The photonuclear cross section of Boron-10 from the No Core Shell Model

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What can low-energy nuclear physics do for fundamental physics?

Electromagnetic + weak observables

Can constrain nuclear Hamiltonian precisely.
Is the neutrino-C12 cross section solved?
Electron-scattering form factors,
and many other things…

Unitarity of the CKM matrix: (other work I did)

If CKM matrix is not unitary it could signal beyond standard model physics in the form of new generations of quarks.
Places limits on the existence of Scalar currents.

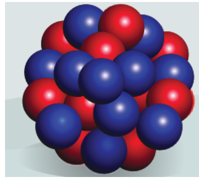
Permanent electric dipole moment (EDM) of light nuclei (He-3, Li-6): (I'd like to do)

If experimentally measured would imply Parity and Time-reversal would be violated.
Note this is not necessarily the θ-term in the QCD Lagrangian.

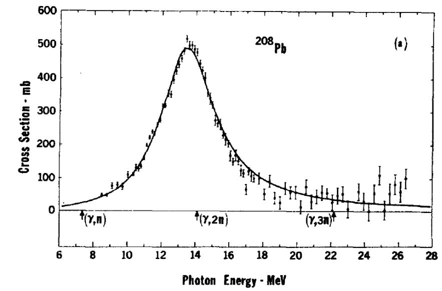
Neutrinoless double-beta decay[1] (e.g. Ge-76).

If observed implies that the neutrino is its own anti-particle, i.e. Majorana.
Furthermore one could say something about the actual masses of the neutrinos directly not just the differences (i.e. mass hierarchy).

[1)] See Avignone III, Elliott, Engel in RMP **80** (2008) for a review



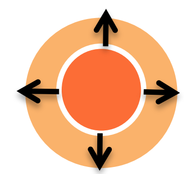
Motivation



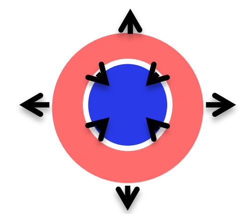
Nuclei can be excited by external fields such as electromagnetic waves (e.g. electric or magnetic dipoles). These **probes are** also excellent **tests of the nuclear Hamiltonian** and also can give **insight into collective motion** of nuclei.

Examples of collective modes:

The **monopoles are breathing modes** of the nucleus.
The proton or neutron fluids can either be in phase (isoscalar) or out of phase (isovector).
The **compressibility** of finite nuclei can be determined from moments of the isoscalar monopole strength function.

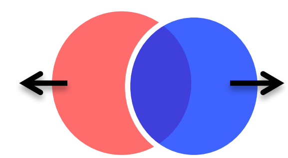


Isoscalar monopole



Isovector monopole

The **electric-dipole (E1)** has the well known **Giant Dipole Resonance** and can be used to study **deformation in nuclei**.

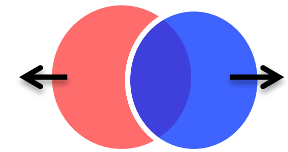


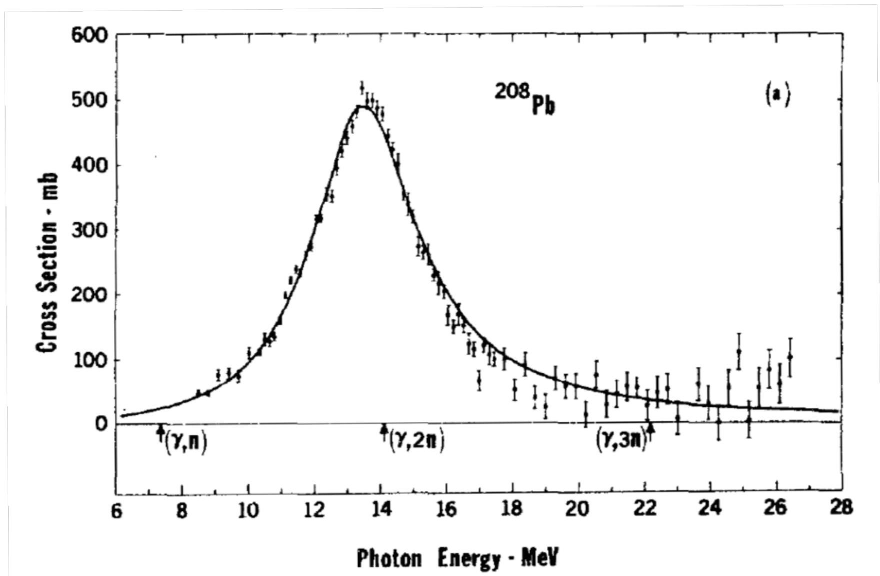
Electric dipole

The **magnetic-dipole (M1)** can be used to study the **scissor modes** in heavier nuclei.

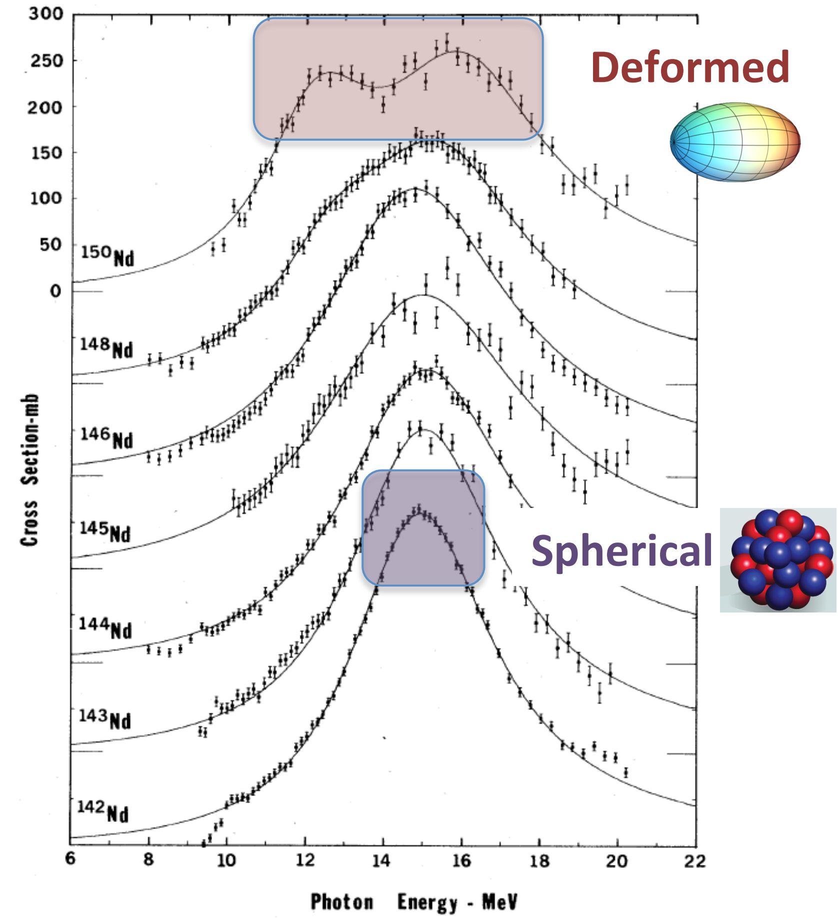
Phenomenological excitation mechanisms of nuclei
Proton fluid in red, Neutron fluid in blue

Examples of Giant Dipoles





The **Giant Dipole Resonance (GDR)** is excited in nuclei by gamma rays. The interesting point is that <u>almost all nuclei exhibit GDR resonances at the same excitation energy</u>.



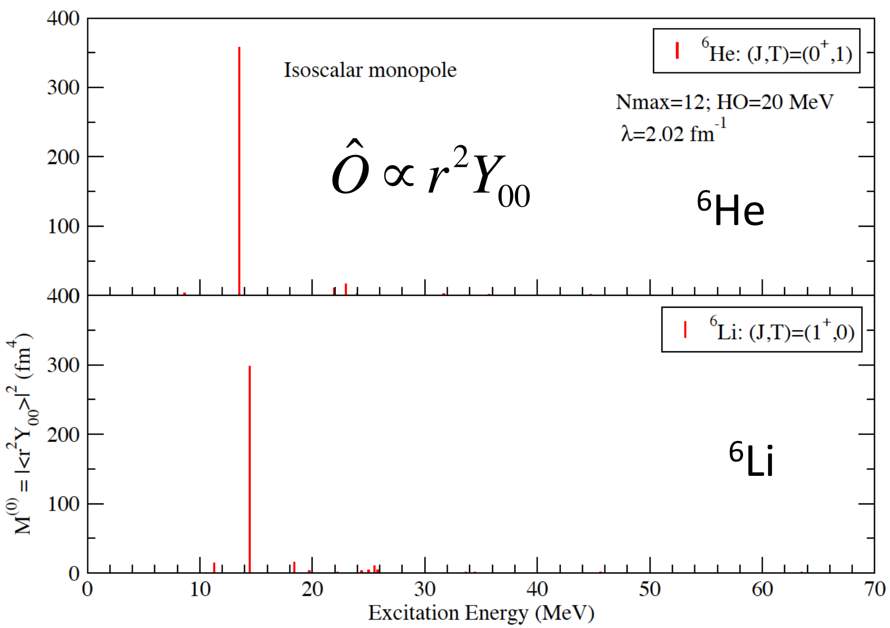
In spherical nuclei the GDR is one peak. In **deformed nuclei the GDR splits along the principal axis of oscillation**. Thus studies of GDR gives us one clue to deformation in nuclei.

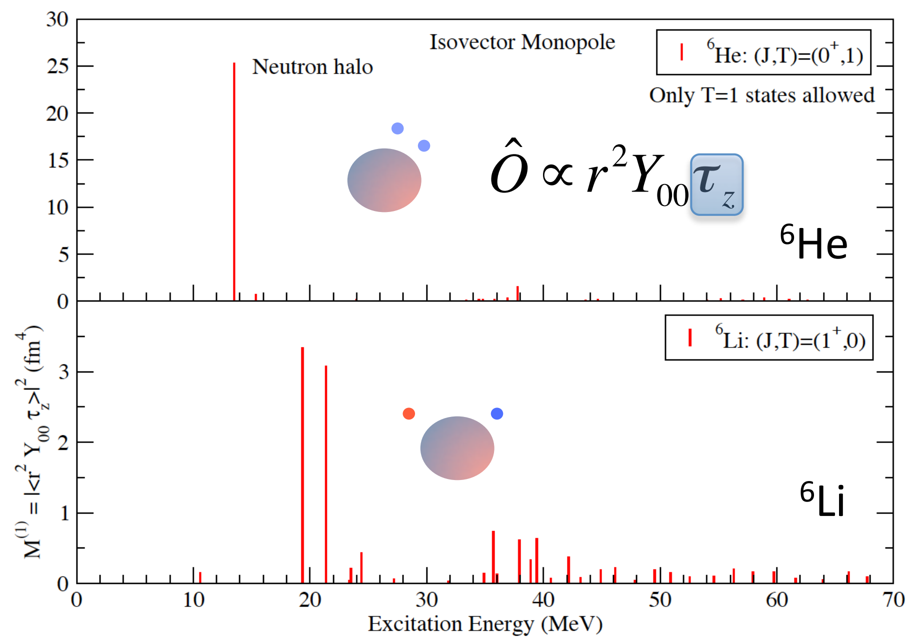
Can theory reproduce these results?

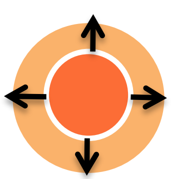
Examples of monopoles

By **collective states** we mean that roughly **50% or more of the total strength** is found in one or a few nearby states.

The strength function is: $$S(\omega) = \sum_f \left| \langle \Psi_f | \hat{O} | \Psi_i \rangle \right|^2 \delta(E_f - E_i - \omega)$$

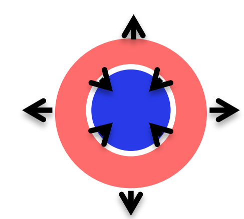




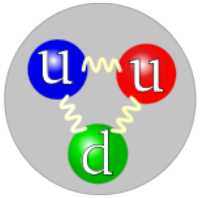


Isoscalar monopole strength function for ^{6}He and ^{6}Li g.s

Isovector monopole strength function for ^{6}He and ^{6}Li g.s



QCD

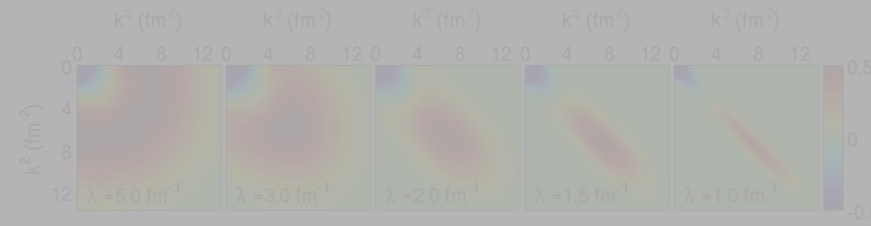


Fundamental theory of strong interactions

(Chiral) Effective Field Theory

FT introduces relevant
r nuclear scales:
N nd pions

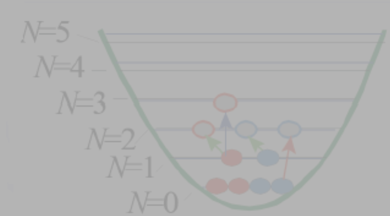
Softened interaction



Similarity group renormalization decouples the high- and low-momentum components of interaction.

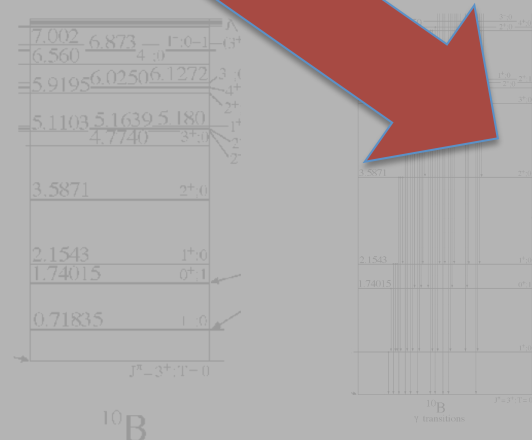
No Core Shell Model

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij} + \sum_{i<j<k}^{A} V_{\mathrm{NNN},ijk}$$



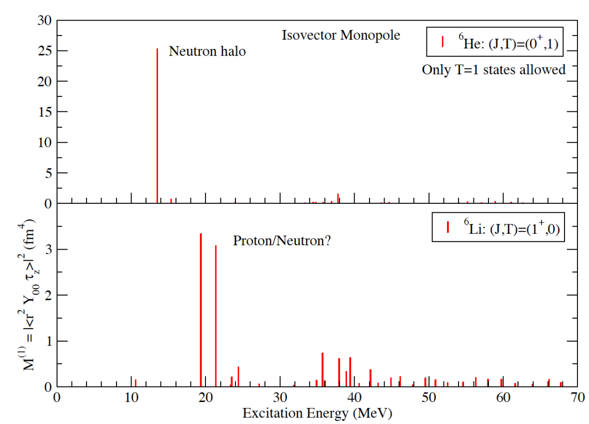
Configuration-interaction type diagonalization in a harmonic oscillator basis.

Eigenva vefunctions



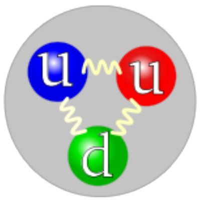
NCSM can give us spectrum and transition rates of light (A < 16) nuclei

Applications: Strength function



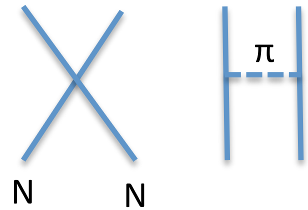
Isovector monopole for ^{6}He and ^{6}Li g.s

QCD



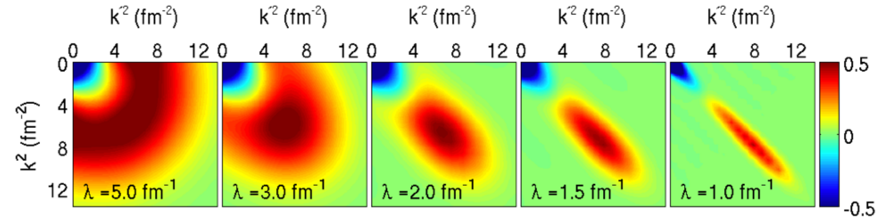
Fundamental theory of strong interactions

(Chiral) Effective Field Theory



EFT introduces relevant dof for nuclear scales: nucleons and pions

Softened interaction



Similarity group renormalization decouples the high- and low-momentum components of interaction.

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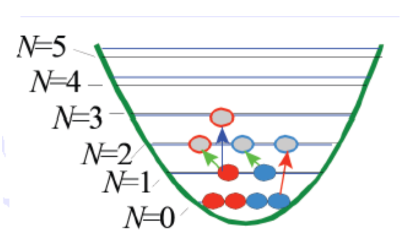
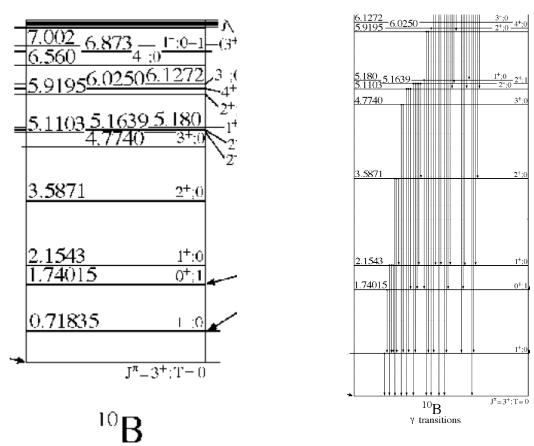
$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij} + \sum_{i<j<k}^{A} V_{\mathrm{NNN},ijk}$$



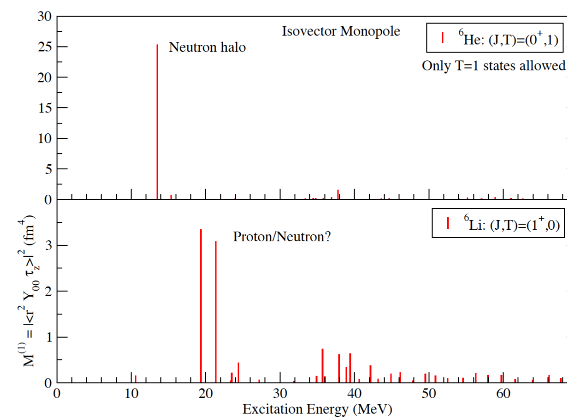
Configuration-interaction type diagonalization in a harmonic oscillator basis.

Eigenvalues, Wavefunctions



NCSM can give us spectrum and transition rates of light nuclei (A < 16).

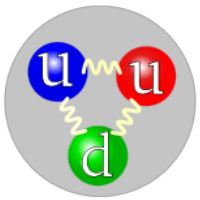
Applications: Strength function



Isovector monopole for ^{6}He and ^{6}Li g.s

Nuclear forces from QCD

QCD

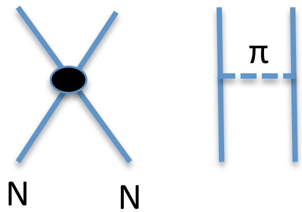


Quarks and gluons interact by exchanging gluons.

At **high momenta** (energy) QCD is perturbative. The coupling constant **α_s is small** and one can use perturbation theory to evaluate processes.

At **low momenta** (energy) QCD is non-perturbative. The coupling constant **α_s is large** making perturbation expansions inappropriate.

Chiral Effective Field Theory

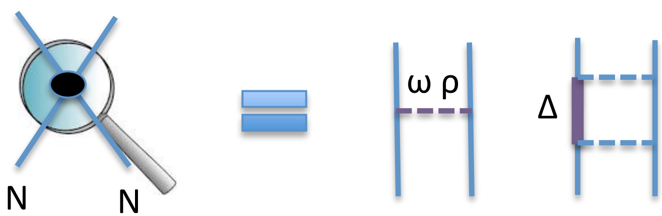


EFT introduces relevant degrees of freedom: nucleons and pions

Effective Field theory is the **bridge between QCD and nuclei.**

A **low-momentum theory** is built from the **symmetries of QCD** that has as degrees of freedom **nucleons interacting via pions.**

The **(unresolved) high-momentum** components are encoded in **low-energy constants** that are determined from experiments.

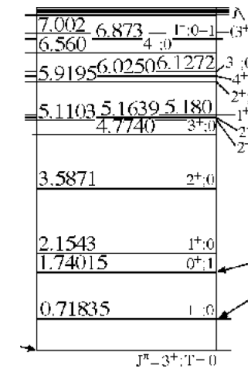


Contact interactions contain all the physics of high-momenta; other mesons or excitations of nucleon (Delta isobar)

No-Core Shell Model

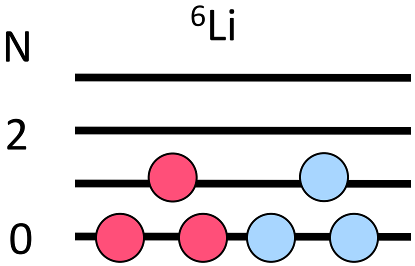
Hamiltonian is translationally invariant.

$$H_A = \frac{1}{A}\sum_{i<j}^{A}\frac{(\vec{p_i}-\vec{p_j})^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij}$$

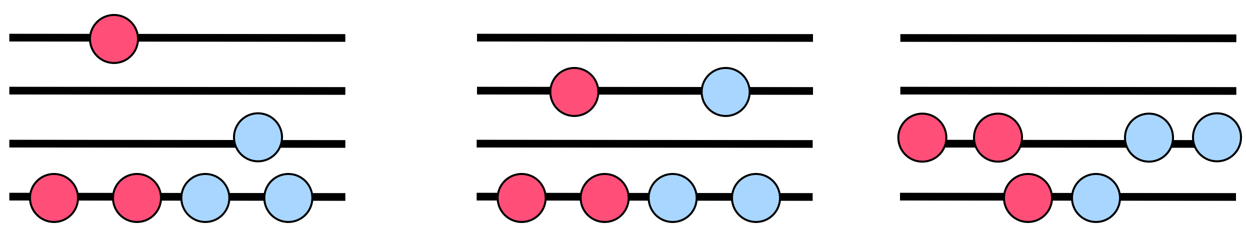


^{10}B

Diagonalizing the Hamiltonian → Observables



Unperturbed cfg.



(Some) Valid Nmax=2 cfgs for positive parity states.

NCSM has two parameters: **Nmax** and **hΩ**

We form an **anti-symmetric basis** made up of Slater determinants. Single particle states are taken as the **harmonic oscillator states**. The Hamiltonian is expressed in this basis and is diagonalized. This gives the energy spectrum and wavefunctions.

Strength functions from Lanczos

Diagonalization the Hamiltonian leads to energies and wavefunctions of the Boron-10 system.

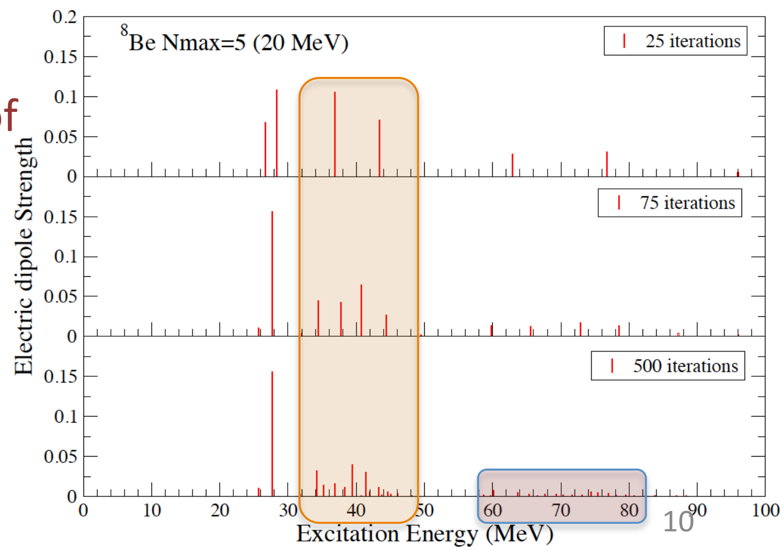
$$H\left|\Psi_i\right\rangle = E_i\left|\Psi_i\right\rangle$$

Acting on the initial wavefunction with the desired operator creates a **starting pivot** for Lanczos which will now **only connect to other states as allowed by selection rules of the operator**.
Example: E1 operator connects states that have ΔT=1, ΔL=1, Δπ=-1.

$$\hat{O}\left|\Psi_i\right\rangle = \left|p\right\rangle$$

It's important to use the **translationally invariant operator**, e.g E1 in B10: $\hat{O} = \left(\frac{1}{2}\right)\sum_{i=1}^{A} rY_{10}\tau_z$

The **pivot** becomes the starting vector for a new calculation using the Lanczos method and the Hamiltonian. Lanczos will generate the 2n-1 moments of the strength function.

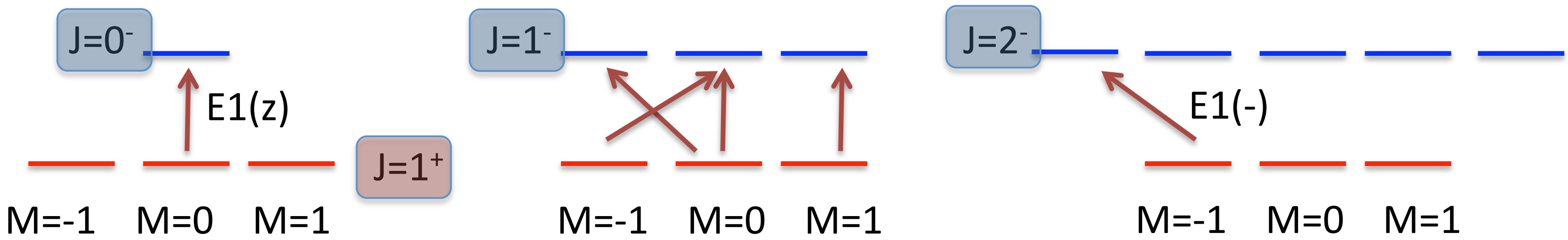


The strength function

$$S(\omega) = \sum_f \left|\left\langle \Psi_f \left|\hat{O}\right|\Psi_i\right\rangle\right|^2 \delta(E_f - E_i - \omega)$$

Creating the reduced (BE1) strength function

We want to calculate the **reduced** strength function of the E1 operator; in other words **BE1**. You want to **average** over the initial state and **sum** over the possible final states taking into account all the different polarizations of the electric-field.



Mathematically all these components can be determined by using the **reduced matrix element**

$$S(\omega) = \frac{1}{2J+1}\sum_{J',M'}\sum_{q}\sum_{M}|\langle J'M'|E1_q|JM\rangle|^2\delta(E'-E-\omega)$$

$$= \sum_{J'}\frac{\boxed{B(E1;J\to J')}}{3}\delta(E'-E-\omega),$$

$$B(E1;J\to J') = \frac{|\langle J'||E1||J\rangle|^2}{2J+1}$$

$$= \frac{2J'+1}{(2J+1)\boxed{|\langle JM10|J'M'\rangle|^2}}\times|\langle J'M'|E1_z|JM\rangle|^2$$

This procedure **requires** that you know what the **angular momentum of each excited state** is (as shown by the Clebsch in the BE1 expression).

Technical details regarding pivot

We want to calculate the **reduced** strength function of the E1 operator; in other words **BE1**. This **requires** that you know what the **angular momentum of each excited state** is.

$$S(\omega) = \frac{1}{2J+1}\sum_{J',M'}\sum_{q}\sum_{M}|\langle J'M'|E1_q|JM\rangle|^2\delta(E'-E-\omega)$$
$$= \sum_{J'}\frac{\boxed{B(E1;J\to J')}}{3}\delta(E'-E-\omega),$$

$$B(E1;J\to J') = \frac{|\langle J'||E1||J\rangle|^2}{2J+1}$$
$$= \frac{2J'+1}{(2J+1)\boxed{|\langle JM10|J'M'\rangle|^2}}\times|\langle J'M'|E1_z|JM\rangle|^2$$

Applying the E1 (rank 1) operator to an angular momentum state J results in a superposition of angular momentum states in the pivot.

$$\hat{O}(E1)\left|J\right\rangle = a\left|J-1\right\rangle + b\left|J\right\rangle + c\left|J+1\right\rangle$$

Impossible to determine J of **unconverged** excited states when Lanczos now runs

But **pre-diagonalizing the pivot with J^2** gives us three **pivots with good J**.
These individual pivots only produce states with the same J throughout the ex. spectrum hence we can determine what the appropriate Clebsch is in the BE1 formula.

We calculate the strength function for each of the **three pivots with good J** and then form the BE1 strength function. The total BE1 strength function is the sum of the three parts.

Electric dipole (E1) for Boron-10

Interaction details:

Chiral N3LO (NN only) SRG λ=2.02 fm^{-1}
No Coulomb force included.
Isoscalar interaction (Vpp = Vnn = Vpn)

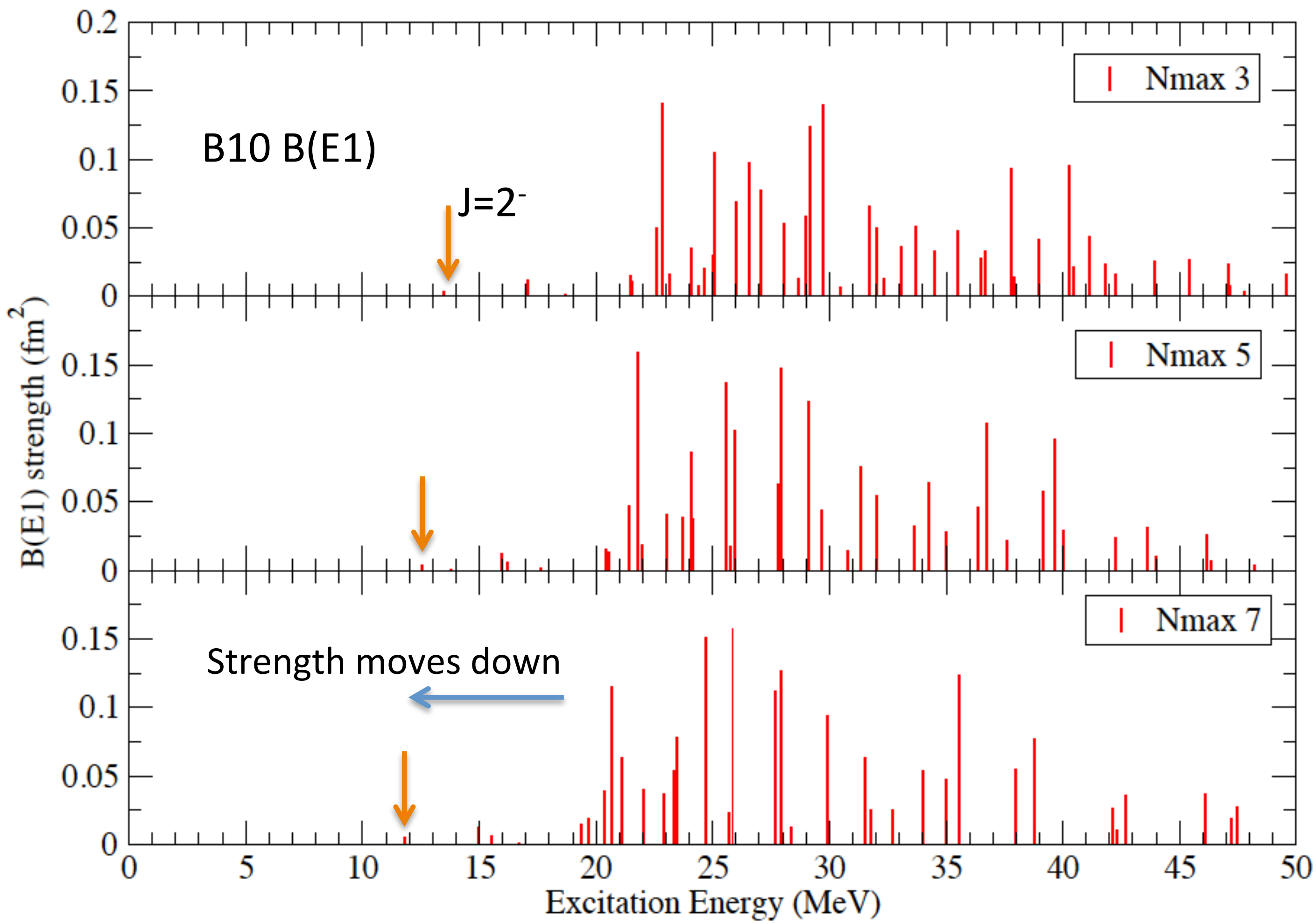
λ and hω combination

At λ=2.02 fm^{-1} and hω=20 MeV one reproduces the binding energies as well as the neutron separation energy of Helium isotopes (^4He, ^6He, ^8He).

NCSM details

Nmax=3-7 (now 9) including both parities in the basis.
500 Lanczos iterations for calculating the spectrum (converge 10 lowest states).
150-500 Lanczos iterations to calculate strength function.
Both M=0 and M=1 basis is used to create BE1 values (ask me about details).

BE1 strength with increasing basis size



Strength distribution shape is robust in Nmax.

Slowly moves down in energy as a function of Nmax.

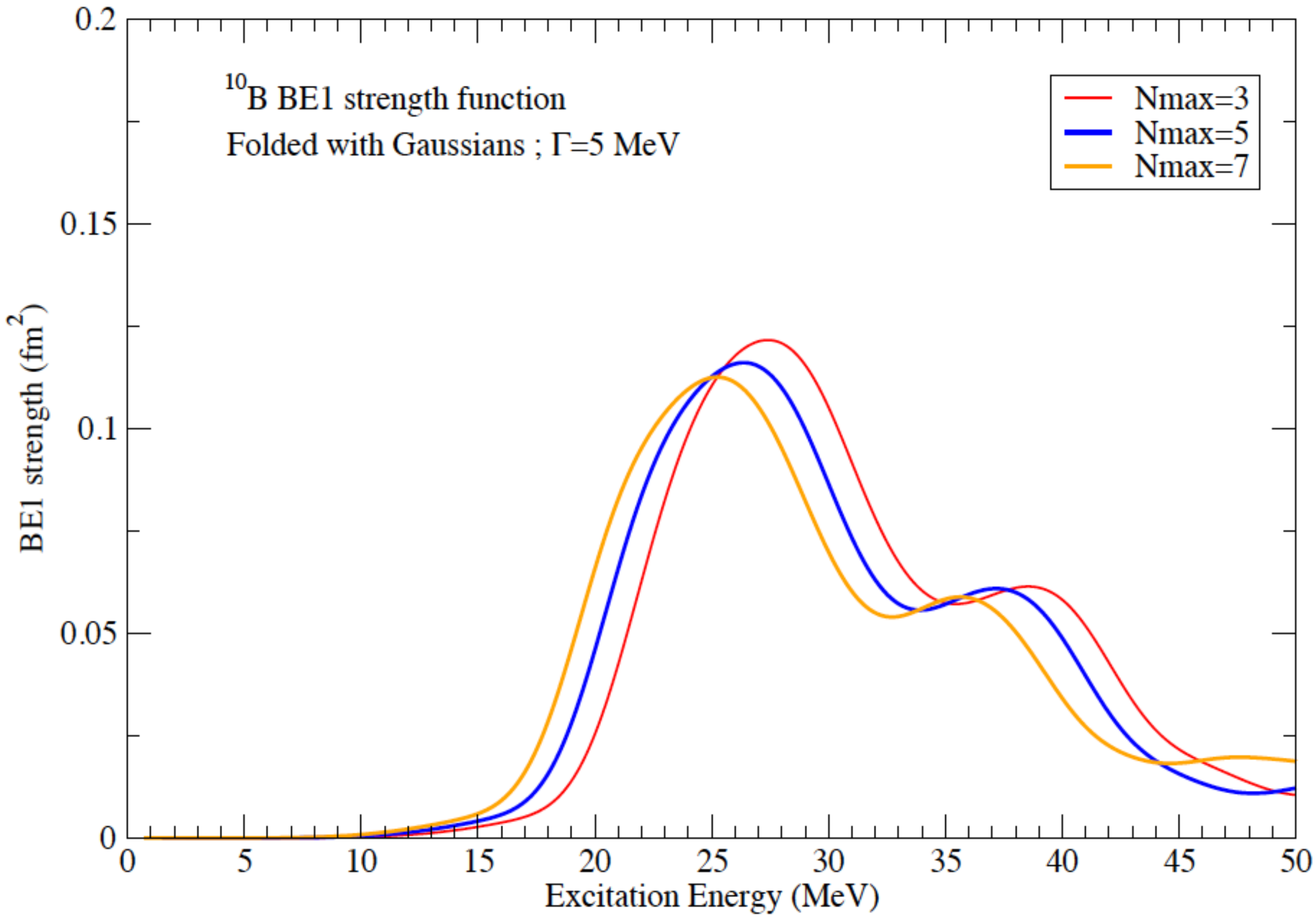
How to extrapolate this distribution?

We used the **2nd $J=2^-$ state** and used

$$E(x) = a\exp(-bx) + c$$

to extrapolate.

BE1 strength with increasing basis size



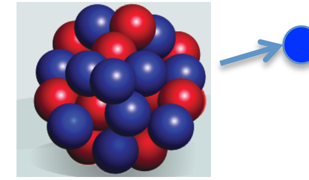
Strength distribution shape is robust in Nmax.

Slowly moves down in energy as a function of Nmax.

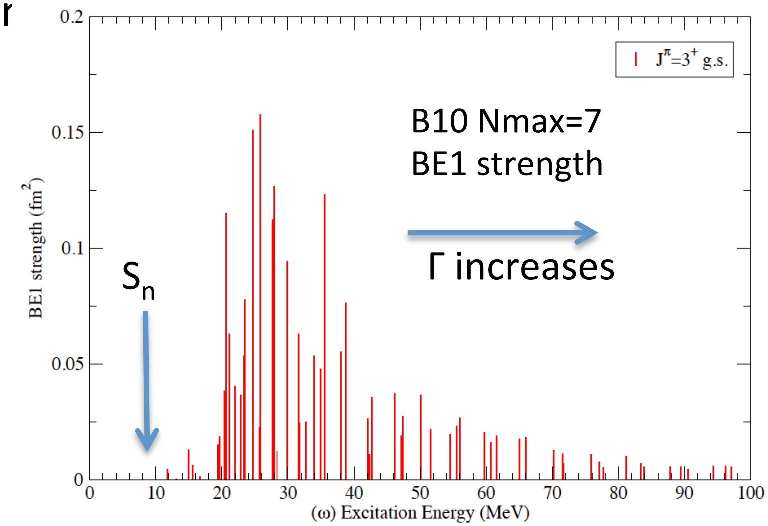
How to extrapolate this distribution?

Perhaps it is best to extrapolate centroids?

Neutron escape widths



Experimentalists measure cross sections. How do we **compare** our results **with data**?
Need to introduce finite widths in our strength function



To assign widths to our discrete states we use **neutron escape widths** because these typically have the **fastest decay rates**.

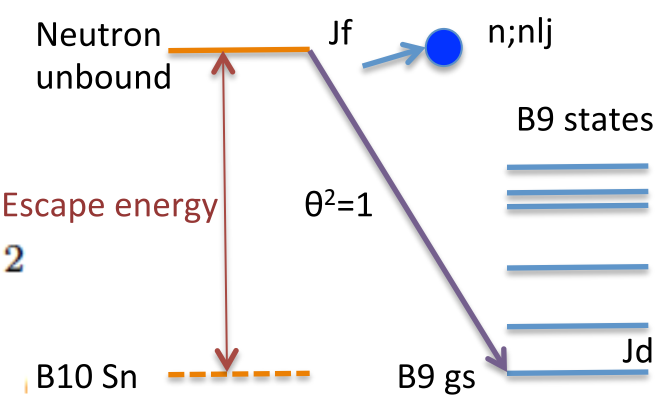
$$\Gamma(\omega - S_n) = 2\gamma_{sp}^2 P_l \Theta_l^2$$

$$\gamma_{sp}^2 = \left(\frac{\hbar c}{\mu R}\right)^2$$

Sp width

$$P_l = kRV_l \qquad V_2 = \frac{(kR)^4}{9 + 3(kR)^2 + (kR)^4}$$

Penetration factor depends on partial wave. Wave vector: $k = \sqrt{2\mu E_n}$



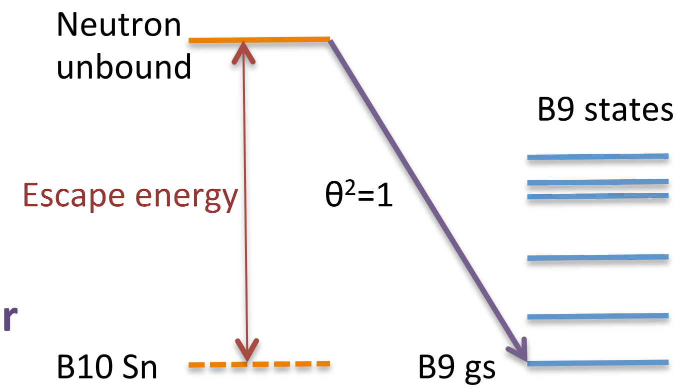
$$\theta_l^2 = \left| \left[\langle {}^9B; J_d T_d | \times \langle n; (nlj; 1/2) | \right]^{J_f T_f} | {}^{10}B; J_f T_f \rangle \right|^2$$

The spectroscopic factor is the overlap of a ^{9}B+n (coupled) wavefunction with the initial ^{10}B state

Is the spectroscopic factor unity to gs?

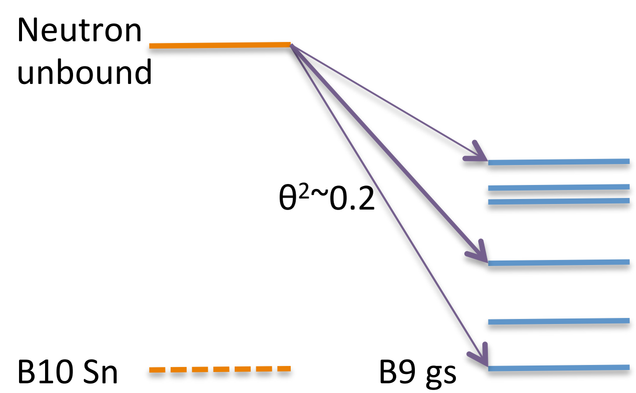
To assign widths to our discrete states we use **neutron escape widths** because these typically have the **fastest decay rates**.

$$\Gamma(\omega - S_n) = 2\gamma_{sp}^2 P_l \boxed{\Theta_l^2}$$



We initially assumed the square of the **spectroscopic factor** is unity for a neutron-unbound state going to the B9 ground-state.
But is that actually the case?

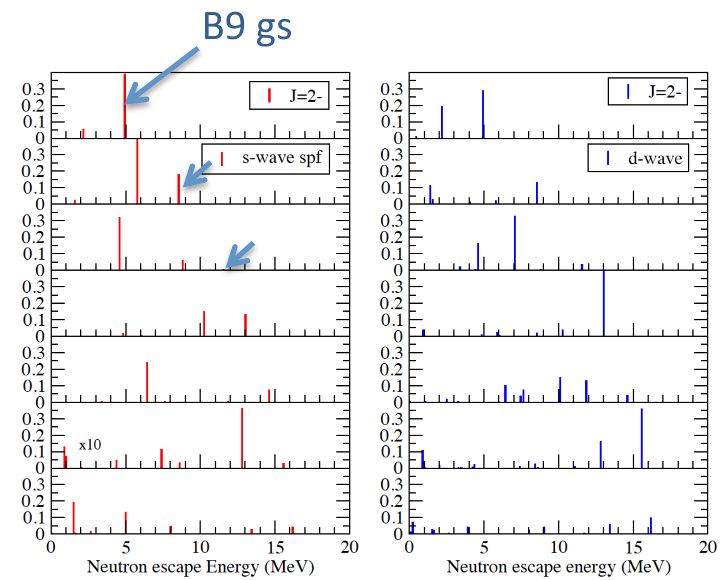
In reality the B10 neutron-unbound state decays into a number of B9 excited states and perhaps the ground-state. This being the case we effectively weigh the neutron-escape energy that we use in the penetration factors by the fragmented spectroscopic factors.



In general it is very difficult to calculate all the spectroscopic factors for every state in B10 in our BE1 strength function. We thus **build a model from small space calculations** that captures the fragmentation of the spectroscopic factors using empirical Nmax=3 and Nmax=5 data.

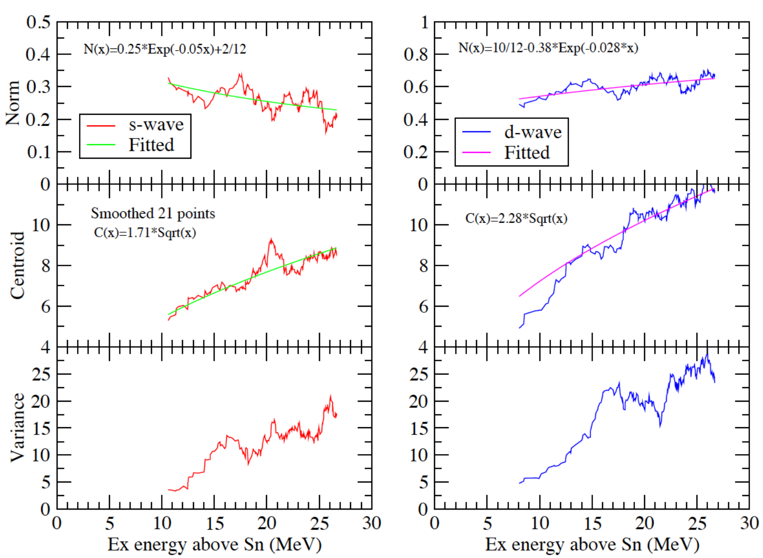
Spectroscopic factor model

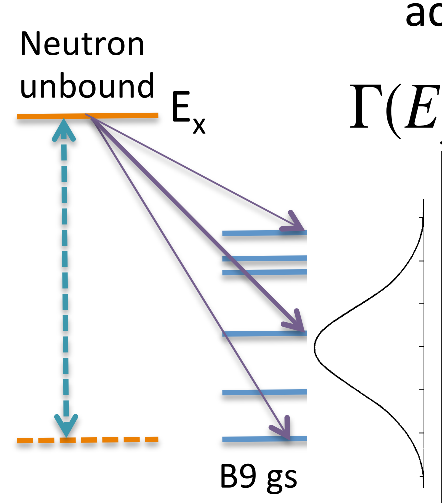
Performed Nmax=3 calculations for 250 states in B10 that can decay into 100 B9 states. Needed about 3000 Lanczos iterations to get those 250 states converged (good JT).



Assume all we need is the **norm, centroid and variance** of the distribution to describe the spectroscopic factors. Use a normalized Gaussian for this purpose.

These quantities are fitted with functions that fit the (smoothed) data as well as respect physical constraints.





Recalculate the width of a state according to

$$\Gamma(E_x) = 2\gamma_{sp}^2 \sum_l \int_0^{E_x} dE\, P_l(E)\, \Theta_l^2(E)$$

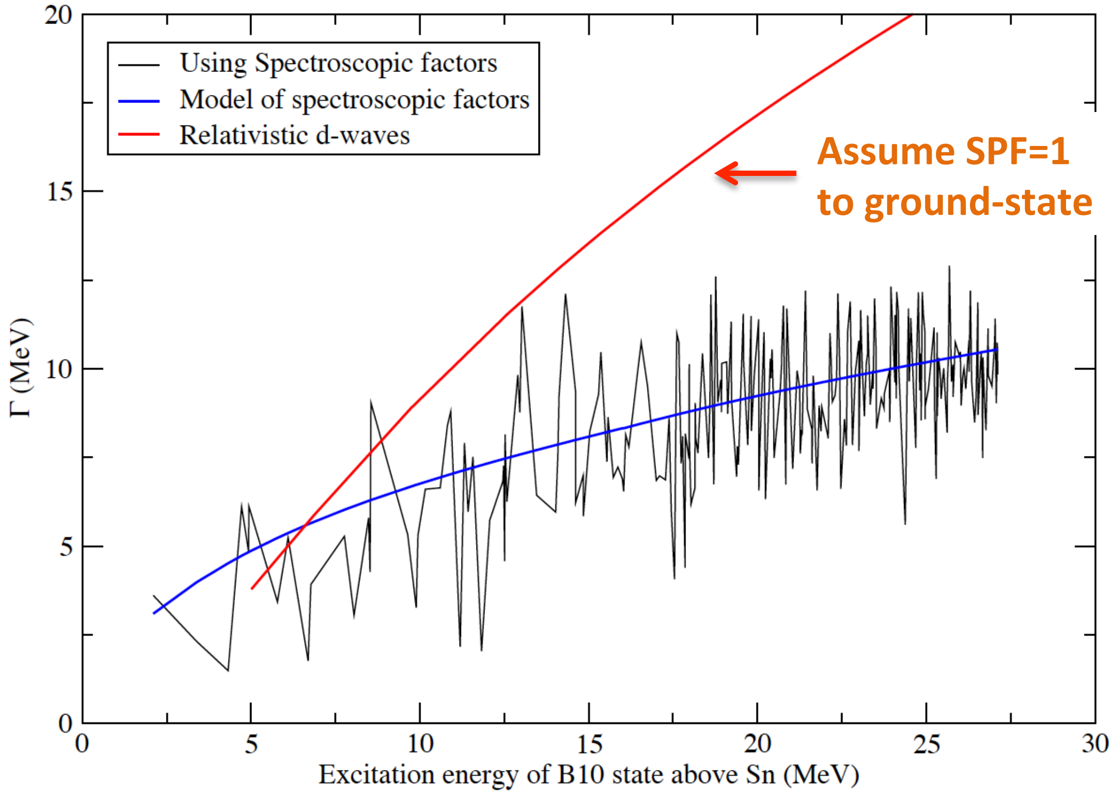
Integrate over all B9 states below B10 state.

Spectroscopic model

Spectroscopic model vs actual widths

The spectroscopic model works quite well when you compare what the prediction of the width is compared to the actual width (from Nmax=3 or Nmax=5).

The spectroscopic model significantly reduces the widths of our states as compared to the "relativistic" widths we were using before (where we assumed SPF = 1). The model also reproduces on average the width of the states calculated from spectroscopic factors.



$$\Gamma(\omega - S_n) = 2\gamma_{sp}^2 P_l \Theta_l^2$$

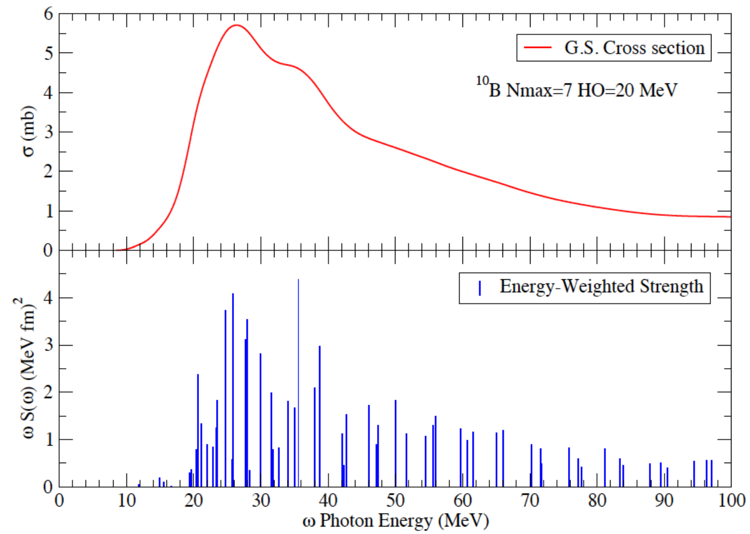
$$\Gamma(E_x) = 2\gamma_{sp}^2 \sum_l \int_0^{E_x} dE P_l(E)\Theta_l^2(E)$$

Cross-sections

The cross section is determined in Bohr& Mottelson vol II

$$\sigma(\omega) = \frac{2}{\pi} \sum_f \boxed{S_f E_f} \boxed{\frac{\omega^2 \Gamma_f}{(\omega^2 - E_f^2)^2 + (\omega \Gamma_f)^2}}$$

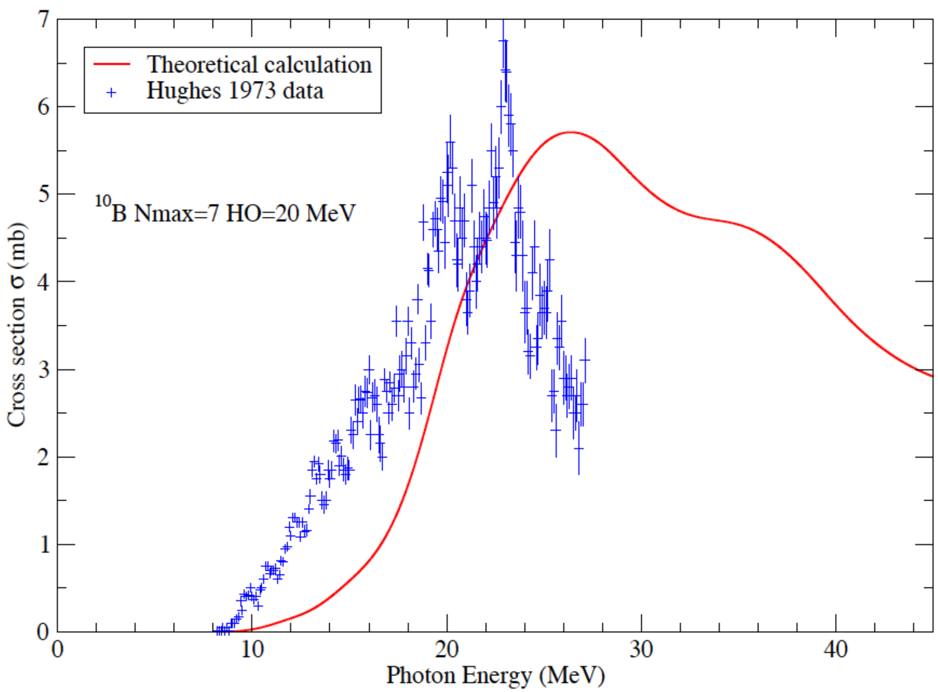
Energy weighted strength   Lorentzian line-shape



The cross section at Nmax=7 shows promising features when compared to experimental data. Note the two-humped structure.

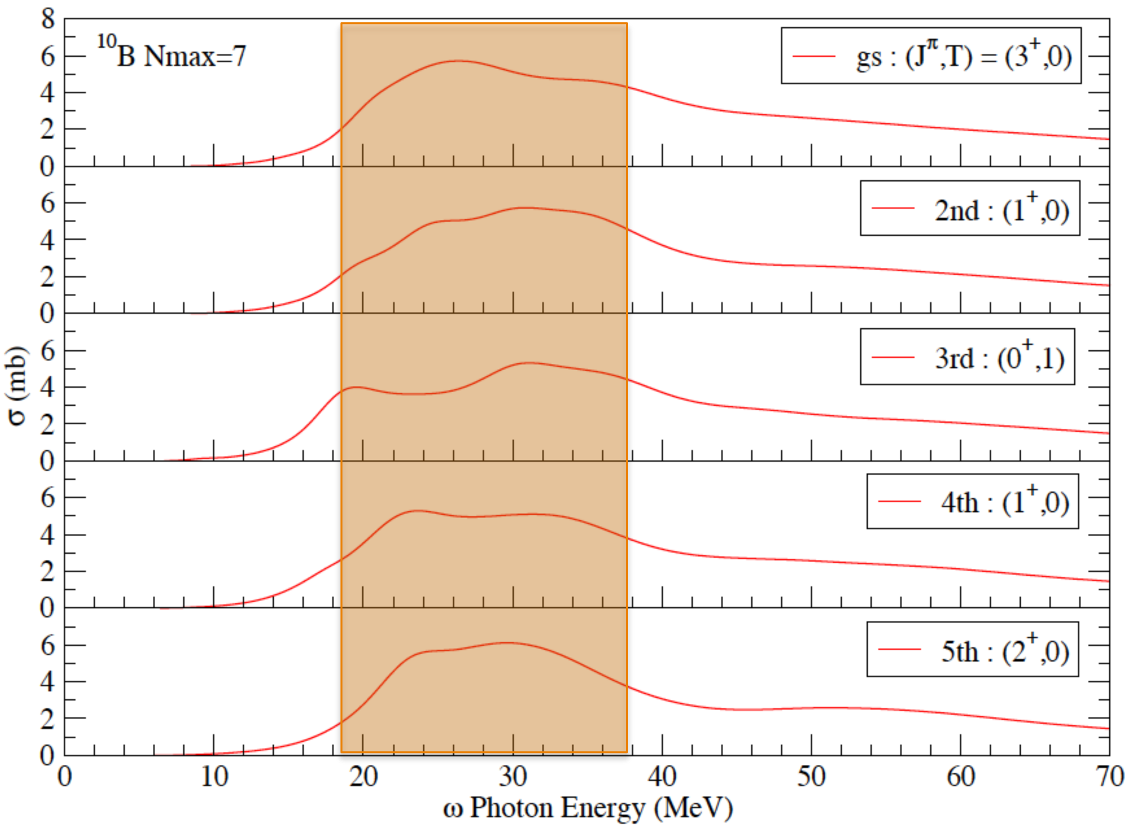
Nmax=9 calculation (in progress) may provide much better agreement with data.

How should cross section be extrapolated?



Brink hypothesis

In 1955 Brink hypothesized that if ground-state supports a dipole resonance then so will the excited states. The GDR should appear at the same energy provided you take into account the excitation energy of the excited state (i.e. all energies are relative to the states considered).

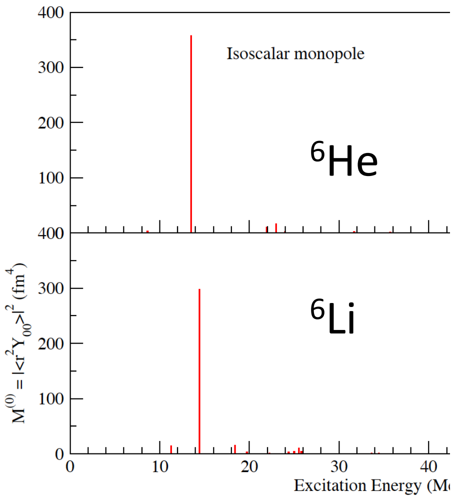


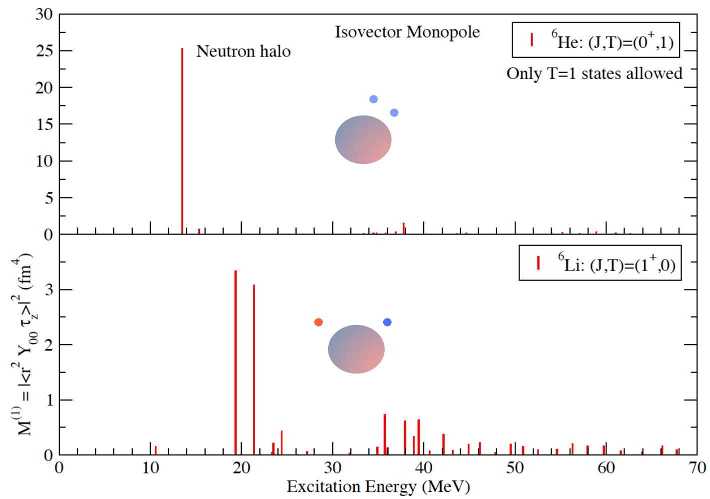
Our calculations confirm Brink hypothesis for first 10 states of B10.
The same trend is found for other nuclei we have looked at.

Conclusions

Collective motion in light nuclei can give us insight into **resonance phenomena**.

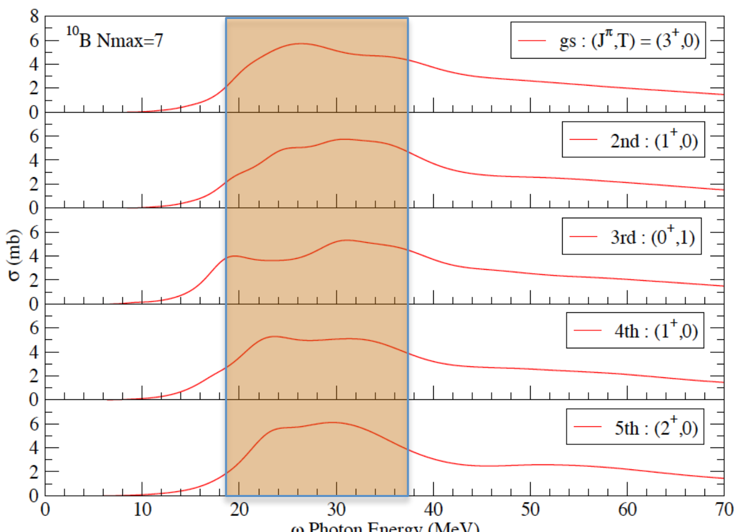
Ab initio strength functions can give insights into how light nuclei exhibit collective motion.





Monopoles tell us about **breathing modes** and determine **compressibility K** in finite nuclei.

Compressibility can be found from moments of strength function; A = 4, K ~ 30



^{10}B E1 cross sections; 5 lowest states

Strength functions are easily found by using **moment-generating method of Lanczos.**

Developed tools to calculate strength functions for various operators (Monopoles, E1, M1 etc)

Used **neutron escape widths** to assign finite widths to unbound states.

Calculated **cross section** by folding in Lorentzian line-shapes with appropriate widths.

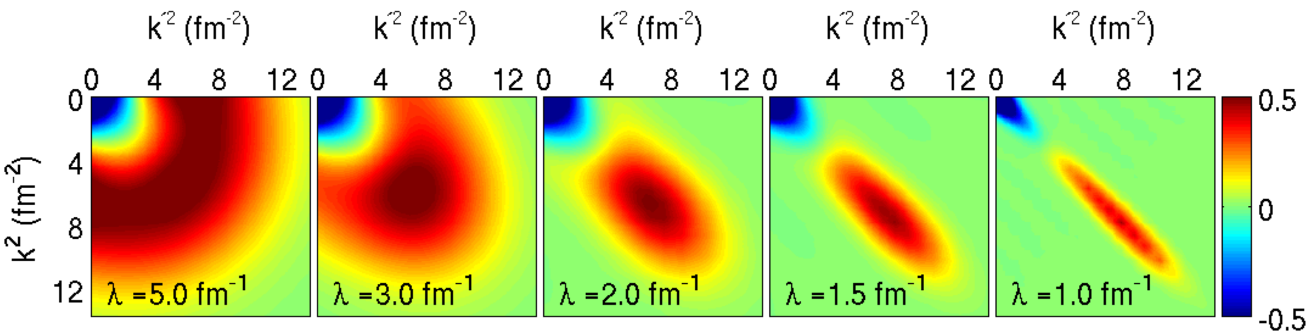
Brink hypothesis tested in ^{10}B and ^{6}Li

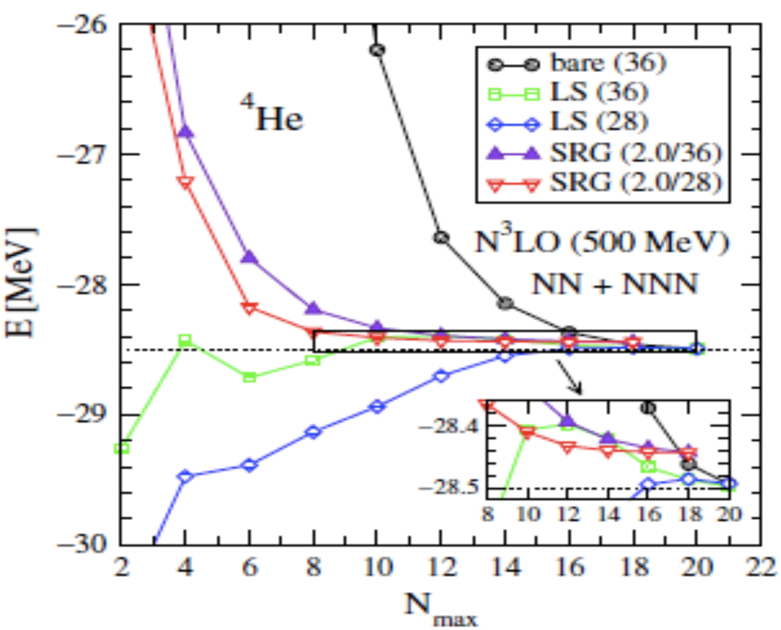
END of presentation

Softened interactions

PRL **103**, 082501 (2009)
Jurgenson, Navratil, …

$$H_\lambda = U_\lambda H_{\lambda=\infty} U_\lambda^\dagger \qquad \frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T, H_\lambda], H_\lambda]$$



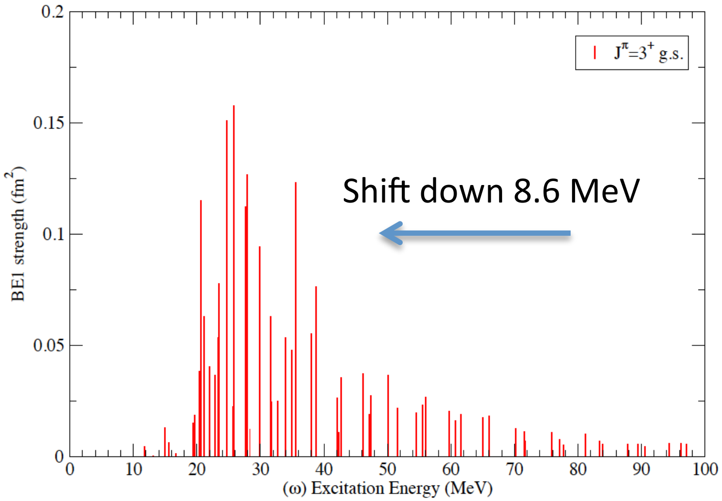


Initial bare chiral interactions contain **strong repulsive** (high-momentum) components that make many-body calculations **difficult to converge**.

Softening the interaction **decouples the high- and low-momentum** components so that we are left with an interaction appropriate for many-body techniques

Preserves long-distance (low-momentum) parts of the interaction (i.e. one-pion exchange)

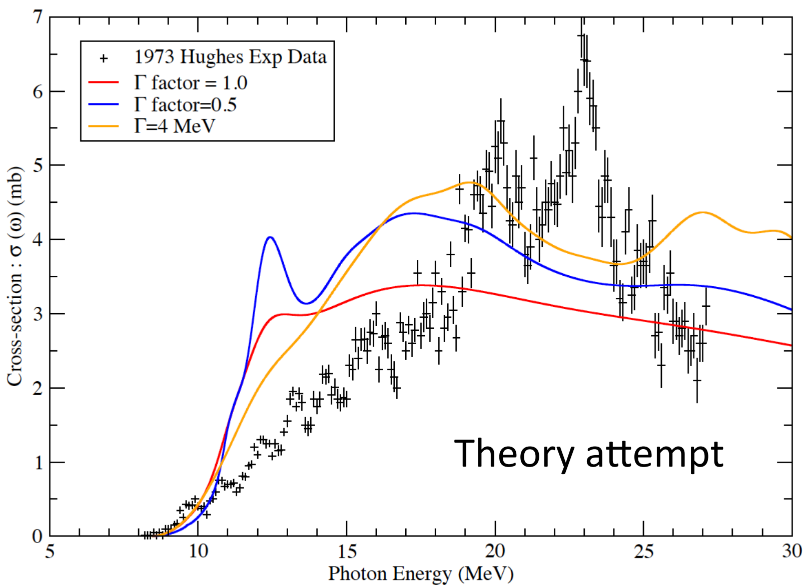
Theory (using relativistic widths) vs experiment



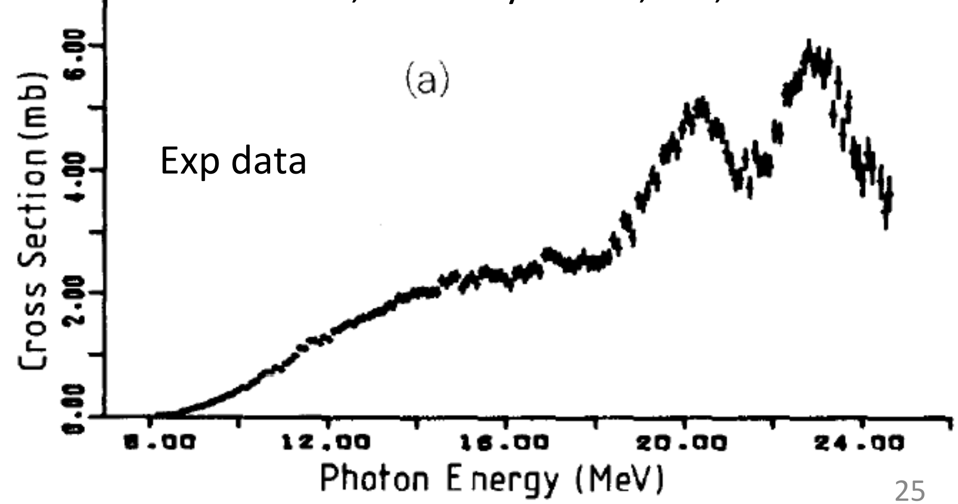
To **converge** the BE1 strength function we **extrapolated the 2nd J=2- by "Nmax" extrapolation.** $E(x) = a\exp(-bx) + c$

The **result** is that the J=2- state **moves down 8.368 MeV**. We move the whole BE1 spectrum down by that much.

The extrapolated cross section has a broad peak which peaks at ~ 20 MeV. Widths may be too big?



Ahsan *et al*, Nucl Phys A469,381, 1987

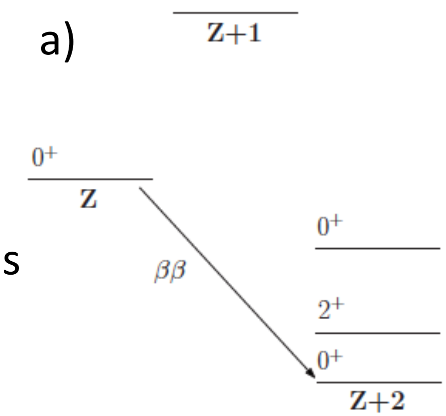


Neutrinoless double beta-decay

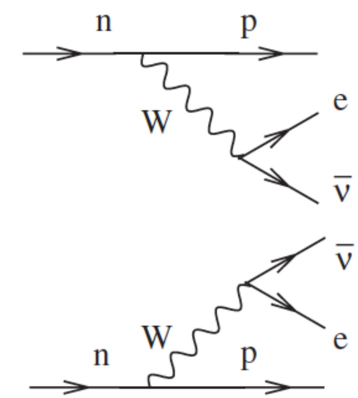
Ordinary double beta decay (2ν) does indeed happen in nature. It is a “second-order” process meaning it is “rare” (i.e. long half-lives on the order of 10^{21} years).

On the right in a) I show the typical energy level diagram of ββ-decay. The parent is an even-even nucleus which implies it is more tightly bound (by pairing) than the Z+1 nucleus but less-bound than the Z+2 nucleus.

a)

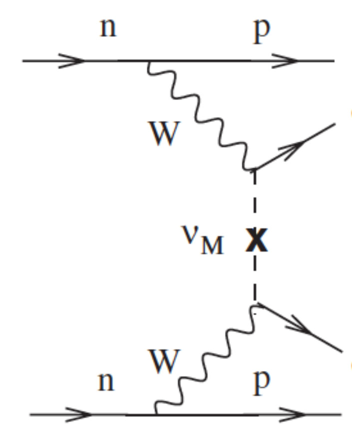


b)



$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}(Q_{\beta\beta}, Z)|M_{2\nu}|^2$$

Neutrinoless double beta decay or ββ (0ν) requires that neutrinos have mass (which they do) and that they are their own anti-particle (Majorana).



The minimal model simply requires that light-neutrinos are exchanged amongst the W bosons. Note the process is lepton-number violating and depends on the masses of the neutrinos.

The masses enter through: $\langle m_{\beta\beta}\rangle \equiv \left|\sum_k m_k U^2_{ek}\right|$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$