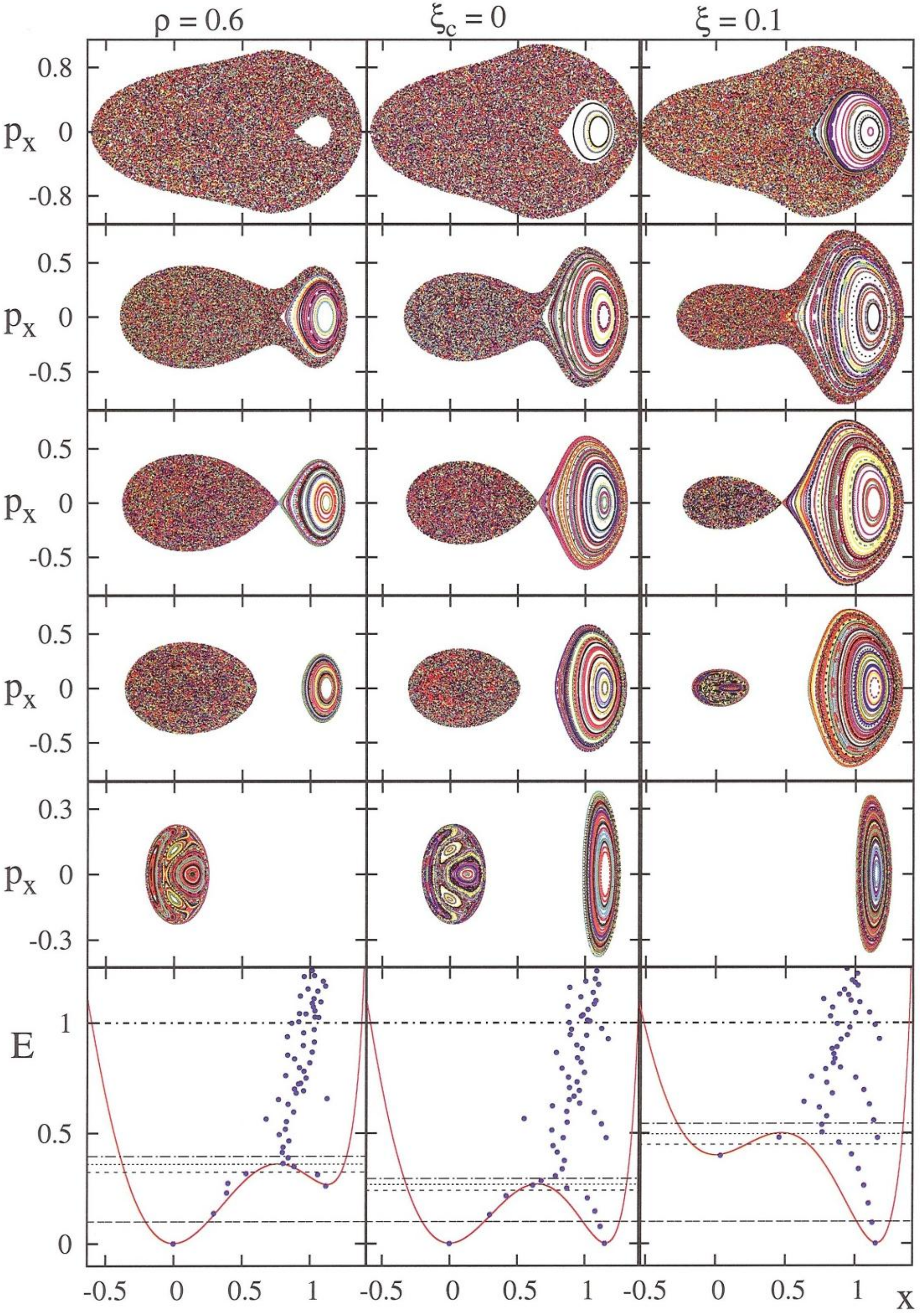
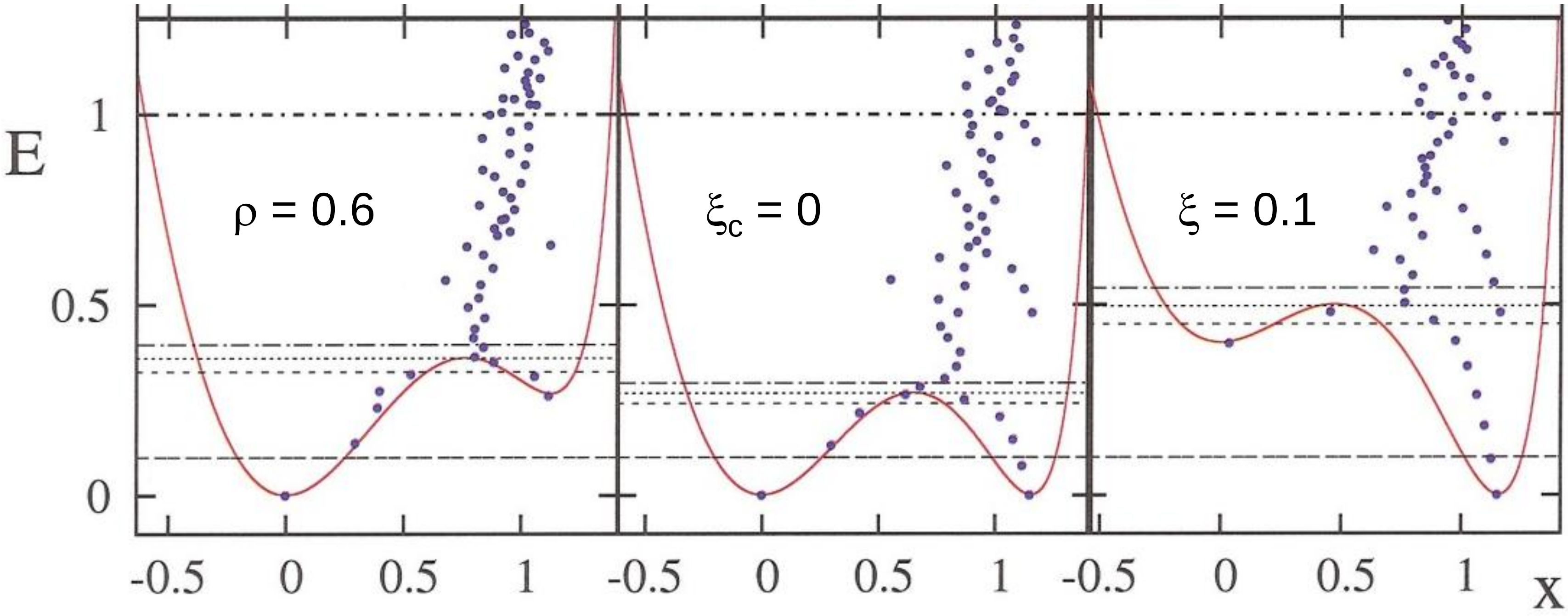
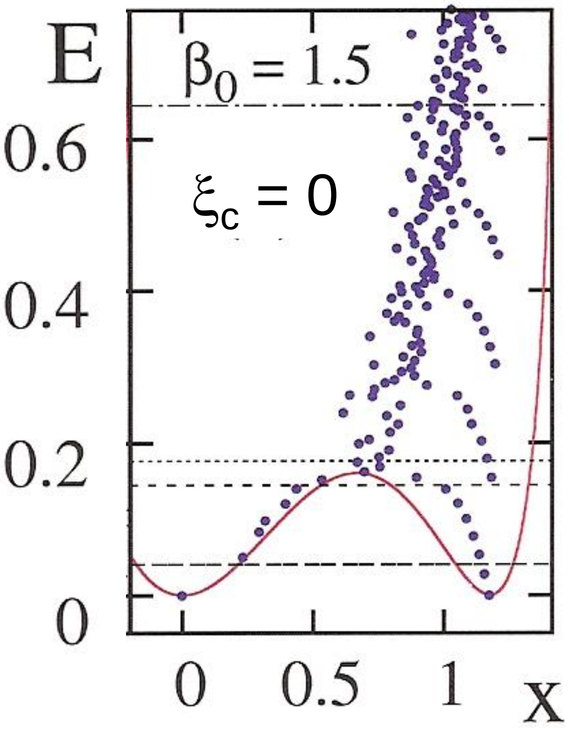
$\rho = 0.2$
$\rho^* = 0.5$
E
$E_{lim}$
0
1
2
3
-1
0
1
x
$\xi^{**} = 1/3$
$\xi = 1/2$
$\xi = 2/3$
$\xi_{SU(3)} = 1$
E
$E_{lim}$
0
1
2
3
4
-0.5
0
0.5
1
x

# Peres lattices of L=0 states in the coexistence region





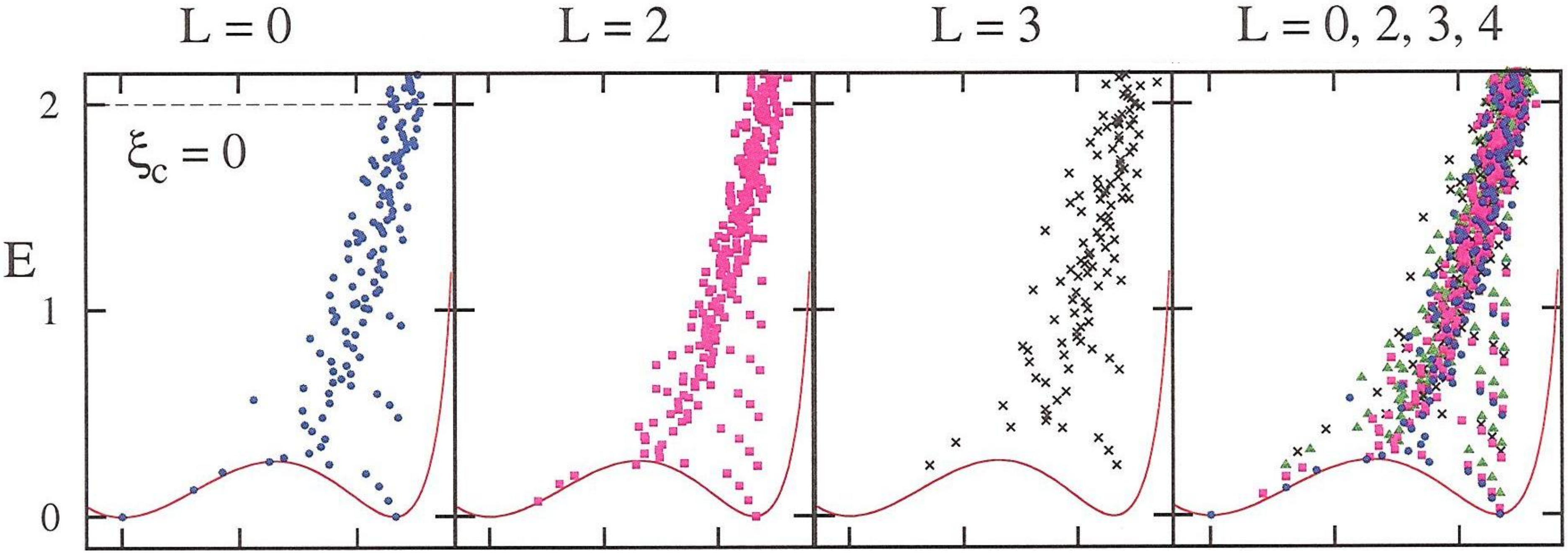


Regular sequences of L=0 states localized within or above the deformed well, related to the regular islands in the Poincare sections

The number of such sequences is larger for deeper wells

Remaining states form disordered (chaotic) meshes of points at high energy

# Peres Lattices L ≥ 0 states

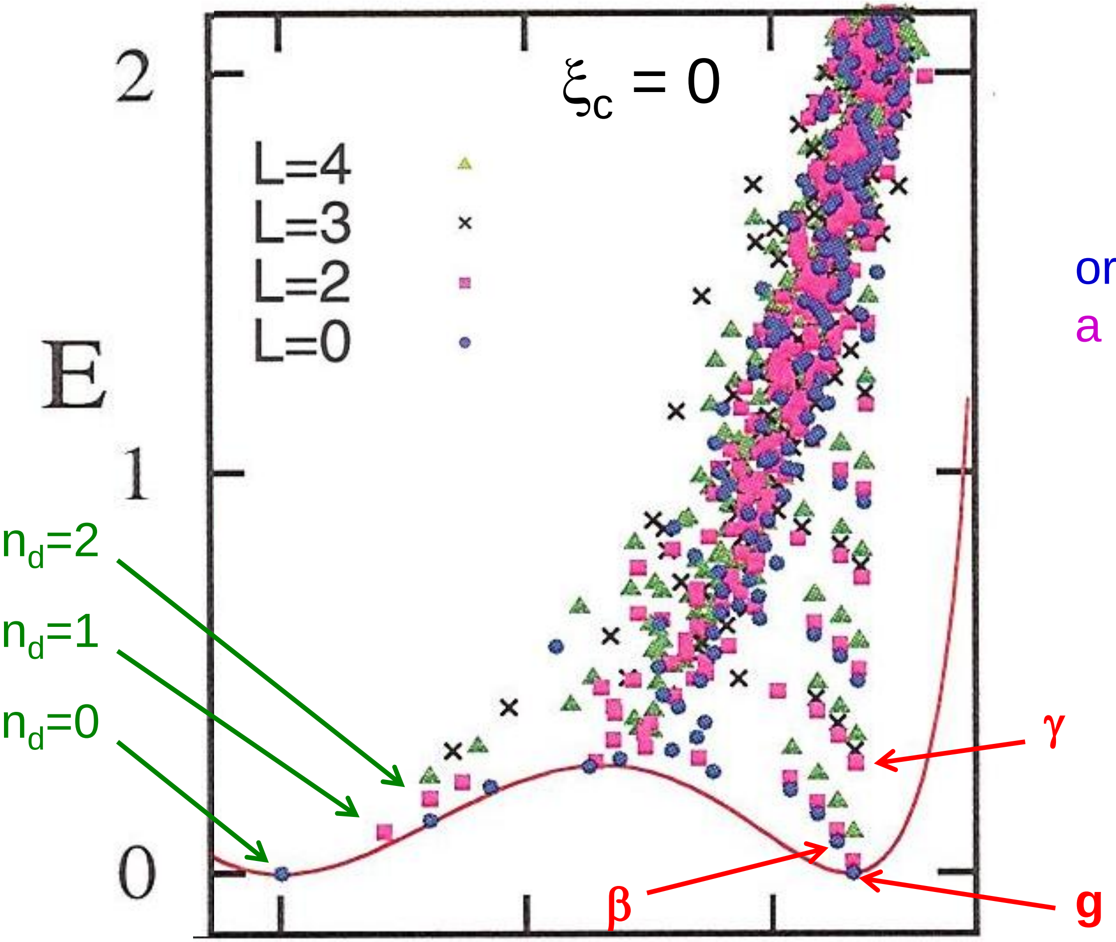


Rotational K-bands L = K,K+1,K+2,…

K=0 L=0,2,4,… g(K=0), $\beta^n$(K=0), $\beta^n \gamma^2$(K=0), $\beta^n \gamma^{4(}$K=0), etc…

K=2 L=2,3,4… $\beta^n \gamma$(K=2), $\beta^n \gamma^3$(K=2), $\beta^n \gamma^5$(K=2), etc…

Spherical $n_d$-multiples (nd=0, L=0),(nd=1,L=2),(nd=2,L=0,2,4)



ordered structure amidst
a complicated environment

- Whenever a deformed (or spherical) min. occurs in $V(\beta)$, the Peres lattices exhibit:

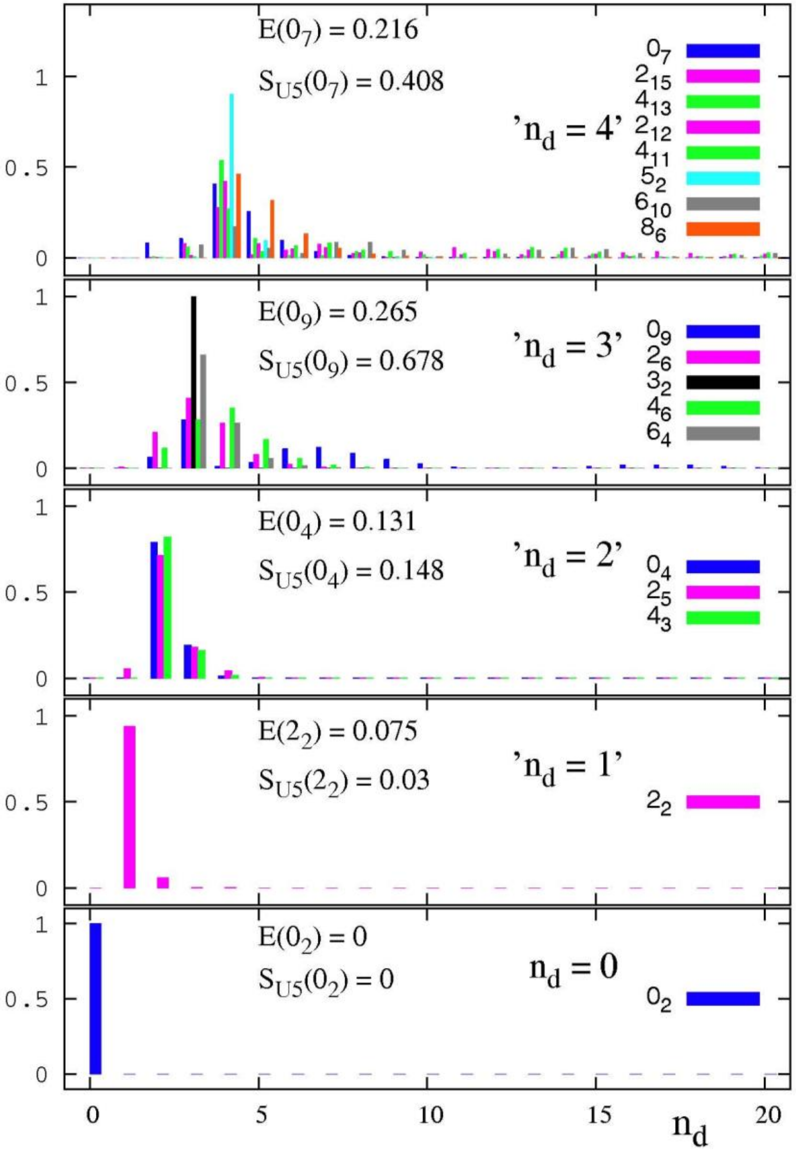
  - regular sequences of states (rotational K-bands) localized in the region of the deformed well, persisting to energies >> barrier
  - or regular spherical-vibrator states ($n_d$ multiplets) in the spherical region

  well separated from the remaining states which form disordered meshes of points

# w.f. decomposition in the U(5) basis

$$|L_i\rangle = \sum_{n_d,\tau,n_\Delta} C^{(L_i)}_{n_d,\tau,n_\Delta} |N, n_d, \tau, n_\Delta, L_i\rangle \qquad P^{(L)}_{n_d} = \sum_{\tau,n_\Delta} |C^{(L)}_{n_d,\tau,n_\Delta}|^2$$

$\xi_c = 0$



right →

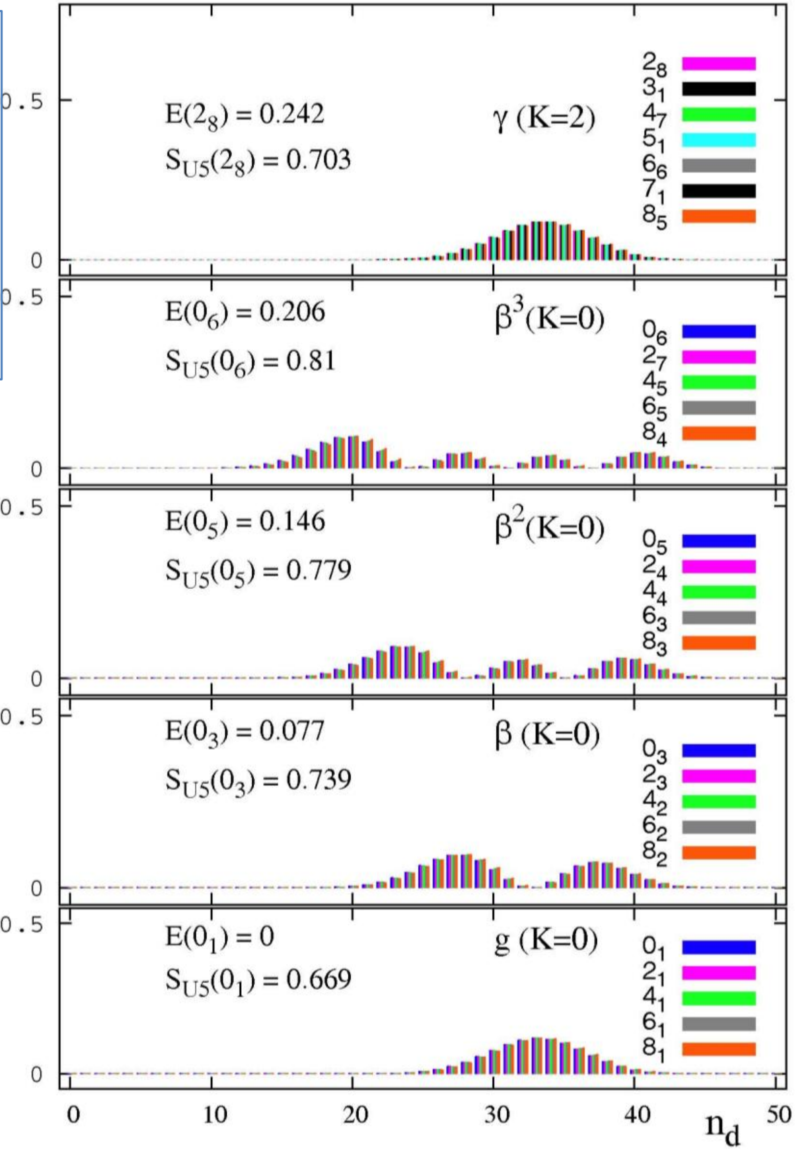
deformed states

broad $n_d$ distribution

← left

spherical states

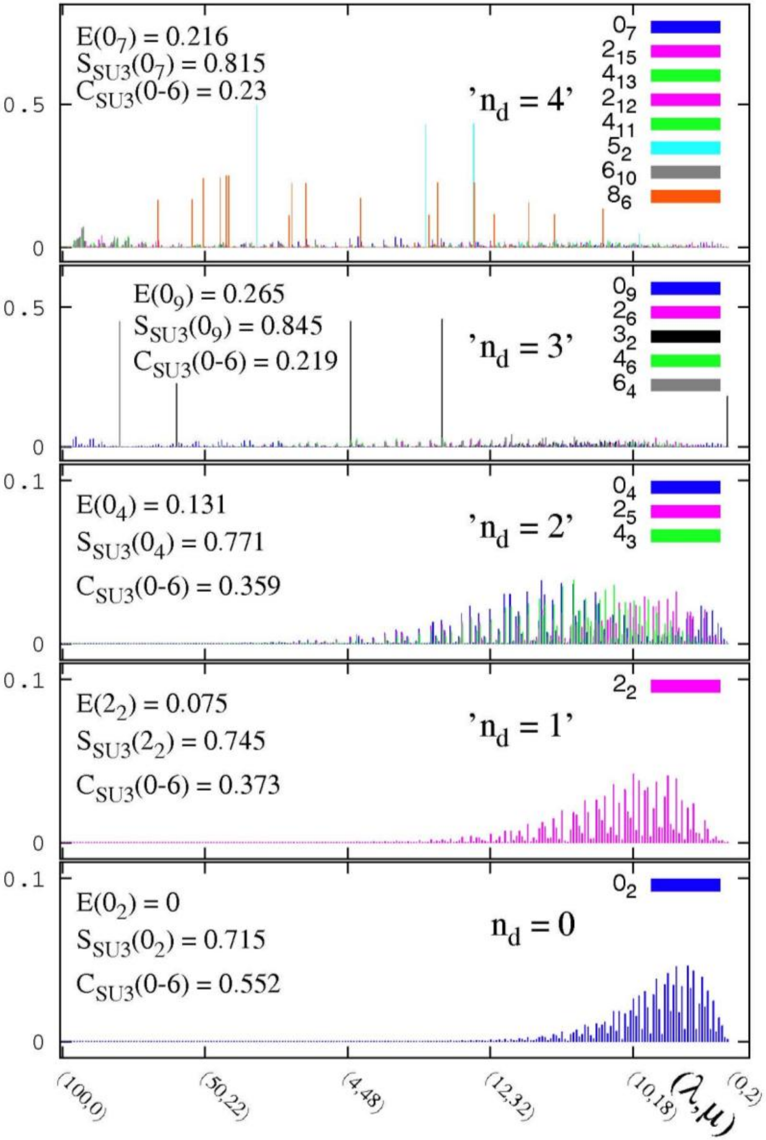
dominant single $n_d$ component



# w.f. decomposition in the SU(3) basis

$$|L\rangle = \sum_{(\lambda,\mu),K} C^{(L)}_{(\lambda,\mu),K} |N,(\lambda,\mu),K,L\rangle \qquad P^{(L)}_{(\lambda,\mu)} = \sum_{K} |C^{(L)}_{(\lambda,\mu),K}|^2$$

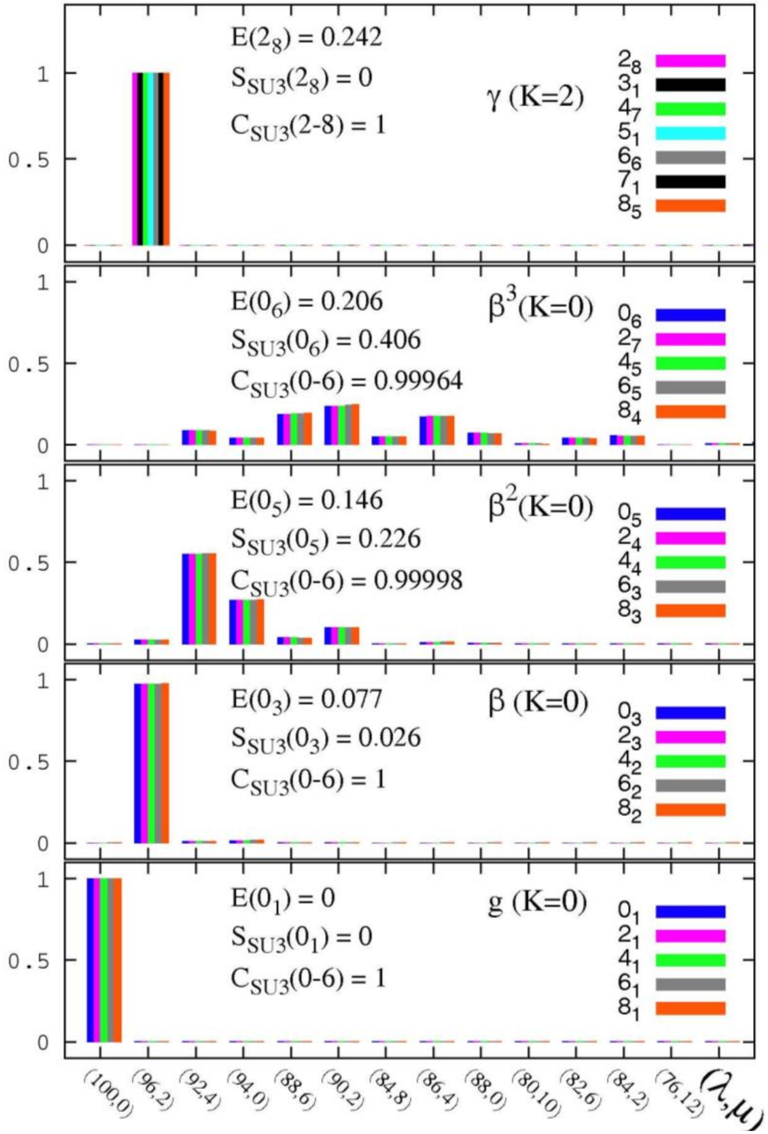
$\xi_c = 0$



right →
deformed
states

coherent
SU(3)
mixing

← left
spherical
states



# Symmetry analysis

- Exact dynamical symmetry (DS)
- Partial dynamical symmetry (PDS)
- Quasi dynamical symmetry (QDS)

# Dynamical Symmetry

$$\begin{tikzcd} G_{\mathrm{dyn}} \arrow[r, phantom, "\supset"] \arrow[d] & G \arrow[r, phantom, "\supset \cdots \supset"] \arrow[d] & G_{\mathrm{sym}} \arrow[d] \\ {[N]} & \langle\Sigma\rangle & \Lambda \end{tikzcd}$$

- *Solvability of the* ***complete*** *spectrum*
- *Quantum numbers for* ***all*** *eigenstates*

$$\hat{H} = \sum_G a_G \hat{C}_G$$

Eigenstates: $|[N]\langle\Sigma\rangle\Lambda\rangle$ Eigenvalues: $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$

# Dynamical Symmetry

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- *Solvability of the* ***complete*** *spectrum*
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$$\hat{H} = \sum_{G} a_G \, \hat{C}_G$$

Eigenstates: $|[N]\langle\Sigma\rangle\Lambda\rangle$ Eigenvalues: $E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$

# Partial Dynamical Symmetry

- Only **some** states solvable with good symmetry

Leviatan, Prog. Part. Nucl. Phys. **66**, 93 (2011)

# Construction of Hamiltonians with PDS

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym}$$

$[N]$ $\langle\Sigma\rangle$ $\Lambda$

n-particle annihilation operator

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \Lambda\rangle = 0$$

for **all** possible $\Lambda$ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \rangle = 0$$

| **Lowest weight state** ⟩

- Condition is satisfied if $\langle\sigma\rangle \otimes \langle\Sigma_0\rangle \not\ni [N-n]$

n-body $\hat{H}' = \sum_{\alpha,\beta} A_{\alpha\beta} \hat{T}^{\dagger}_{\alpha} \hat{T}_{\beta}$

$$\hat{H}_{PDS} = \hat{H}_{DS} + \hat{H}'$$

DS is **broken** but **solvability** of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ Is **preserved**

Garcia-Ramos, Leviatan, Van Isacker, PRL **102**, 112502 (2009)

# SU(3) PDS

$$U(6) \supset SU(3) \supset SO(3)$$

**[N]** **(λ,μ)** **K** **L**

$$\hat{B}^{\dagger}_{[n](\lambda,\mu)\ell m} \qquad \left.\begin{array}{l} P^{\dagger}_{0} = d^{\dagger}\cdot d^{\dagger} - 2\,(s^{\dagger})^{2} \\ P^{\dagger}_{2,\mu} = 2\,s^{\dagger}d^{\dagger}_{\mu} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu} \end{array}\right\}$$

**(λ,μ)** = (0,2)

$$\begin{aligned} P_{\ell,\mu}|[N](2N,0)L\rangle &= 0 \\ P_{\ell,\mu}|N;\beta=\sqrt{2}\,\rangle &= 0 \end{aligned} \qquad \begin{aligned} |N;\beta=\sqrt{2}\,\rangle &= (N!)^{-1/2}(b^{\dagger}_{c})^{N}|0\rangle \\ b^{\dagger}_{c} &= (\sqrt{2}\,d^{\dagger}_{0} + s^{\dagger})/\sqrt{3} \end{aligned}$$

**(λ,μ)** = (2N,0)

**SU(3) PDS**

$$\boxed{H \;=\; h_0 P^{\dagger}_{0} P_0 + h_2 P^{\dagger}_{2}\cdot\tilde{P}_2}$$

**(λ,μ)** = (0,0)⊕(2,2)

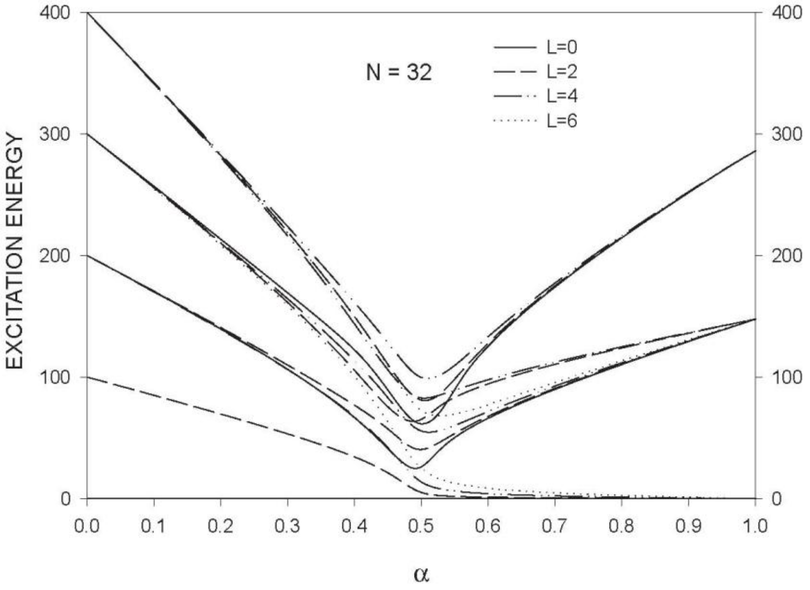
$$H(h_0=h_2) = \left[-\hat{C}_{SU(3)} + 2\hat{N}(2\hat{N}+3)\right]$$

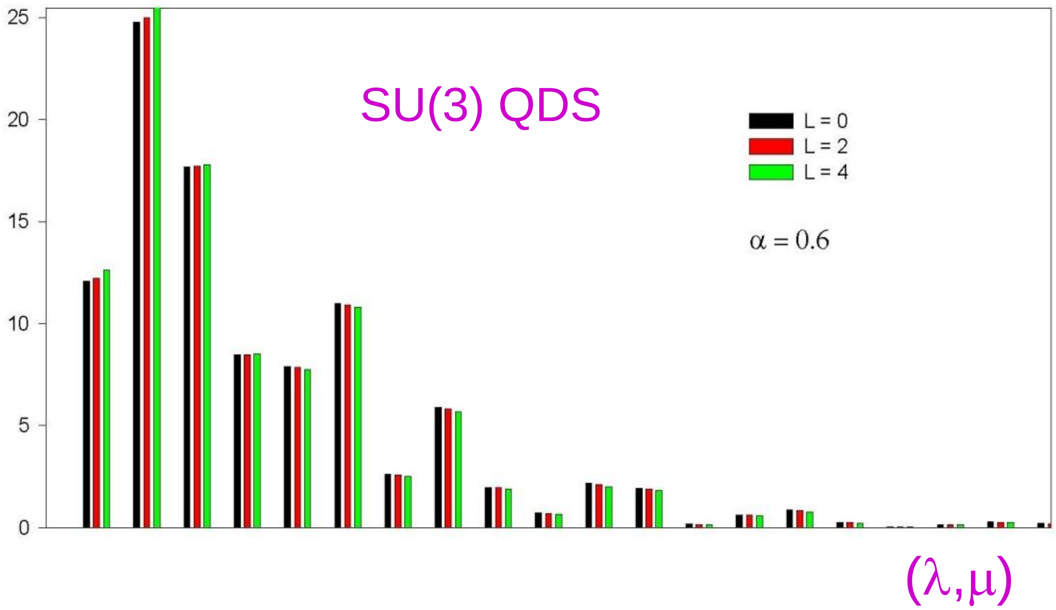
$$P_0|[N](2N-4k,2k),K=2k,L\rangle = 0 \qquad k=1,2,\ldots$$

- **Solvable** bands: g(**K**=0) , γ$^{k}$(**K**=2k) **good SU(3) symmetry (2N-4k,2k)** $E_k = 6h_2(2N+1-2k)$
- Other bands: **mixed**

*Leviatan, PRL **66**, 818 (1996)*

# Quasi Dynamical Symmetry (QDS)





$$H = (1- \alpha)\, H_{U(5)} + \alpha\, H_{SU(3)}$$

away from the critical point
selected states display properties
similar to the closest DS

w.f. display strong but coherent mixing

SU(3) mixing is similar for all L-states
in the ground band

QDS ↔ intrinsic states ↔ adiabaticity

Rowe et al., NPA (2004, 2005)

# Symmetry properties of the QPT Hamiltonian

$$\hat{H}_1(\rho)/\bar{h}_2 = 2(1-2\rho^2)\hat{n}_d(\hat{n}_d-1) + 2R_2^\dagger(\rho)\cdot\tilde{R}_2(\rho)$$ spherical

$$\hat{H}_2(\xi)/\bar{h}_2 = \xi P_0^\dagger P_0 + P_2^\dagger\cdot\tilde{P}_2$$ deformed

# Symmetry aspects

- Exact dynamical symmetry (DS) ALL states solvable

$H_1(\rho = 0)$ U(5) DS $|[N]\; n_d\; \tau\; L\rangle$

$H_2(\xi = 1)$ SU(3) DS $|[N]\; (\lambda\;,\mu\;)\; K\; L\rangle$

- Partial dynamical symmetry (PDS) SOME states solvable

$H_1(\rho \neq 0)$ U(5) PDS $|[N]\; n_d = \tau = L = 0\rangle$

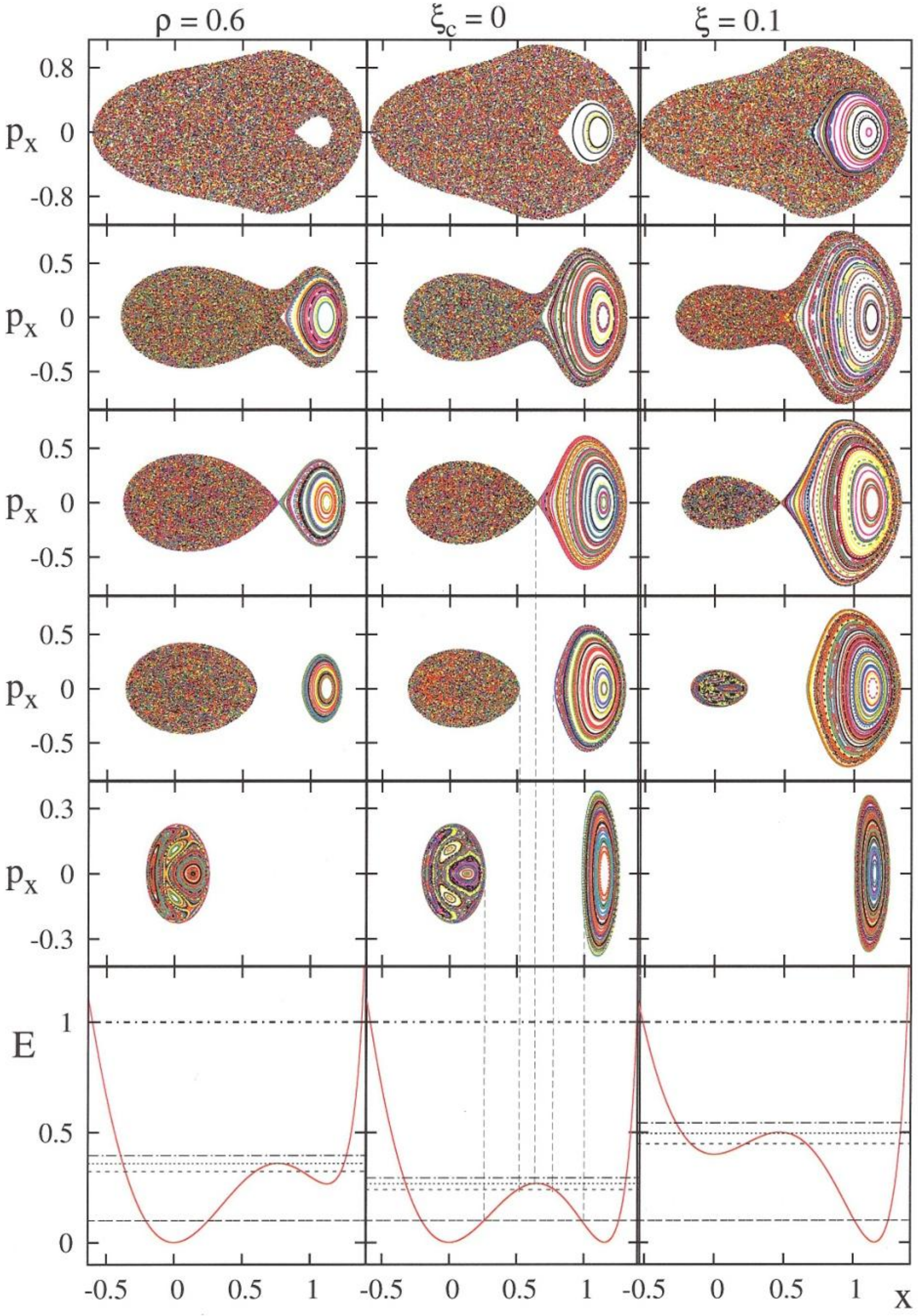
$|[N]\; n_d = \tau = L = 3\rangle$

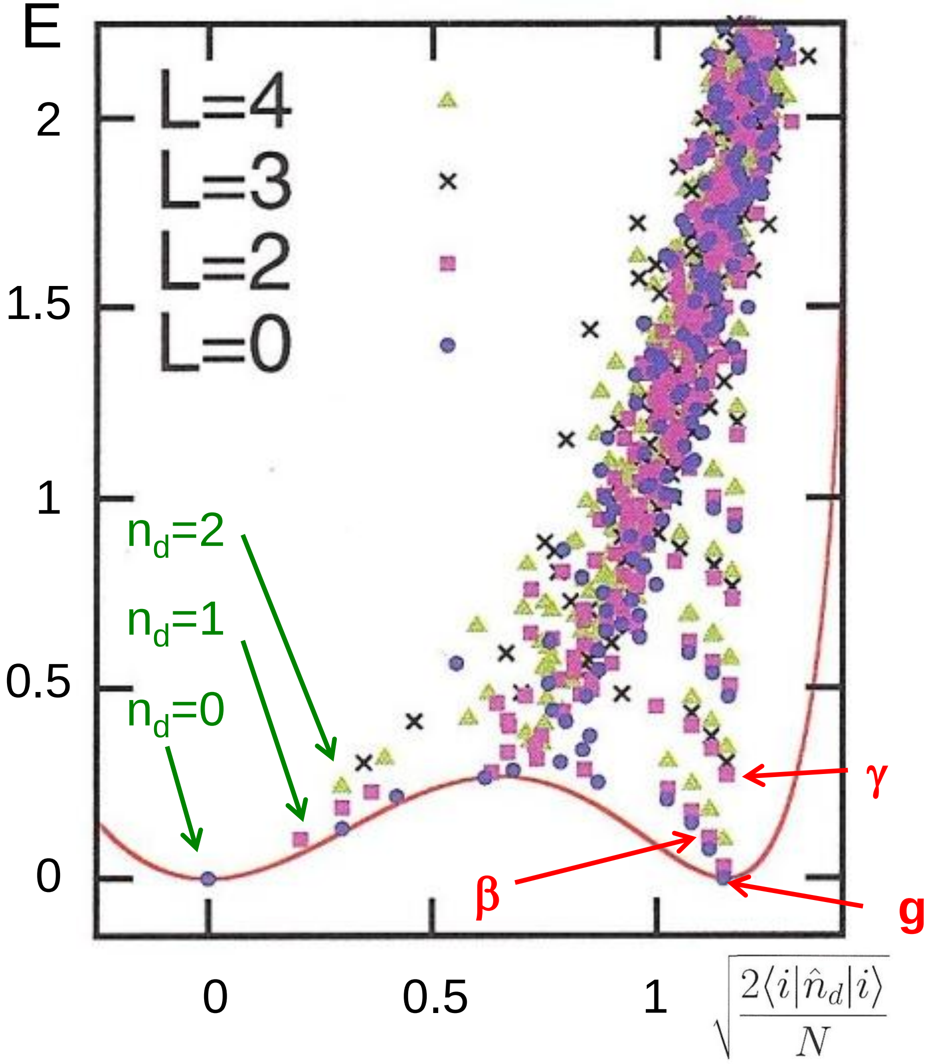
$H_2(\xi \neq 1)$ SU(3) DS $|[N]\; (2N,0)\; K\; L\rangle$ $L = 0,2,4,\ldots,\; 2N$ g(K=0)

$|[N]\; (2N\text{-}4k,2k\;)\; K\; L\rangle$ $L = K,\; K+1,\ldots,(2N\text{-}2k)$ $\gamma^k(K=2k)$

- Quasi dynamical symmetry (QDS) “APPARENT” symmetry

subset of observables exhibit properties of
a DS in spite of strong symmetry-breaking



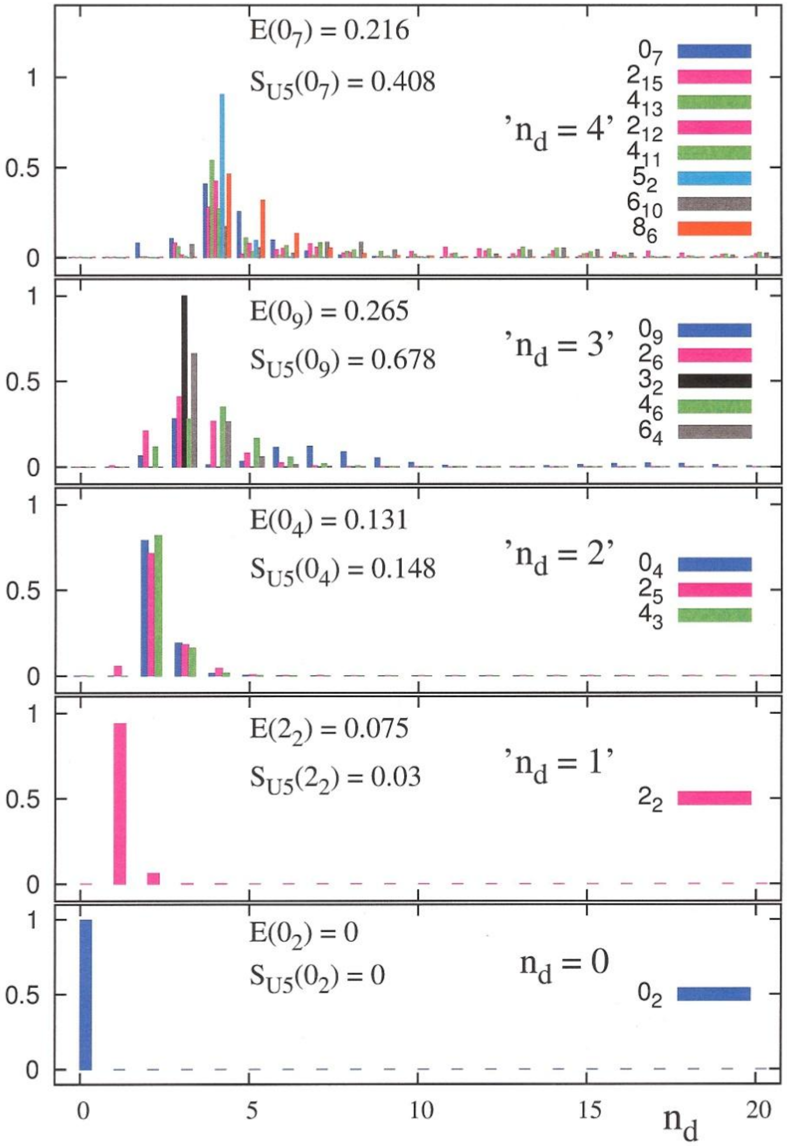


Regular U(5)-like spherical $n_d$ multiplets

Regular SU(3)-like deformed K-bands

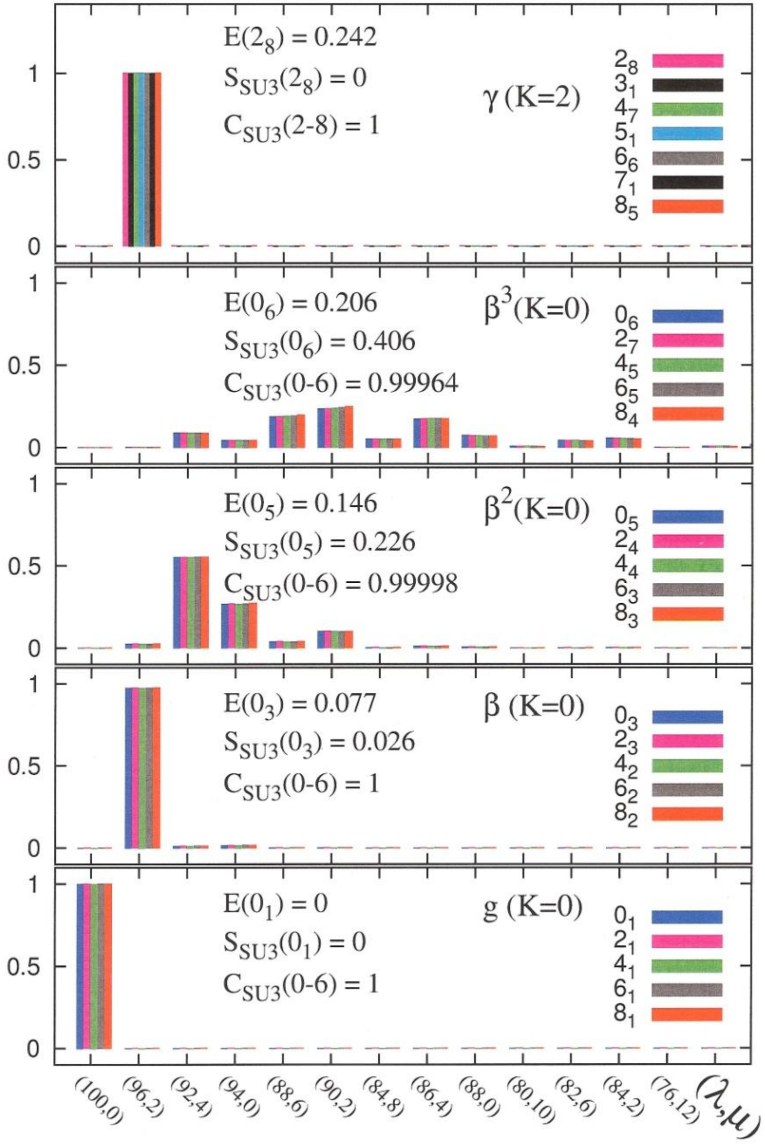
Macek, Leviatan, PRC **84**, 041302(R) (2011)
Leviatan, Macek, PLB **714**, 110 (2012)

Persisting spherical $n_d$ multiplets



(approximate) U(5) PDS

Persisting deformed K-bands



SU(3) QDS

Macek, Leviatan, (2014)

# Measures of PDS

$|L\rangle = \sum_i C_i \, |N, \alpha_i, L\rangle$ $\quad C^{(L)}_{n_d,\tau,n_\Delta} \; , \; C^{(L)}_{(\lambda,\mu),K}$ U(5) , SU(3) decomposition

Shannon entropy

Probability distribution

$$S_{\mathrm{U5}}(L) = -\sum_{n_d} P^{(L)}_{n_d} \ln P^{(L)}_{n_d}$$

$$P^{(L)}_{n_d} = \sum_{\tau,n_\Delta} |C^{(L)}_{n_d,\tau,n_\Delta}|^2$$

$$S_{\mathrm{SU3}}(L) = -\sum_{(\lambda,\mu)} P^{(L)}_{(\lambda,\mu)} \ln P^{(L)}_{(\lambda,\mu)}$$

$$P^{(L)}_{(\lambda,\mu)} = \sum_{K} |C^{(L)}_{(\lambda,\mu),K}|^2$$

$S_G(L) = 0$ pure states

# Measures of QDS

SU(3) decomposition $|L\rangle = \sum_{(\lambda,\mu),K} C^{(L)}_{(\lambda,\mu),K} |N,(\lambda,\mu),K,L\rangle \qquad P^{(L)}_{(\lambda,\mu)} = \sum_K |C^{(L)}_{(\lambda,\mu),K}|^2$

$C^{(L)}_{(\lambda,\mu),K}$ ≈ independent of L, highly correlated

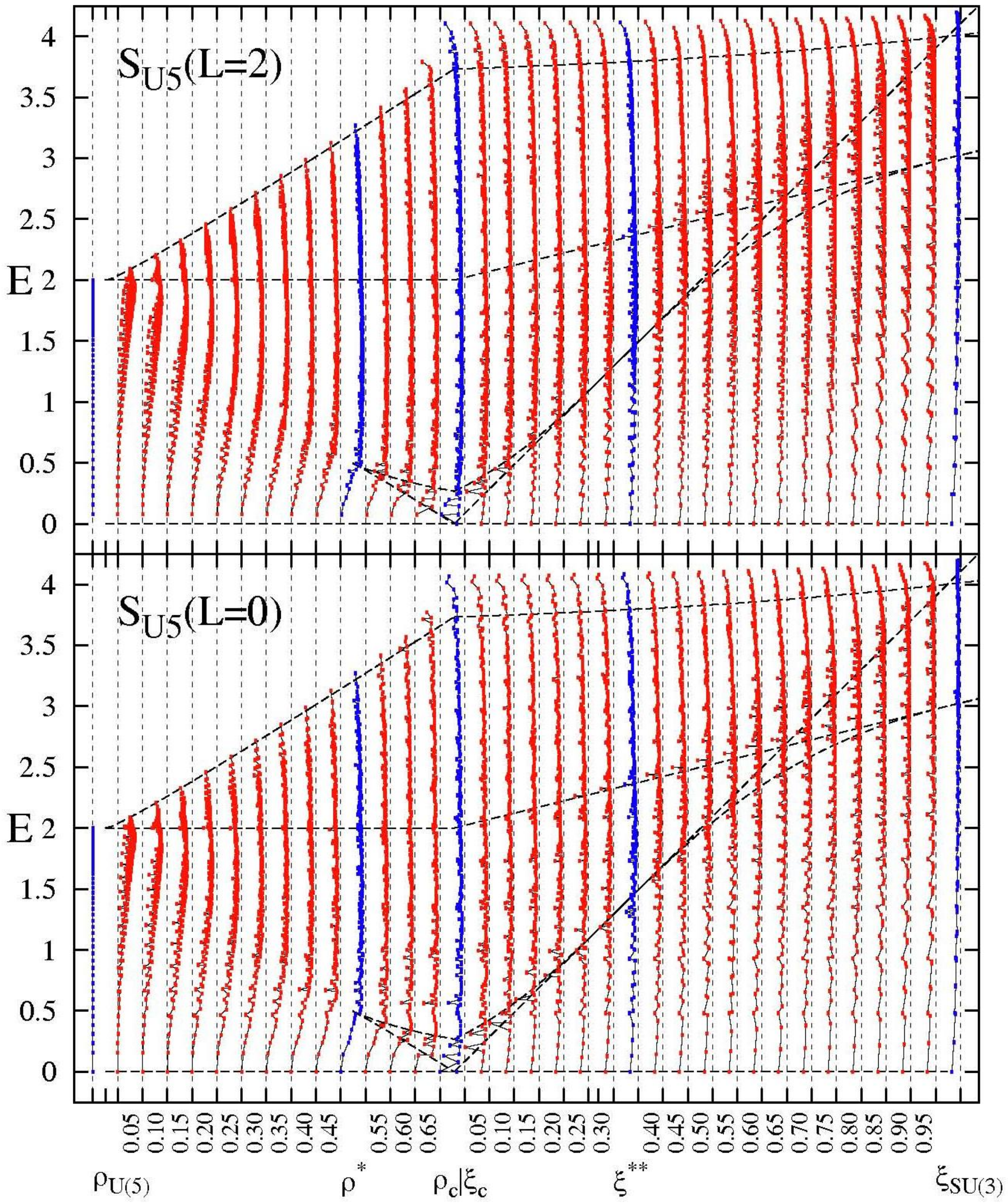
- Pearson correlation $\pi(X,Y) = \frac{1}{n-1}\sum_{m=1}^{n} \frac{(X_m - \overline{X})}{\sigma_X}\frac{(Y_m - \overline{Y})}{\sigma_Y}$

$\pi(X,Y) = 1$ perfect correlation
$\pi(X,Y) = 0$ no linear correlation

$$\boxed{C_{\mathrm{SU3}}(0_i{-}6) \equiv \max_j\{\pi(0_i,2_j)\}\ \max_k\{\pi(0_i,4_k)\}\ \max_\ell\{\pi(0_i,6_\ell)\}}$$

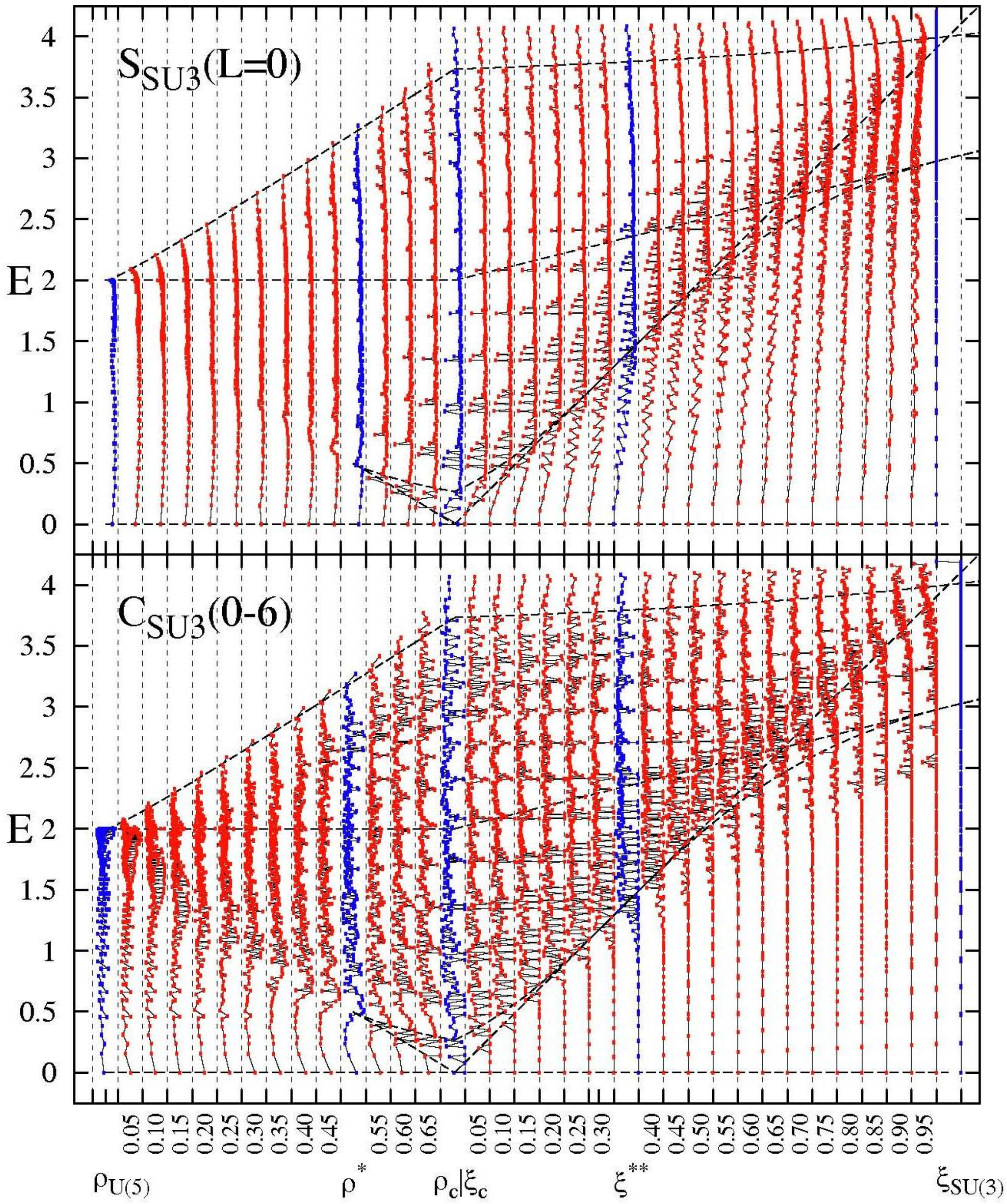
$C_{SU3}(0\text{-}6) \approx 1$ L = 0, 2, 4, 6 correlated and form a band SU(3) QDS

- PDS and QDS monitor the remaining regularity in the system



U(5) PDS

$S_{U5}(L) = 0$ U(5)-purity

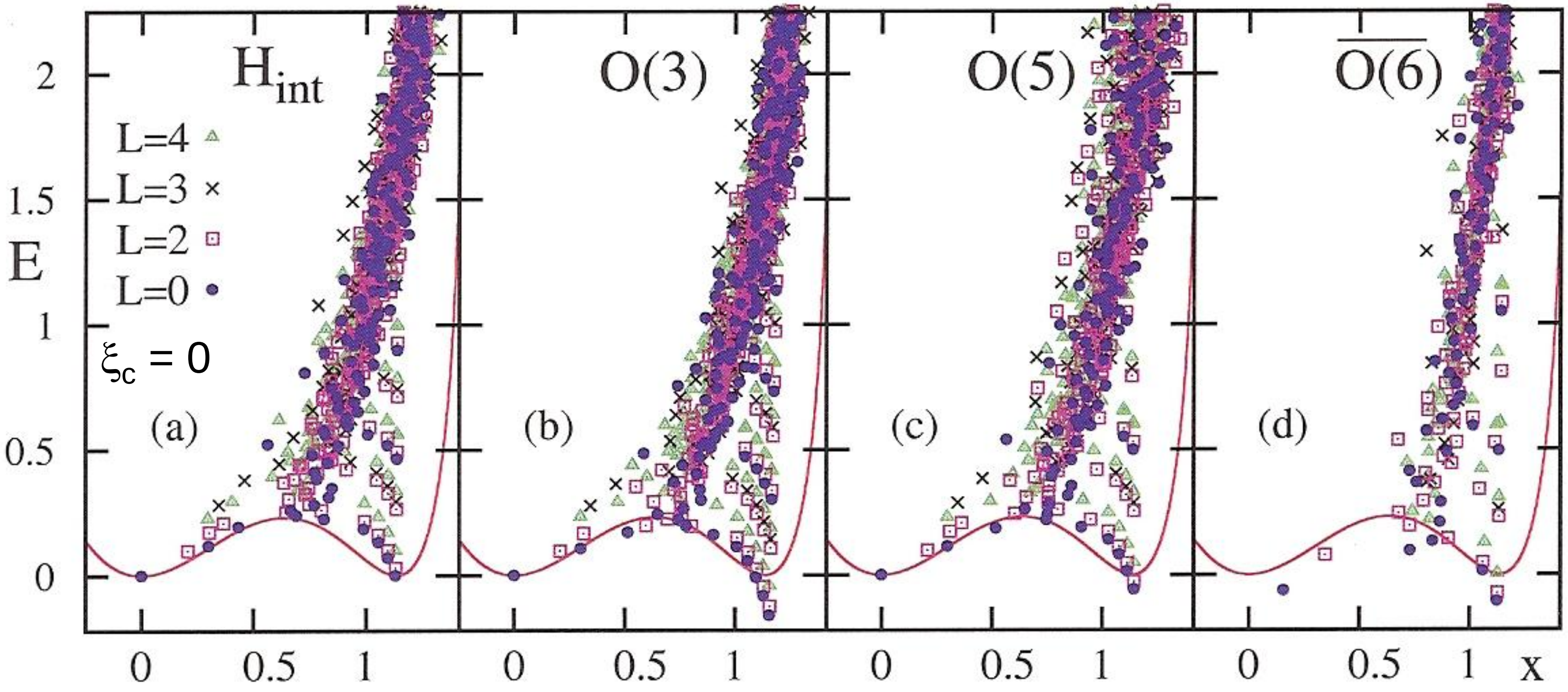


$S_{SU3}(L) = 0$    SU(3)-purity

SU(3) QDS

$C_{SU3}(0\text{-}6) \approx 1$

coherent SU(3) mixing



Collective Hamiltonian

$$\hat{H}_{\rm col} = \bar{c}_3[\,\hat{C}_{O(3)} - 6\hat{n}_d\,] + \bar{c}_5[\,\hat{C}_{O(5)} - 4\hat{n}_d\,] + \bar{c}_6[\,\hat{C}_{\overline{O(6)}} - 5\hat{N}\,]$$

Collective rotations associated with Euler angles, $\gamma$ and $\beta$ d.o.f.

O(3) & O(5) preserve the ordered band-structure, $\overline{\text{O(6)}}$ disrupts it

# Summary

- The competing interactions that drive a $1^{st}$ order QPT can give rise to an intricate interplay of order and chaos, which reflects the structural evolution

- The dynamics inside the phase coexistence region exhibits a very simple pattern

- A classical analysis reveals a robustly regular dynamics confined to the deformed region and well separated from a chaotic dynamics ascribed to the spherical region

- A quantum analysis discloses several low-E regular $n_d$ -multiplets in the spherical region and several regular K-bands extending to high E and L, in the deformed region. These subsets of states retain their identity amidst a complicated environment of other states

- The regular sequences exhibit U(5)-PDS or SU(3) QDS

- Deviations from this marked separation is largely due to kinetic rotational terms

"simplicity out of complexity" ↔ Quasi symmetries

# Thank you